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Statistical approximation of multicriteria problems of stochastic programming

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The article validates an approximation technique for solving multiobjective stochastic optimization problems. As a generalized model of a stochastic system to be optimized, a vector "input–random output" system is considered. Random outputs are converted into a vector of deterministic performance/risk indicators. The problem is to find those inputs that correspond to Pareto-optimal values of output indicators. The problem is approximated by a sequence of deterministic multicriteria optimization problems, where, for example, the objective vector function is a sample average approximation of the original one, and the feasible set is a discrete sample approximation of the feasible inputs. Approximate optimal solutions are defined as weakly Pareto efficient ones within some vector tolerance. Convergence analysis includes establishing the convergence of the general approximation scheme and establishing the conditions of convergence with probability one under proper regulation of sampling parameters.

Contemporary approach to the optimal decision making is based on the modeling and the optimization of systems. Any complex system can be described by an "input–output" model $y = g(x, \omega)$, where x denotes the input parameter vector from some feasible set X, ω is a vector of uncertain parameters from a set Ω, y is an output vector from a set Y, and g is some mapping of $\mathrm{X} \times \Omega$ into Y. The model g may be given by mathematical relations, a simulation computer program or as an output result of an optimization or other solver. The vector ω of uncertain parameters can be either deterministic or random, with distribution P. In the first case, the optimization problem reads: $\min_{\omega\in\Omega} f(x, g(x, \omega)) \to \max_{x\in X}$, which corresponds to the so-called minimax decision-making approach. The second case is related to the stochastic programming [1, 2], where the "input-output" pairs are evaluated by means of some utility functional $f(x, y, \omega)$, and a corresponding optimization problem is formulated as $F(x) = \mathrm{E}f(x, g(x, \omega)) \to \max_{x\in X}$, where E denotes the expectation over the distribution P. Note that the above stochastic programming problem already contains a vector criterion $\vec{f}(x) = \{f(x, g(x, \omega)), \omega \in \Omega\}$, with a large number of components, which are combined in many cases into one or more scalar indicators. The most commonly used indicator is the average value $F(x) = \mathrm{E}f(x, g(x, \omega))$, along with the variance functions, probability, quantile (VaR), and other risk indicators [3, 4]. Optimization of such indicators requires substantial computational resources, and, in particular, the usage of parallel computing.

However, the efficient and unambiguous choice of the utility function f is not always possible. In this case, we have to deal directly with the vector model $y = g(x, \omega)$, which maps an input x into a random vector output $g(x, \omega)$. To make rational decisions, we have to define a preference relation on the set of random vectors $g(x, \omega)$. This can be done in various ways [3, 4], e. g., we can calculate an average output $\mathrm{E}g(x, \omega)$ and then to use a natural deterministic preference relation on the space Y.

Unlike standard one-criterion stochastic programming problems [1, 2], the problem of output vector optimization can contain nonconvex, nonsmooth, and discontinuous functions. So, the

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traditional stochastic programming methods like the gradient-type procedures [1] might not be applicable. In this case, the random search methods, for example, evolutionary or hybrid algorithms should be applied [5]. In case of a small dimension n of the set $X \subset \mathbb{R}^n$, a simple multiobjective random search (MRS) method can appear to be competitive. First, this method randomly generates a cloud of points in the feasible region, and then nondominated points are selected. Next, chiefly in a vicinity of the chosen nondominated points, new random points are generated, and again nondominated points are selected, and so on. Efficiency of the search is boosted due to the fact that the new points are generated in perspective areas. Remark that the MRS method naturally admits the parallel and interactive implementation, which is essential for stochastic multiobjective problems. In case of stochastic multicriteria optimization, the additional difficulty consists in proper numerical evaluation of the vector objective function that contains multidimensional integrals (expectations). In the present paper, we assume that the objective functions are estimated by statistical sampling and focus on the method convergence analysis. The considered MRS method belongs to nonscalarizing approaches [4] and, in spite of the used sample average approximations, is substantially different from the scalarization method of [6]. The results of numerical experiments and applications of the MRS method to insurance optimization problems are reported in [7, 8].

2. $\vec{\epsilon}$-dominance and $\vec{\epsilon}$-efficiency. The following concept appears to be useful for the control over the accuracy, strength, and directing preferences in $\mathbb{R}^m$.

Definition 1. ($\vec{\epsilon}$-dominance and $\vec{\epsilon}$-efficiency/optimality). The vector $\vec{z}_1 \in \mathbb{R}^m$ $\vec{\epsilon}$ dominates a vector $\vec{z}_2 \in \mathbb{R}^m$, if $\vec{z}_1 > \vec{z}_2 + \vec{\epsilon}$ (componentwise), where $\vec{\epsilon} \in \mathbb{R}^m$. The subset $Z^*(\vec{\epsilon})$ of the set $Z \subset \mathbb{R}^m$ is called $\vec{\epsilon}$-efficient/optimal if, for any $\vec{z} \in Z^*(\vec{\epsilon})$, there is no $\vec{z}^* \in Z$, $\vec{z}^* \neq \vec{z}$, such that $\vec{z}^* > \vec{z} + \vec{\epsilon}$.

The concept of $\vec{\epsilon}$-efficiency was introduced in [9]. In case of $\vec{\epsilon} > 0$, it generalizes the standard notion of the ϵ-optimality of a scalar optimization. It also includes the notion of weak Pareto optimality, which corresponds to $\vec{\epsilon} = 0$. Further various generalizations of the $\vec{\epsilon}$-efficiency are discussed in [10]. By adding $\vec{\epsilon}$ to a vector $\vec{z}$, the importance of components of $\vec{z}$ can be controlled (e. g., the importance of criteria in vector optimization). Namely, an increase of the ϵ_i component decreases the importance of the z_i component. Moreover, in contrast to [9, 10], we allow $\vec{\epsilon} \notin \mathbb{R}^m_+$. If $\vec{\epsilon}$ contains negative components, then the $\vec{\epsilon}$-dominance of $\vec{z}_1$ over $\vec{z}_2$ admits that some components of $\vec{z}_1$ can be somewhat smaller than the corresponding components of $\vec{z}_2$. Remark that if the point $\vec{z}^* \in Z$ is $\vec{\epsilon}_k$-efficient for some sequence $\{\mathbb{R}^m \ni \vec{\epsilon}_k \to 0, k = 1, 2, \ldots\}$, then $\vec{z}^*$ is called a generalized efficient point (see [11, Definition 5.53]).

Let us recall some notations and definitions [12, Section 4 A], which concern the convergence of a sequence of sets $\{Z_i \subset \mathbb{R}^m, i = 1, 2, \ldots\}$:

$\limsup_i Z_i = \{z \colon \exists\, z_{i_k} \in Z_{i_k}, z = \lim_k z_{i_k}\}$,

$\liminf_i Z_i = \{z \colon \exists\, z_i \in Z_i, z = \lim_i z_i\}$,

$\lim_i Z_i = \liminf_i Z_i = \limsup_i Z_i$.

Lemma 1. (Properties of the $\vec{\epsilon}$-optimal mappings). *Let the sequence of sets $\{Z_i \in \mathbb{R}^m\}$ converge to a compact set $\{Z \subset \mathbb{R}^m\}$, $\lim_i Z_i = Z$. Denote, by $Z_i^*(\vec{\epsilon})$ and $Z^*(\vec{\epsilon})$, the subsets of $\vec{\epsilon}$-nondominated points in Z_i and Z, respectively. Let $\lim_i \vec{\epsilon}_i = \vec{\epsilon}$. Then, for any $\vec{\epsilon}' \leqslant \vec{\epsilon}$, $\vec{\epsilon}' \neq \vec{\epsilon}$, the relations*

$$Z^*(\vec{\epsilon}') \subseteq \liminf_i Z_i^*(\vec{\epsilon}_i) \subseteq \limsup_i Z_i^*(\vec{\epsilon}_i) \subseteq Z^*(\vec{\epsilon})$$

hold true, where the last inclusion, in particular, indicates that the mapping $\vec{\epsilon} \to Z^*(\vec{\epsilon})$ *is upper semicontinuous. Moreover, in case of a convex set* Z*, we have* $\liminf_i Z_i^*(\vec{\epsilon}_i) = \limsup_i Z_i^*(\vec{\epsilon}) = Z^*(\vec{\epsilon})$.

3. Approximation of multicriteria optimization problems. Consider a general multicriteria optimization problem,

$$\vec{F}(x) = \{f_1(x), \dots, f_m(x)\} \to \max_{x \in X \subset \mathbb{R}^n}, \tag{1}$$

where the functions $f_i(x)$, $i = 1, \dots, m$, are assumed to be (semi)continuous on a compact set $X \subset \mathbb{R}^n$, and the preference relation in the criteria space $\mathbb{R}^m$ is set out by the nonnegative cone $\mathbb{R}^m_{++} = \{x \in \mathbb{R}^m : x_i > 0, i = 1, \dots, m\}$. The problem is to find a weak Pareto-optimal set X^* and a subset $X^*(\vec{\epsilon})$ of $\vec{\epsilon}$-efficient points of the set $\vec{\epsilon} \in \mathbb{R}^m$.

It is easy to see that the mapping $\vec{\epsilon} \to X^*(\vec{\epsilon})$ is upper semicontinuous for the upper semicontinuous vector function $\vec{F}(\cdot)$, i. e., $\limsup_i X^*(\vec{\epsilon}_i) \subseteq X^*(\vec{\epsilon})$ for any sequence $\vec{\epsilon}_i \to \vec{\epsilon}$.

Let us consider also the approximations for problem (1):

$$\vec{F^i}(x) = \{f_1^i(x), \dots, f_m^i(x)\} \to \max_{x \in X_i \subset \mathbb{R}^n}, \qquad i = 1, 2, \dots, \tag{2}$$

where the sequence of sets $\{X_i\}$ converges to the set X, $\lim_i X_i = X$, and the sequence of vector functions $\{\vec{F^i}(x), x \in X_i\}$ converges to the vector function $\vec{F}(x)$, $x \in X$ (in the sense of Definition 2 or 3). Denote, by $X_i^*(\vec{\epsilon},)$ the set of $\vec{\epsilon}$-nondominated points of problem (2), i. e., $X_i^*(\vec{\epsilon})$ is a $\vec{\epsilon}$-efficient subset of X_i.

Concerning $\vec{F}$ and $\{\vec{F^i}\}$, we assume the certain continuity and convergence properties outlined in the following definitions.

Definition 2. (Continuous convergence of a sequence of vector functions). A sequence of vector functions $\{\vec{F^i}(x), x \in X_i\}$ is called continuously convergent to a vector function $\vec{F}(x)$, $x \in X$, if a) $\lim_i X_i = X$, b) for any sequence $X_i \ni x_i \to x$, $\lim_i \vec{F^i}(x_i) = \vec{F}(x)$ holds (componentwise).

Definition 3. (Graphical convergence from below of a sequence of vector functions). A sequence of vector-functions $\{\vec{F^i}(x), x \in X_i\}$ is called graphically convergent from below to a vector function $\vec{F}(x)$, $x \in X$, if a) $\lim_i X_i = X$, b) for each sequence $X_i \ni x_i \to x$, $\limsup_i \vec{F^i}(x_i) \leqslant \vec{F}(x)$ holds (componentwise), and c) for any point $x \in X$, there is a sequence $X_i \ni x_i \to x$ such that $\lim_i \vec{F^i}(x_i) = \vec{F}(x)$ (componentwise).

The concepts of continuous and graphical convergence of multivalued mappings and functions (in the latter case, epi- and hypo-convergence) were comprehensively studied in [12, Sections 6E, 6G, 7B]. Definitions 2 and 3 differ from the corresponding notions in [12] by that the domains X_i and X of functions $\vec{F^i}$ and $\vec{F}$ in Definitions 2 and 3 are explicitly outlined, but the functions in [12, Definition 5.41] are considered on a common domain X or $\mathbb{R}^n$. In addition, Definition 3 extends the definition of the graphical convergence of scalar functions [12, 7(3), 7(9), Definition 7.1] to vector functions.

Examples of the sequences of vector functions that are graphically convergent from below.

E1. Obviously, if a sequence $\{\vec{F^i}(x), x \in X_i\}$ converges continuously to $\{\vec{F}(x), x \in X\}$, i. e., $\lim_i X_i = X$ and $\lim_i \vec{F^i}(x_i) = \vec{F}(x)$ for any sequence $X_i \ni x_i \to x$, then $\{\vec{F^i}(\cdot)\}$ converges to $\vec{F}(\cdot)$ graphically.

E2. Obviously, if all scalar components, except the first one, of $\{\vec{F}^i(\cdot)\}$ converge continuously to the corresponding scalar components of $\{\vec{F}(\cdot)\}$, and if the first component of $\{\vec{F}^i(\cdot)\}$ hypo-converges to the first component of $\vec{F}(\cdot)$, then $\{\vec{F}^i(\cdot)\}$ graphically converges from below to $\vec{F}(\cdot)$.

E3. If $\vec{F}^i(x) = \vec{F}(x, y_i)$, $x \in X$, $Y \ni y_i \to y$, where $\vec{F}(x, y)$ is componentwise upper semi-continuous on $X \times Y$ and is continuous at $y \in Y$ for any $x \in X$, then $\{\vec{F}^i(\cdot, y_i)\}$ graphically converges from below to $\vec{F}(\cdot, y)$. In particular, this case includes a stationary sequence of upper semicontinuous vector functions $\vec{F}^i(x) = \vec{F}(x)$, $x \in X_i = X$.

E4. Example of the construction of a continuously convergent sequence of functions. Let a function $\vec{F}(x)$, $x \in X$ be continuous on a closed set X, and let a sequence of functions $\{\vec{F}^i(x), x \in X_i \subseteq X\}$ be such that $\Delta_i := \sup_{x \in X_i} \|\vec{F}^i(x) - \vec{F}(x)\| \to 0$ with $i \to \infty$. Then, for any sequence $(X_i \ni) x_i \to x$, we obtain

$$\|\vec{F}^i(x_i) - \vec{F}(x)\| \leqslant \|\vec{F}^i(x_i) - \vec{F}(x_i)\| + \|\vec{F}(x_i) - \vec{F}(x)\| \leqslant \Delta_i + \|\vec{F}(x_i) - \vec{F}(x)\| \to 0.$$

E5. If the objective vector function of problem (1) has the form of an expectation, $\vec{F}(x) = \mathrm{E}\vec{F}(x, \omega)$, $x \in X$, then the sample average approximations $\vec{F}^i(x) = (1/M_i) \sum_{k=1}^{M_i} \vec{F}(x, \omega_k)$ can be used instead of $\vec{F}(x)$, where $\{\omega_k, k = 1, 2, \ldots\}$ are i. i. d. observations of the random parameter ω. Terms of the uniform and, therefore, continuous convergence of the empirical estimates $\vec{F}^i(x)$ to $\vec{F}(x)$ on the set X can be found in [2, Section 7.2.5]. Next, we present the sufficient conditions of continuous convergence of discretely defined empirical functions $\vec{F}^i$ to a continuous expectation function $\vec{F}$. Assume that the functions $\vec{F}(x, \omega)$ are uniformly bounded on X, $\|\vec{F}(x, \omega)\| \leqslant M$, and, at each point $x \in X_i$ of a discrete set $X_i \subset X$ (with the number of elements N_i), an empirical estimate $\vec{F}^i(x) = (1/M_i) \sum_{k=1}^{M_i} \vec{F}(x, \omega_k)$ such that

$$\Pr\{\|\vec{F}^i(x) - \vec{F}(x)\| > \delta\} \leqslant C \exp\left\{-\frac{2M_i\delta^2}{M^2}\right\} \qquad \forall\, \delta > 0$$

is independently constructed. Such estimates follow, e. g., from the Hoeffding inequality [2, Section 7.2.8] with $C = 2m$. Then $\Delta_i = \max_{x \in X_i} \|\vec{F}^i(x) - \vec{F}(x)\| \to 0$ with probability one, if, for example, $M_i \geqslant \alpha N_i$, $\alpha > 0$, and the numerical sequence $\{N_i\}$ strictly monotonically increases to infinity. Indeed, the assertion follows from the fact that, for any $\delta > 0$, the relations

$$\sum_{i=1}^{\infty} \Pr\{\Delta_i > \delta\} \leqslant \sum_{i=1}^{\infty} CN_i \exp\left\{-2\frac{M_i\delta^2}{M^2}\right\} \leqslant \sum_{i=1}^{\infty} CN_i \exp\left\{-2\frac{N_i\alpha\delta^2}{M^2}\right\} < +\infty$$

hold true.

Theorem 1 (Convergence of solutions of the approximate problems (2)). *Let a sequence of sets $\{X_i\}$ converge to a compact set X, $\lim_i X_i = X$, and let a sequence of functions $\{\vec{F}^i(x), x \in X_i\}$ graphically converge from below to a vector function $\vec{F}(x)$, $x \in X$. Let $\lim_i \vec{\epsilon}_i = \vec{\epsilon}$. Then, for each $\vec{\epsilon}' < \vec{\epsilon}$,*

$$X^*(\vec{\epsilon}') \subseteq \liminf_i X_i^*(\vec{\epsilon}_i) \subseteq \limsup_i X_i^*(\vec{\epsilon}_i) \subseteq X^*(\vec{\epsilon}).$$

4. Multicriteria random search (MRS) algorithm and its convergence. The next multicriteria random search algorithm uses the random discrete approximations $X_i = \bigcup_{k=1}^{i} \widetilde{X}_k$ of a feasible set X and estimates $\vec{F}^i(x)$, $x \in X_i$, of the objective function of (1). The algorithm generates a random sequence of approximate solutions X_i^*, $i = 1, 2, \ldots$, of task (1) as follows.

At the first iteration, the first generation of N_1 points $\widetilde{X}_1$ is randomly generated in the set X, the estimates $\vec{F}^1(x)$ of the objective function $\vec{F}(x)$ are built for all points $x \in \widetilde{X}_1$, and, in the set $\{\vec{F}^1(x), x \in \widetilde{X}_1\}$, a subset $\{\vec{F}^1(x), x \in X_1^*(\vec{\epsilon})\}$ of all $\vec{\epsilon}_1$-nondominated points is chosen.

Suppose that, at iteration i, we have already built the set X_i^*. Then (preferably in a vicinity of the set X_i^*) a new generation of N_i random points $\widetilde{X}_i$ is generated, the estimates $\vec{F}^i(x)$ of the objective function $\vec{F}(x)$ for all $x \in X_i = \cup_{k=1}^{i} \widetilde{X}_k$ are built, and, from the set $\{\vec{F}^i(x), x \in X_i\}$, the subset $\{\vec{F}^i(x), x \in X_i^*\}$ of $\vec{\epsilon}_i$-nondominated points is chosen. Then we proceed to iteration $i+1$. The process continues indefinitely long or ends at reaching the limit of iterations.

Below, we formulate the conditions of convergence of the MRS algorithm. The next statement follows from the Borel–Cantelli lemma.

Lemma 2. *Let a sequence of random sets* $\{\widetilde{X}_i, i = 1, 2, \ldots\}$ *be such that* $\widetilde{X}_i \subseteq X$ *with probability one. Let, with nonzero probability* $p_i(x, \delta) > 0$, *the set* $\widetilde{X}_i$ *intersect with any* δ*-vicinity of any point* $x \in X$, *and let* $\sum_i p_i(x, \delta) = +\infty$ *hold. Then, with probability one,* $\limsup_i \widetilde{X}_i = X$, *and, thus,* $\lim_i \bigcup_{k=1}^{i} \widetilde{X}_k = X$.

Lemma 3. *Let* $\widetilde{X}_i \subseteq X$, $\limsup_i \widetilde{X}_i = X$, $\lim_i \vec{\epsilon}_i = \vec{\epsilon}$, *and let* $\vec{F}(x)$ *be continuous on* X *and* $\Delta_i = \sup_{x \in X_i} \|\vec{F}^i(x) - \vec{F}(x)\| \to 0$. *Then, for each* $\vec{\epsilon}' < \vec{\epsilon}$, *we have*

$$X^*(\vec{\epsilon}') \subseteq \liminf_i X^*{}_i(\vec{\epsilon}) \subseteq \limsup_i \widehat{X}_i(\vec{\epsilon}) \subseteq X^*(\vec{\epsilon}).$$

The above lemma actually covers the case of the sample average approximation of a vector objective function outlined in Example **E5**.

As a consequence of Lemmas 2 and 3, we obtain, by Theorem 1, the following result on the convergence of a multicriteria random search algorithm to the $\vec{\epsilon}$-nondominated set of problem (1).

Theorem 2 (convergence of the MRS algorithm). *Let the vector function* $\vec{F}(x)$ *be continuous on a compact set* $X \subset \mathbb{R}^m$, *let the random sets* $\widetilde{X}_i$ *with positive probability* $p_i(x, \delta) > 0$ *intersect with any* δ*-vicinity of each point* $x \in X$, *and let* $\sum_i p_i(x, \delta) = +\infty$. *Denote* $X_i = \bigcup_{k=1}^{i} \widetilde{X}_k$. *Let* $\{\vec{F}^i(x), x \in X_i \subset X\}$ *be a sequence of random vector functions such that, with probability one,* $\Delta_i = \sup_{x \in X_i} \|\vec{F}^i(x) - \vec{F}(x)\| \to 0$. *Then, with probability one, a) all cluster points of* $\{X_i^*(\vec{\epsilon}_i)\}$ *belong to* $X^*(\vec{\epsilon})$ *and b) for each point* $x^* \in X^*(\vec{\epsilon}')$, $\vec{\epsilon}' < \epsilon$, *there is a sequence of points* $\{x_i \in X_i^*(\vec{\epsilon}_i)\}$ *convergent to* x^*.

In particular, if $\vec{\epsilon} > 0$, then, for each weakly Pareto optimal point $x^* \in X^* \subseteq X$, there is a sequence of points $\{x_i \in X_i(\epsilon)\}$ convergent to x^*. By virtue of the upper semicontinuity of the mapping $X^*(\vec{\epsilon})$ at $\vec{\epsilon} = \vec{0}$ for a sufficiently small vector $\vec{\epsilon}$, the set $X^*(\vec{\epsilon})$ will appear in an arbitrarily small neighborhood of the weakly Pareto optimal points $X^* = X^*(\vec{0})$.

5. Conclusions. The article describes an approximation technology for the multicriteria stochastic optimization of "input–random output" systems. In practice, these models are highly nonlinear and nonconvex. Their functioning can generally be evaluated by deterministic vector indicators such as the means, quantiles, probabilities of reaching/exiting specified areas, etc. Collapsing the vector indicator into a scalar one in view of its optimization is not always possible. So, the task is to find such inputs that correspond to the Pareto-optimal performance vector indicators. This paper validates a methodology of solving such problem by a random search with the selection of Pareto-optimal points.

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Статистична апроксимація багатокритеріальних задач стохастичного програмування

Обгрунтовано апроксимаційний підхід до розв'язання задач багатокритеріальної стохастичної оптимізації. В якості узагальненої моделі стохастичної системи, що оптимізується, використовується векторна модель типу "вхід–випадковий вихід". Випадкові виходи перетворюються в вектор детермінованих показників ефективності і ризику. Проблема полягає в тому, щоб знайти ті входи, які відповідають Парето-оптимальним значенням вихідних показників. Ця задача наближається послідовністю задач детермінованої багатокритеріальної оптимізації, де, наприклад, цільова вектор-функція є вибірковим середнім наближенням вихідної функції, а допустима множина є дискретним наближенням можливих входів. Наближені оптимальні розв'язки визначаються як слабо ефективні (з деякою точністю) за Парето. Аналіз збіжності включає в себе обгрунтування збіжності загальної апроксимаційної схеми і встановлення умов збіжності з ймовірністю одиниця при адекватному регулюванні параметрів вибірки.

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Статистическая аппроксимация многокритериальных задач стохастического программирования

Обосновывается аппроксимационный подход к решению задач многокритериальной стохастической оптимизации. В качестве обобщенной модели оптимизируемой стохастической системы используется векторная модель типа "вход–случайный выход". Случайные выходы преобразуются в вектор детерминированных показателей эффективности и риска. Проблема состоит в том, чтобы найти те входы, которые соответствуют Парето-оптимальным значениям выходных показателей. Эта задача приближается последовательностью задач детерминированной многокритериальной оптимизации, где, например, целевая вектор-функция является выборочным средним приближением исходной функции, а допустимое множество является дискретным приближением возможных входов. Приближенные оптимальные решения определяются как слабо эффективеные (с некоторой точностью) по Парето. Анализ сходимости включает в себя обоснование сходимости общей аппроксимационной схемы и установление условий сходимости с вероятностью единица при адекватном регулировании параметров выборки.