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## On the reflection of solitary waves from steep slopes

*The results of experimental investigations of the interaction of surface solitary waves with steep slopes are presented. It is found that the “dispersive tail” is formed during the interaction of a wave with steep slopes. The spectrum of the “tail” (wave train) has a pronounced maximum. The dependences of the spectral maximum frequency on the water depth and the slope angle are analyzed. The features of the formation of a “tail” are discussed.*

**Keywords:** solitary waves, steep slopes, dispersive train of waves.

The emergence and transformation of tsunami waves is a major reason causing the occurrence of solitary waves. The tsunami waves may arise as a result of underwater earthquakes, eruptions of underwater volcanoes, landslides, etc. [1, 2]. Solitary waves can be also generated by a ship moving with near-critical speed in the shallow water basin [3].

The interest in studying the interaction of solitary waves with a slope is due to the fact that many of the characteristics of this process are similar to the corresponding characteristics of the interaction process of tsunami waves with slope, onshore facilities.

It is known that the vertical hydraulic constructions practically completely reflect incident waves. A significant rise of the water level before constructions greatly exceeds the initial wave height. The maximum of level rise is an important characteristic in assessing the intensity of incident waves, including solitary waves and tsunami waves [4]. Decreasing the inclination angle of a shore slope leads to the changes of the run-up value and the energy of the reflected wave. The complexity of the mathematical modeling of the solitary wave reflection is associated with a significant non-linearity of the process and the need to adequately describe the motion of the water up and down the sloping border.

This paper focuses on the study of the interaction of the surface solitary wave with steep slopes. The process of run-up and run-down of a wave is considered, and the formation of a “dispersion tail” (wave train) and its characteristics are analyzed.

**The experimental setup.** Experiments are carried out in a wave channel 16 m long, 0.3 m wide, and 0.7 m high. Depths of water are equal to 11, 14, and 17 cm.

The side walls of the channel are made of glass.

Solitary waves were generated by falling a heavy body with dimensions  $0.23 \times 0.3 \times 0.75$  m at the end of the channel and creating of a local rise of the water level. The further evolution of the perturbation leads to the formation of a solitary wave. Note that this method of generating a wave was first used by Russell (1834) [5]. In the experiments, the method was modified by using a special arrangement to cut off the “dispersive train”, which arose at the generation of a solitary wave. Models of coastal slopes with different angles of inclination in relation to the horizon were mounted at the opposite end of the channel. Logging the free surface deformation during the passage of a solitary wave was made by distributed capacity gauges. The system for the collection and the processing of experimental data allows executing the fast polling of sensors, transforming a signal into the numerical form due to an analog-digital converter and

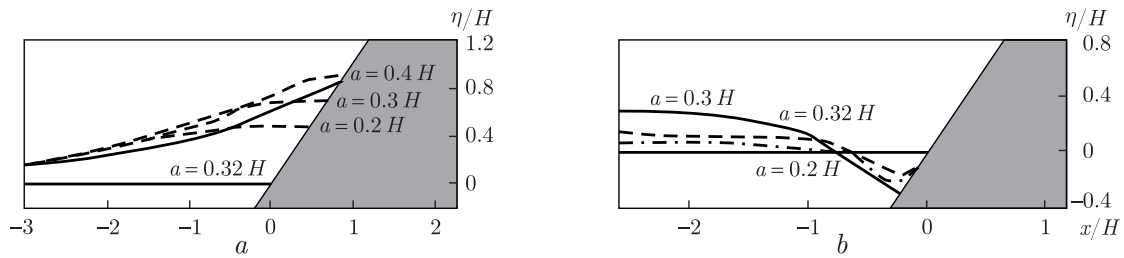


Fig. 1. Profiles of a solitary wave interacting with the slope: *a* – corresponds to run-up; *b* – corresponds to run-down processes

the rapid analysis performance of the process, using a PC. A detailed description of the setup is presented in [6].

**The results of research.** Measurements showed that the waves generated in the experiment are typical nonlinear solitary waves. The wave profile is described by the theoretical relationship, which is known as a solution of the Korteweg–de Vries equation  $\eta = a \cdot \operatorname{sech}^2[(3a/4H^3)^{1/2}x]$ . Here, *a* is the wave amplitude, *H* is the unperturbed depth of water in the channel.

The interaction of a solitary wave with the slope is characterized by three stages: run-up, run-down, and secondary run-up [7]. It should be noted that solitary waves do not break down, when they move onto the slope with an angle of inclination of more than  $10^\circ$  [8].

The results of processing the digital photographs of the processes of run-up and run-down of a solitary wave (solid curves) are represented in Fig. 1. They correspond to the following experimental parameters: the amplitude of the wave is equal to  $a = 0.32H$ , inclination angle is equal to  $45^\circ$ , and water depth is equal to 11 cm. The results of numerical calculations of the wave profiles [9] in the run-up and run-down processes are also figured (dashed curves). It can be seen that the good enough correspondence of results takes place.

The run-down process begins after the moment when the run-up maximum is attained (Fig. 1, *a*). The region with a high level of the free surface is produced before the slope during the run-down of water. It initiates the formation of a reflected solitary wave. At the same time, a part of water trapped by the slope continues to move down and causes the appearance of a depression zone (Fig. 1, *b*). Note that the aeration of a flow can occur in this zone, especially for small slope angles. The process of secondary run-up begins after the instant of time, when the maximum displacement of the water level in the depression zone is reached. Then liquid particles move up and overshoot the equilibrium level. This leads to the generation of a “dispersion train”, which moves following the reflected solitary wave. It should be noted that the appearance of similar secondary waves (with small amplitude) is observed at a frontal collision of solitary waves [10].

It was found that the patterns of run-up and run-down of a wave for other values of inclination angle of the slope differ only quantitatively, and the shapes of water surfaces are similar. The comparison of the results with available data showed a quite good correspondence.

Displacements of the free surface recorded by a gauge that located at a distance of 2m from the end wall of the channel are displayed in Fig. 2. Figure 2, *a* corresponds to the collision of the solitary wave with a vertical wall, and Fig. 2, *b* corresponds to the case where the wave interacts with the slope (inclination angle of  $45^\circ$ ). The water depth is equal to 11 cm. It can be seen that, initially, the sensor detects a deformation of the free surface caused by the incident solitary wave (curve 1, Fig. 2, *a*), and then the reflected wave passes the gauge (corresponding data

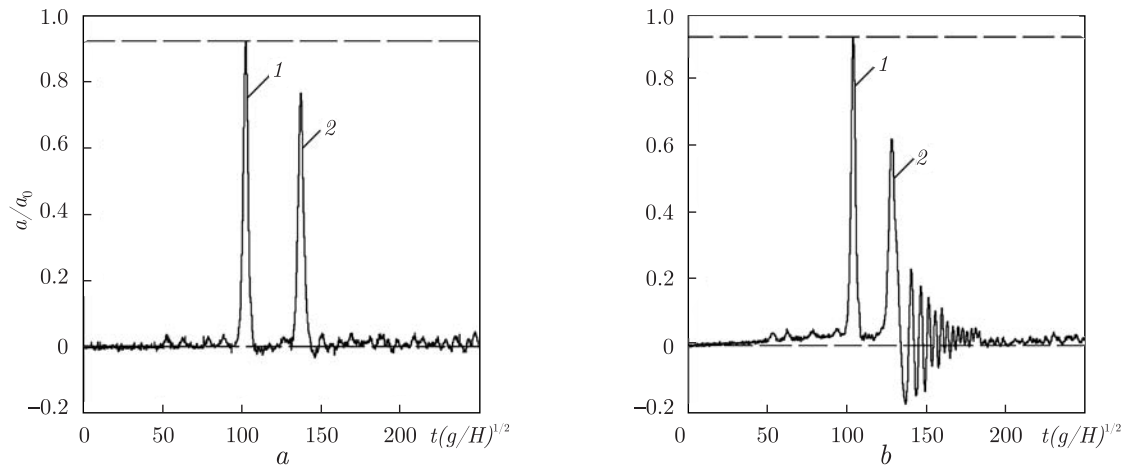


Fig. 2. Changes of the free surface at the propagation of solitary waves

are designated by curve 2). As is seen, the wave is reflected from the vertical wall without any changes, except a slight decrease in its amplitude. The difference of the amplitudes before and after the reflection is not more than 5%. It was pointed in [11] that a decrease of the amplitude after the reflection does not exceed 10% of the amplitude of the incident wave.

As is evident from Fig. 2, there is no “dispersion train” following the incident solitary wave. This is due to using the arrangement that cuts off the “tail”. The special gate is installed at the given distance downstream from a generator. The solitary wave has already formed in this region, and the dispersive train propagates after the wave. The gate is dropped at the instant, when the solitary wave passes the given point. Such procedure allows the generation of solitary waves with very weak tailing perturbations. These peculiarities of the modified method of solitary wave generation are especially important, when the processes of wave reflection from slopes are studied, in particular, for the consideration of properties of the train. It should be noted that the use of a similar arrangement shows the efficiency of the generation of an internal solitary wave by the “step-pool method” [12].

A noticeable decrease of the reflected wave amplitude takes place, when the solitary wave interacts with an inclined slope (curve 2, Fig. 2, b). Moreover, we can see the appearance of a dispersive train moving behind the reflected wave, as distinct from the case where the wave reflection occurs from the vertical wall without formation of a train (see Fig. 2, a). Note that the speed of a reflected wave is greater than the speed of a wave train. In the course of the time, the wave will detach from the train.

It was revealed that a decrease in the inclination angle of the slope results in an increase of the dispersion train amplitude. The maximum amplitude of the dispersive train is about half the amplitude of the reflected wave at an angle of  $17^\circ$ . The further decrease of the angle leads to that a reflected solitary wave is not formed, and the reflected waves have the shape of a wave train propagating upstream.

The analysis of the frequency characteristics of a “dispersion train” shows that there is a pronounced spectral peak amplitude. It is found that the spectra of dispersion trains formed at the reflection of a solitary wave from the slopes with angles of  $60^\circ$ ,  $45^\circ$ , and  $30^\circ$  are similar in form, their spectral peaks are observed at  $\nu = 1.4 \div 1.5$  Hz for the depth of water in the channel  $H = 11$  cm, and the spectra expand slightly with decreasing the angle. The spectral

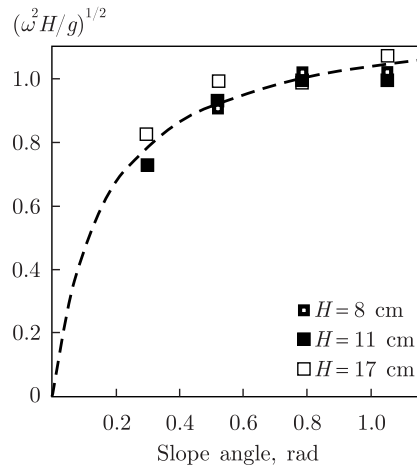


Fig. 3. Dependence of the spectrum maximum frequency on the inclination angle of the slope

maximum frequency falls with increasing the water depth. In particular, for the depth  $H = 17$  cm, it is within the interval  $\nu = 1.2 \div 1.3$  Hz. Reducing the angle down to  $17^\circ$  results in that the maximum of the spectrum is shifted down to 1.1 Hz for the depth  $H = 11$  cm and to 1.0 Hz for the depth  $H = 17$  cm.

Dependences of the frequency of the spectral maximum on the angle of a coastal slope and the water depth are represented in Fig. 3. Here,  $\omega = 2\pi\nu$ . The analysis shows that a significant change of the angle has a relatively small influence on the frequency. On the other hand, it is found that a change of the depth of water leads to a noticeable change of the frequency (dimensional).

It is shown that the modified method for the generation of solitary waves produces solitary waves described by the Korteweg–de Vries equation, with low intensity of a “dispersion tail”. Using this method allowed us to obtain new data on the characteristics of waves reflected from steep slopes. The determination of the reflected wave parameters gives a significant error in the presence of the “tail”.

It is revealed that the formation of a “tail” (dispersion train) is resulted from the trapping effect of the incident wave by the slope and the subsequent appearance of the depression zone of the free surface in front of the slope.

It is ascertained that the spectrum of the wave train has a pronounced maximum. The frequency of the maximum of the wave train spectrum weakly depends on the angle of slopes, and an increase of the water depth results in a significant decrease of the frequency.

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### **Про відбиття поодиноких хвиль від крутих схилів**

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*Наведено результати експериментальних досліджень взаємодії поверхневих поодиноких хвиль з крутими береговими схилами. Виявлено формування “дисперсійного хвоста” при взаємодії хвилі з крутими схилами. Спектр “хвоста” (хвильового цуга) має явно виражений максимум. Аналізуються залежності частоти спектрального максимуму від глибини води і кута схилу. Обговорюються особливості процесу формування “хвоста”.*

**Ключові слова:** поодинокі хвилі, круті схили, дисперсійний цуг хвиль.

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### **Об отражении уединенных волн от крутых склонов**

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*Приведены результаты экспериментальных исследований взаимодействия поверхностных уединенных волн с крутыми береговыми склонами. Обнаружено возникновение “дисперсионного хвоста” при взаимодействии волн с крутыми склонами. Спектр “хвоста” (волнового цуга) имеет явно выраженный максимум. Анализируются зависимости частоты спектрального максимума от глубины воды и угла склона. Обсуждаются особенности процесса формирования “хвоста”.*

**Ключевые слова:** уединенные волны, крутые склоны, дисперсионный цуг волн.