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A Computational Strategy for the Localization and Fracture of Laminated Composites. Part 2. Life Prediction by Mesoscale Modeling for Composite Structures

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Вычислительная методика локализации разрушения ламинатов. Сообщение 2. Расчет долговечности композитных конструкций с помощью мезомасштабного моделирования

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Описанный в сообщении 1 одномерный подход развит на случай двухмерного ламинатного композита T300/914, подвергнутого статическому двухосному растяжению и сдвигу. Решение данной задачи осуществляется с помощью эволюционных моделей с эффектом задержки повреждений при ограниченной скорости их накопления. Размер зоны локализации повреждения в плоскости слоев ламината зависит от временной характеристики модели, связанной с задержкой повреждения и скоростью нагружения. Показано, что совместное использование мезомодели и эволюционных моделей задержки повреждений позволяет оценить размер зоны локализации повреждения и точно оценить время разрушения.

Ключевые слова: прогноз, локализация, задержка, разрушение, повреждение, мезомоделирование, композит.

Introduction. We use the mesoscale modeling developed by [1–3], which has length dimensions related to internal characteristics of composites. The model naturally introduces the thickness of the basic folds, thereby establishing a length. In the plane of each layer, the model with delay effect allows one to fix the missing dimensions. The basic idea derives from the theoretical equations of the mesoscale modeling, particularly, the properties of the evolution model with delay effect presented on the example of a beam in tension. We show that the evolution laws for damage delay allow one to define a rupture zone. This property essential to the simulation phase of fracture is highlighted in the case of a beam in tension with several defects of rigidity.

1. **New Approach for the Mesoscale Modeling of Layered Structures**. In the models introduced for the study of layered structures [4, 5], the first aspect of this approach is the use of an intermediate scale between the microscale associated with the basic constituents of the composite (fiber, matrix) and the macroscale due

to the structure. At this scale, called mesoscale, a layered structure is represented by a stack of uniform in the layer thickness and interlaminar interfaces; the layer and the interface are the two entities called meso-constituents.

1.1. ***Study of the Localization of the Mesoscale Model***. The mesoscale modeling may be regarded as a localization limiter. It is developed for layered structures and allows one to set the localization zone thickness in each ply [6], to determine the thickness of the zone where in each ply the mesoscale model is constructed by imposing the damage variable to be uniform by thickness. This property of mesoscale model has an essential role in the simulation problems of fracture [7, 8]. The localization area in the plane of the layers is determined by using the model with delay effect.

1.2. ***Contribution of the Model with Delay***. The structure is subjected to bulk forces f_d and surface F_d in part $\partial_2\Omega$ of the border. On the complementary part $\partial_1\Omega$ is imposed a displacement U_d. The equations characterizing the mechanical problem are written as

$$\mathrm{div}(\dot{\sigma}) + \dot{\vec{f}}_d = \vec{0}, \tag{1}$$

$$\dot{\vec{u}} = \dot{\vec{U}}_d \qquad \partial_1\Omega, \tag{2}$$

$$\dot{\sigma}\vec{n} = \dot{\vec{F}}_d \qquad \partial_2\Omega, \tag{3}$$

$$\dot{\sigma} = E(1-d)\dot{\varepsilon}^e - \dot{d}E\varepsilon^e, \tag{4}$$

$$\dot{d} = f(\varepsilon^e, d). \tag{5}$$

The above five equations represent, respectively, the equilibrium, the conditions and the forces imposed on the structure, the relationship between stress and strain and the model of damage evolution with delay effect. We seek a point at which there are two different solutions that satisfy the rate problem defined by the previous equations for a given state,

$$\mathrm{div}(\Delta\dot{\sigma}) = \vec{0}, \tag{6}$$

$$\Delta\dot{\vec{u}} = \vec{0} \qquad \partial_1\Omega, \tag{7}$$

$$\Delta\dot{\sigma}\vec{n} = \vec{0} \qquad \partial_2\Omega, \tag{8}$$

$$\Delta\dot{\sigma} = E(1-d)\Delta\dot{\varepsilon}^e. \tag{9}$$

$\Delta\dot{X}$ represents the difference between two solutions in rate with the appropriate equilibrium, behavior and boundary conditions.

The principle of virtual power leads to

$$\int_{\Omega} \Delta\dot{\sigma}\Delta\dot{\varepsilon} = 0, \tag{10}$$

where $\Delta\dot{\sigma}$ and $\Delta\dot{\varepsilon}$ represent the difference between two solutions in stress rate and strain rate. The difference between two solutions in stresses shows no discontinuity of the velocity of damage because the rate is a function of state

$$\Delta\dot{\sigma} = E(1-d)\Delta\dot{\varepsilon}. \tag{11}$$

Just as $\Delta\dot{\sigma} = E(1-d)\Delta\dot{\varepsilon}\Delta\dot{\varepsilon} = 0,$ with $\Delta\dot{\varepsilon} \neq 0$ for two distinct solutions exist

$$\int_{\Omega} (1-d)\Delta\dot{\varepsilon}\Delta\dot{\varepsilon} = 0, \qquad \Delta\dot{\varepsilon} \neq 0 \qquad \Rightarrow d = 1. \tag{12}$$

The limit point corresponds to the loss of uniqueness, it occurs at the breaking point $(d = 1)$.

2. **Illustration of Uniqueness in Two-Dimensional Case**. The degradation of the transverse and shear moduli is taken into account. To illustrate the uniqueness in the two-dimensional case, we assume a structure whose variables d, d', σ, and ε^e are set at time t and we seek at $t + dt$ a point from which there is a discontinuity of the solution velocity to find out if patterns of localization exist.

Model in the transverse direction to fiber

$$\dot{d} = k_2 < F_d - d >_+^{n_2} \quad \text{if} \quad d < 1 \quad \text{and} \quad \sqrt{Y_{d'}} < \sqrt{Y_{s'}} \quad \text{otherwise} \quad d = 1, \tag{13}$$

$$\text{with:} \quad F_d(\sqrt{\underline{Y}_d}) = \frac{\sqrt{\underline{Y}_d} - \sqrt{Y_0}}{\sqrt{Y_c}} < 1 \quad \text{otherwise} \quad F_d(\sqrt{\underline{Y}_d}) = 1. \tag{14}$$

Model along the shear plane

$$\dot{d}' = k_4 < F_{d'} - d' >_+^{n_4} \quad \text{if} \quad d' < 1 \quad \text{and} \quad \sqrt{Y_{d'}} < \sqrt{Y_{s'}} \quad \text{otherwise} \quad d' = 1, \tag{15}$$

$$\text{with:} \quad F_{d'}(\sqrt{\underline{Y}_d}) = \frac{\sqrt{\underline{Y}_d} - \sqrt{Y_0}}{\sqrt{Y_{c'}}} < 1 \quad \text{otherwise} \quad F_{d'}(\sqrt{\underline{Y}_d}) = 1. \tag{16}$$

For this type of loading layer, the relations between stress and strain rates are written explicitly because there is no nonlinearity in the direction of fibers in tension. The velocity relation is obtained from the strain energy.

The velocity relations between strains ans stresses can be written as

$$\dot{\sigma}_{11} = \frac{E_1^0}{1-\nu_{12}\nu_{21}(1-d')}\dot{\varepsilon}_{11}^e + \frac{\nu_{12}E_2^0(1-d')}{1-\nu_{12}\nu_{21}(1-d')}\dot{\varepsilon}_{22}^e - \frac{\nu_{12}\nu_{21}\varepsilon_{11}^e}{(1-\nu_{12}\nu_{21}(1-d'))}\dot{d}' -$$

$$-\frac{\nu_{12}E_2^0\varepsilon_{22}^e}{1-\nu_{12}\nu_{21}(1-d')}\dot{d}'-\frac{\nu_{12}E_2^0(1-d')\varepsilon_{22}^e}{(1-\nu_{12}\nu_{21}(1-d'))^2}\dot{d}', \tag{17}$$

$$\dot{\sigma}_{22}=\frac{\nu_{12}E_2^0(1-d')}{1-\nu_{12}\nu_{21}(1-d')}\dot{\varepsilon}_{11}^e+\frac{E_2^0(1-d')}{1-\nu_{12}\nu_{21}(1-d')}\dot{\varepsilon}_{22}^e-\frac{\nu_{12}E_2^0(1-d')\varepsilon_{11}^e}{(1-\nu_{12}\nu_{21}(1-d'))^2}\dot{d}'-$$

$$-\frac{\nu_{12}E_2^0\varepsilon_{11}^e}{1-\nu_{12}\nu_{21}(1-d')}\dot{d}'-\frac{\nu_{12}E_2^0(1-d')\varepsilon_{22}^e}{(1-\nu_{12}\nu_{21}(1-d'))^2}\dot{d}', \tag{18}$$

$$\dot{\sigma}_{12}=2G_{12}^0(1-d)\dot{\varepsilon}_{12}^e-2G_{12}^0\varepsilon_{12}^e\dot{d}, \tag{19}$$

$$\Delta\dot{\sigma}_{11}=\frac{E_1^0}{1-\nu_{12}\nu_{21}(1-d')}\Delta\dot{\varepsilon}_{11}^e+\frac{\nu_{12}E_2^0(1-d')}{1-\nu_{12}\nu_{21}(1-d')}\Delta\dot{\varepsilon}_{22}^e, \tag{20}$$

$$\Delta\dot{\sigma}_{22}=\frac{\nu_{12}E_2^0(1-d')}{1-\nu_{12}\nu_{21}(1-d')}\Delta\dot{\varepsilon}_{11}^e+\frac{E_2^0(1-d')}{1-\nu_{12}\nu_{21}(1-d')}\Delta\dot{\varepsilon}_{22}^e, \tag{21}$$

$$\Delta\dot{\sigma}_{12}=2G_{12}^0(1-d)\Delta\dot{\varepsilon}_{12}^e, \tag{22}$$

$$\begin{bmatrix}\Delta\dot{\sigma}_{11}\\ \Delta\dot{\sigma}_{22}\\ \Delta\dot{\sigma}_{12}\end{bmatrix}=[K]\begin{bmatrix}\Delta\dot{\varepsilon}_{11}^e\\ \Delta\dot{\varepsilon}_{22}^e\\ \Delta\dot{\varepsilon}_{12}^e\end{bmatrix}.$$

We seek a point at which there are two different solutions which satisfy the velocity problem set from a given state

$$\mathrm{div}(\Delta\dot{\sigma})=\vec{0}, \tag{23}$$

$$\Delta\dot{\vec{u}}=\vec{0} \qquad \partial_1\Omega, \tag{24}$$

$$\Delta\dot{\sigma}\vec{n}=\vec{0} \qquad \partial_2\Omega, \tag{25}$$

$$\Delta\dot{\sigma}=[K]\Delta\dot{\varepsilon}^e. \tag{26}$$

The two solutions must satisfy the same boundary conditions, the principle of virtual power then leads to the following form:

$$\int_D \Delta\dot{\varepsilon}^T\Delta\dot{\sigma}=0 \qquad \text{with} \qquad \Delta\dot{\sigma}=[K]\Delta\dot{\varepsilon}^e. \tag{27}$$

A sufficient condition for the problem to admit two distinct solutions is

$$\int_D \Delta\dot{\varepsilon}^T K \Delta\dot{\varepsilon} = 0 \qquad \text{with} \qquad \Delta\dot{\varepsilon} \neq 0, \tag{28}$$

$$\Rightarrow \det\left(\frac{K + K^T}{2}\right) = 0,$$

where K is symmetrical,

$$\Rightarrow \det(K) = 0. \tag{29}$$

This problem has a unique solution for the loss of uniqueness, which is reached for values of the following constant damage:

$$d' = 1 - \frac{E_1^0}{\nu_{12} E_2^0 \nu_{12}} < 0 \qquad \text{or} \qquad d = 1. \tag{30}$$

The first is, a priori, negative, the second coincides with the transverse fracture.

2.1. ***Development and Sensitivity Analysis of a Beam with a Single Defect***. A beam $[90]_n$ T300/914 composed of two parts (one has the Young modulus slightly lower in order to force is the localization) is subject to a state of pure tension (Fig. 1). The inequality of moduli within the two parts results in the lack of rigidity of the beam.

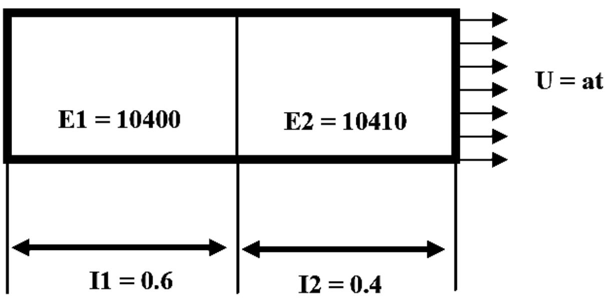

Fig. 1. Beam in tension-imposed displacement.

This example shows the influence of the defect on the rupture zone where the damage model with delay is used. The localization of strain and fracture occurs in the area where the stiffness is the lowest. The proposed criterion of instability and localization can be applied for assessment of the deformation value for which the localization occurs causing the rupture. The inelastic strains are neglected in this problem.

The equations solved using a program in C++ can be written as:
imposed displacement:

$$\sum_{i=1,2} l_i \varepsilon_i = at, \tag{31}$$

equilibrium:

$$E_1(1-d_1)\varepsilon_1 = E_2(1-d_2)\varepsilon_2, \tag{32}$$

behavior:

$$\dot{d}_i = k\left\langle F_{di}(\sqrt{\underline{Y}_d}) - d_i \right\rangle_+^n \qquad i=1,2, \tag{33}$$

with

$$F_{d_i}(\sqrt{\underline{Y}_d}) = \frac{\sqrt{\underline{Y}_d} - \sqrt{Y_0}}{\sqrt{Y_c}} < 1 \qquad \text{otherwise} \qquad F_{d_i}(\sqrt{\underline{Y}_d}) = 1, \tag{34}$$

where k and n are the material constants. In this study, the parameters of the model with delay effect are: $a = 0.01$ mm/s, $k = 100$, and $n = 0.5$.

The criterion of instability and localization is written for both parts of the beam:

$$d_{0i} = 1 + \varepsilon_{0i}^e \frac{\dfrac{\partial f}{\partial \varepsilon^e}}{\dfrac{\partial f}{\partial d}}, \qquad \text{we set,} \qquad g_i(\varepsilon_{0i}^e) = 1 + \varepsilon_{0i}^e \frac{\dfrac{\partial f}{\partial \varepsilon^e}}{\dfrac{\partial f}{\partial d}}, \tag{35}$$

$$\text{with:} \qquad f = k\left\langle F_{di}\sqrt{\underline{Y}_d} - d_i \right\rangle_+^n, \qquad i=1,2. \tag{36}$$

2.1.1. *Results and Discussion*. The localization occurs when the damage and strain of the bar satisfy this criterion. In the present case, the localization and fracture zones are located in element *1* (Fig. 2). We show in this example that the damage is localized in the weakest element of rigidity. The intersection of two curves representing the criterion of instability and damage as a function of strain is the point from which there is localization (Fig. 3). It shows the strain and the value of damage for which the localization occurs. Subsequently, the example of a beam composed of several defects highlights the existence of a fracture zone which may consist of several elements. It is impossible to obtain such a phenomenon with a classical model.

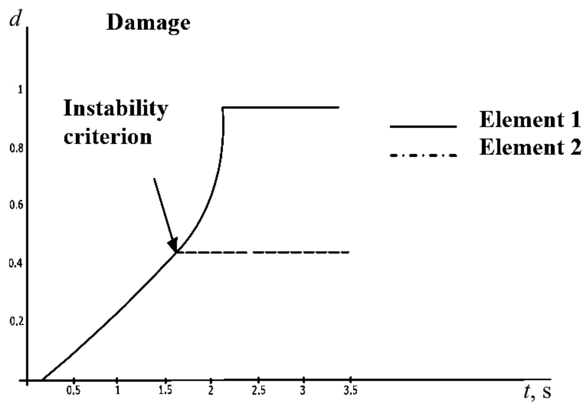

Fig. 2. Damage vs. time in the beam, localization in element *1*.

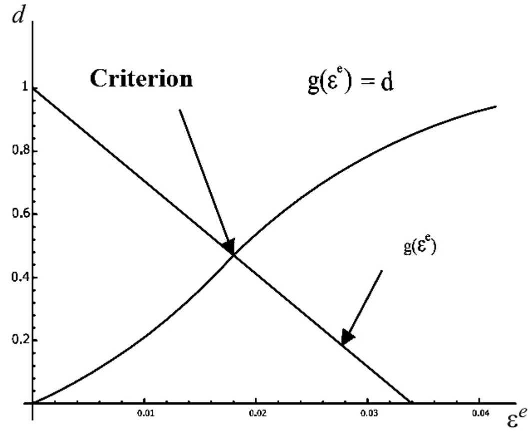

Fig. 3. Damage vs. deformation and instability criterion.

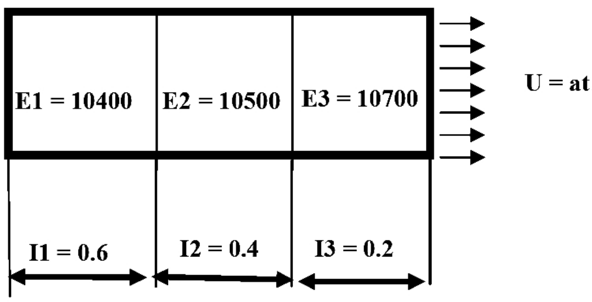

Fig. 4. Beam in tension-imposed displacement.

2.2. ***Development and Sensitivity Analysis of a Bar with Several Defects***. The case of beam $[90]_n$ with several defects is considered in this part (Fig. 4). The inelastic strains are neglected, and at given loading rate the localization occurs in the area of the defect size order. For sufficiently high loading rate the localization also occurs in the same area if it consists of several elements. In both cases, the curves representing the load according to the displacement are identical. This example makes it possible to show that, in the case of model with delay efect, area and localization of fracture are determined by the defects and the effect of damage accumulation rate model. The criterion of instability and localization confirms the existence of the fracture zone. The equations solved can be written as:
imposed displacement:

$$\sum_{i=1,2} l_i \varepsilon_i = at, \tag{37}$$

equilibrium:

$$E_i(1-d_i)\varepsilon_i = E_j(1-d_j)\varepsilon_j \quad \text{with} \quad (i=1,\ j=2) \quad \text{and} \quad (i=2,\ j=3), \tag{38}$$

behavior:

$$\dot{d}_i = k\left\langle F_{di}(\sqrt{\underline{Y}_d}) - d_i \right\rangle_+^n. \tag{39}$$

The criterion of instability and localization is written for all three beam parts:

$$d_{0i} = 1 + \varepsilon_{0i}^{e} \frac{\dfrac{\partial f}{\partial \varepsilon^{e}}}{\dfrac{\partial f}{\partial d}}, \qquad \text{with} \qquad f = k\left\langle F_{di}\sqrt{\underline{Y}_{d}} - d_{i}\right\rangle_{+}^{n}, \qquad i = 1, 2, 3, \quad (40)$$

where k and n are the materials constants. For further calculations we take $k = 1000$ and $n = 1$, the length of the bar is 1 mm.

2.2.1. *Results and Discussion*. For the loading rate ($a = 1$ mm/s) and the parameters of the chosen delay model the localization appears in part l_1 (Fig. 5). The localization criteria are satisfied in the three elements for three different strains. Consequently, the damage evolves differently in each part after the point defined by the first criterion of instability. The damage is localized in the weakest element of rigidity and attains two distinct values of damage in the other two elements.

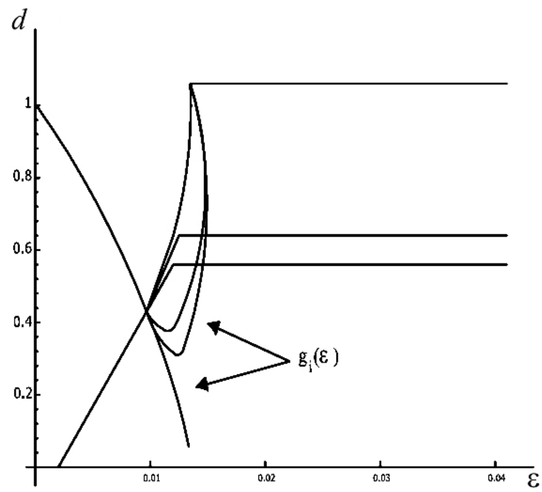

Fig. 5. Damage and criterion with vs. time, localization in the element of length l_1 ($a = 1$ mm/s).

For the loading rate ($a = 25$), the localization occurs in the first two elements (Fig. 6). The localization length is $l_1 + l_2$. The localization criteria are verified in the three elements for two distinct strains. Thus, the damage evolves identically in the two elements whose rigidities are lower. A length scale $l_1 + l_2$ is then associated with the phenomenon of fracture of the beam. For the loading rate $a = 50$ mm/s the localization occurs across the entire beam (Fig. 7). In this case, the three localization criteria determined in each part of the beam are similar and show the same value of damage at the same time.

Finally, we consider a beam similar to the previous one except that its weakest part is cut into three $l_1' = 0.2$, $l_2'' = 0.2$, and $l_3''' = 0.2$, with the identical values of the Young modulus (Fig. 8). This shows the independence of the size of the fracture with respect to the beam discretization. This example shows that for a small perturbation of the Young modulus in the entire part I, the localization occurs throughout the zone length l_1 for the rate close to the previous one ($a = 1$ mm/s). For lower loading rate localization occurs again in the weakest link l_1 ($a = 0.1$ mm/s).

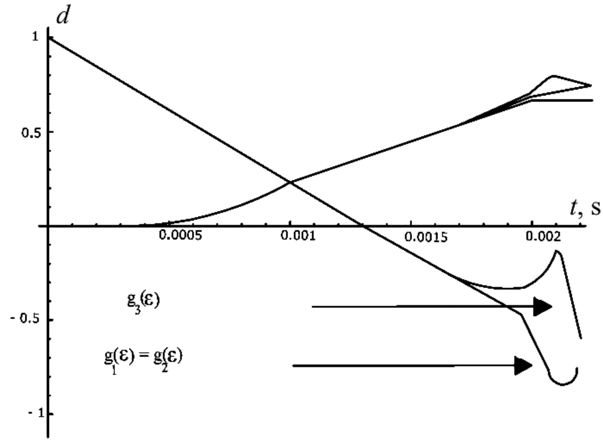

Fig. 6. Damage and criterion vs. time, localization in the elements of length l_1 and l_2 ($a = 25$ mm/s).

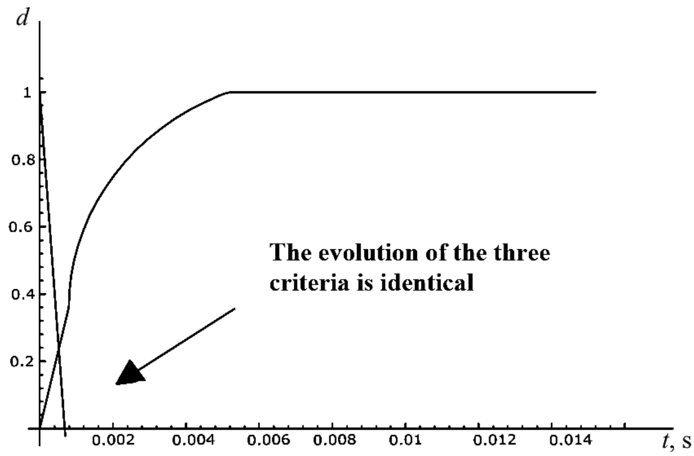

Fig. 7. Damage for all elements and criteria vs. time ($a = 50$ mm/s).

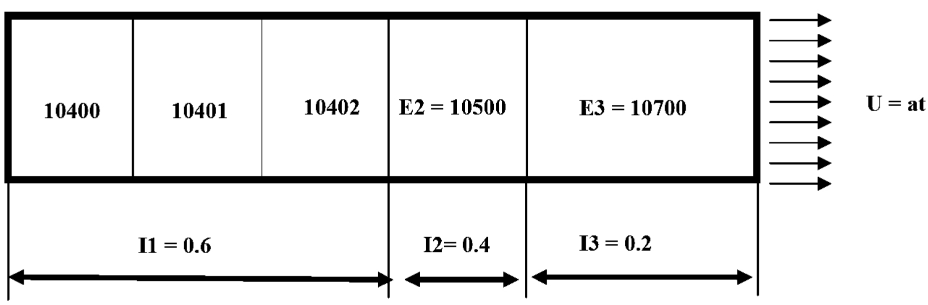

Fig. 8. Beam in tension-imposed displacement.

Starting from a certain value of loading rate of ($a = 1.5$ mm/s), the load–displacement curves obtained with discretization (three fields for l_1) are identical to those obtained with the previous discretization (one domain for l_1). This illustrates that for given loading rate the zone where fracture occurs is independent of the beam discretization (Fig. 9).

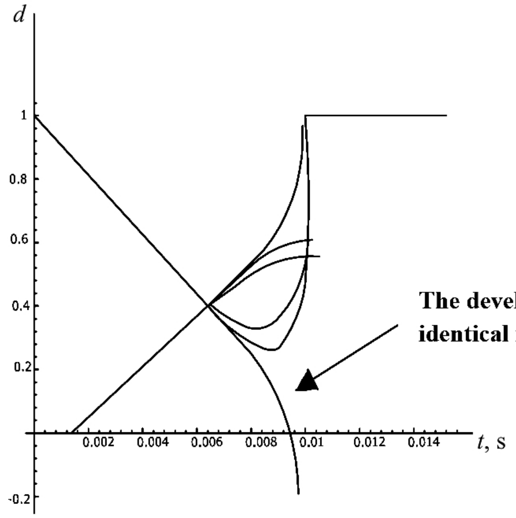

Fig. 9. Damage throughout the first three elements and criteria vs. time ($a = 1.5$ mm/s).

Conclusions. The analysis shows a dependence of the size of the localization area with respect to the loading rate. The model with delay effect envisages formation of a fracture zone, which makes results independent of the discretization considered. For the usual loading rate, the size of the fracture zone is small. To emphasize the existence of the fracture zone in several elements, the mesh must be sufficiently fine. During the phase of localization and fracture, the velocity variations are significant and the inclusion of inertia terms in the equations of equilibrium reduces the influence of loading rate on the fracture zone.

This study has shown the contribution of regulating the messcale model coupled to the model with delay effect. The model with delay effect has a characteristic time which, combined with the loading rate, implies formation of fracture zone in each of the layers constituting the laminated structure. The thickness of the fracture zone in each layer is determined by the thickness of the fold. The calculation of structures using mesoscale modeling are therefore unbiased and unique, and combined with the delay model application can provide assessment up to fracture. The prospects of this study is to identify the size of the fracture zone and take into account the inertia terms in the fracture phase, in order to better represent the physical phenomena.

Резюме

Описаний в повідомленні 1 одновимірний підхід розвинуто на випадок двовимірного ламінатного композита Т300/914, що зазнає статичного двовісного розтягання і зсуву. Розв’язок даної задачі виконується за допомогою еволюційних моделей з ефектом затримки пошкоджень за обмеженої швидкості їх накопичення. Розмір зони локалізації пошкодження у площині шарів ламіната залежить від часової характеристики моделі, яка пов’язана із затримкою пошкодження і швидкістю навантаження. Показано, що спільне використання мезомоделі й еволюційних моделей затримки пошкоджень дозволяє оцінити розмір зони локалізації пошкоджень і точно оцінити час руйнування.

1. L. Gornet, D. Lévêque, et L. Perret, "Modélisation, identification et simulation éléments finis des phénomènes de délaminage dans les structures composites stratifiées," *Mécanique & Industries*, **1**, No. 3, 267–276 (2000).
2. P. Ladevèze, O. Allix, J.-F. Deü, and D. Lévêque, "A mesomodel for localisation and damage computation in laminates," *Comput. Meth. Appl. Mech. Eng.*, **183**, 105–122 (2000).
3. P. Ladevèze and G. Lubineau, "An enhanced mesomodel for laminates based on micromechanics," *Compos. Sci. Tech.*, **62**, 533–541 (2002).
4. A. Boutaous, B. Peseux, L. Gornet, and A. Bélaidi, "A new modeling of plasticity coupled with the damage and identification for carbon fibre composite laminates," *Compos. Struct.*, **74**, 1–9 (2006).
5. P. Ladevèze and G. Lubineau, "On a damage mesomodel for laminates: micromechanics basis and improvement," *Mech. Mater.*, **35**, 763–775 (2003).
6. P. Ladevèze and E. Le Dantec, "A damage modelling of the elementary ply for laminated composites," *Compos. Sci. Technol.*, **43**, 257–267 (1992).
7. P. Ladevèze, "A damage computational method for composite structures," *J. Comput. Struct.*, **44**, 79–87 (1992).
8. O. Allix, L. Gornet, C. Hochard, et P. Ladevèze, "Endommagement des structures stratifiées: analyse des zones à fort gradient," 12 éme Congrès Français de Mécanique (Strasbourg, Sept. 1995), **1**, pp. 325–328.
9. A. Needleman, "Material rate dependance and mesh sensitivity in localization problems," *Comput. Meth. Appl. Mech. Eng.*, **67**, 189–214 (1988).

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