UDC 539.4

A Computational Strategy for the Localization and Fracture of Laminated Composites. Part 1. Development of a Localization Criterion Adapted to Model Damage Evolution Time-Delay

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УДК 539.4

Вычислительная методика локализации разрушения ламинатов. Сообщение 1. Разработка критерия локального повреждения для моделирования развития повреждения с учетом эффекта задержки

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Предложен критерий нестабильности и локализации повреждений в балке из однородного ламината T300/914 для моделирования развития повреждений с учетом эффекта задержки. Результаты, полученные для одномерного случая, свидетельствуют об одновременном появлении зоны разрушения по всей конструкции. Предложено решение, которое базируется на мезомоделировании композитов. Полученные с помощью предложенного подхода расчетные результаты позволяют выполнить точный прогноз потери устойчивости образца при ухудшении параметров его жесткости.

Ключевые слова: прогнозирование, локализация, эффект задержки, повреждение, мезомасштабное моделирование, численный расчет, композиты.

Introduction. From a theoretical point of view, simulation of localization and fracture of layered structures can be described in numerical specific criteria [1–3]. The study of stability and uniqueness of the solution for structures governed by the laws of elasto-plastic behavior has been studied by [4]. Following these works and those of [5], a first possibility has been to establish criteria of uniqueness and localization in order to define a field of use of behavior laws. They have been developed for the non-associated elasto-plastic laws [6], aimed to obtain a criteria of a possible onset of microcracks.

For composite laminates, the ultimate point of rupture is usually close to the point determined using the criteria of uniqueness, this criterion corresponds to the point of instability and can be the endpoint from which there is a potential bifurcation of the solution [7]. The localization criterion [8] allows one to determine, for materials governed by the time independent behavior laws, the point at which localized solutions appear. We note the absence of internal length in these models from the classical mechanics of damage and can not properly simulate the phase fracture [1].

We show in this first part, that evolution law for damage delay allows one to define a rupture zone, this property is essential to the simulation phase of fracture and is highlighted in the case of a beam in tension.

1. **Bibliographic Aspect of Localization Limiters**. Extending the calculations beyond the critical points defined by conventional criteria creates difficulties caused by the loss of uniqueness and localization of strain. To follow a solution beyond the critical points, one essential step is to introduce localization limiters. The laws of behavior called special (limiting localization) can lead the calculations until failure. With these type of laws, it is possible to obtain a fracture zone in the framework of damage mechanics.

One approach to limit the phenomenon localization is to use a non-local model [9], whether in damage, in velocity localization damage or in dissipation. Another approach to regularization is to use a mechanical formula taking into account the higher order terms in the calculation of strains. In the one-dimensional case [6], taking into account the second order term in the strain, allows one to regularize the original problem. The theory of second gradient is also used as a localization limiter [10].

The adjustment can also be obtained by using a model in which the characteristic lengths are used [11]. This method is used by authors [12] for behavior laws of elasto-plastic softening.

All these methods of adjustments compensate for the lack of models by introducing local classical computations in terms of length. These lengths must control the phenomenon of localization and are introduced directly into the constitutive law.

In this work, we used the mesomodelisation developed by [1, 13, 14], which has internal dimensions related to internal characteristics of composites. The model naturally introduces the thickness of the basic folds, thereby establishing a length, in the plane of each layer. The model with delay effect allows one to fix the missing dimensions.

1.1. ***Modeling of the Elementary Layer***. The model is based on an analysis of the microlayer, taking into account the micromechanisms of damage in structures. The ply is considered homogeneous with orthotropic elastoplastic behavior damaged. At the mesolevel different mechanisms of damage in laminated composite structures are well described by simple models of behavior [3, 15]. The mechanisms considered are: transverse microcracking of the matrix, the fiber debonding and inelastic deformation of the matrix. This modeling is adapted to the ply composed of a single-direction carbon-epoxy fiber type T300/914 ($E_1^0 = 150000$ MPa, $E_2^0 = 10800$ MPa, $G_{12} = 5800$ MPa, $\nu_{12}^0 = 0.32$, $b = 2.5$, $Y_0 = 0.0961$ MPa, $Y_c = 13.6161$ MPa, $Y_s' = 0.47$ MPa, and $\gamma = 1.5 \cdot 10^{-4}$ MPa^{-1}).

1.2. ***Strain Energy of the Elementary Layer***. The strain energy of the layer is split into a traction power and energy of compression to take account of the unilateral aspect of model behavior. The kinematics of damage uses three scalar damage variables noted d, d', and d_F, respectively, related to the collapse of shear stiffness, tensile and transverse rupture of the fibers. The damage is considered constant along the thickness of the monolayer

$$E_D^{cp} = \frac{1}{2(1-d_F)}\left[\frac{<\sigma_{11}>_+^2}{E_1^0} + \frac{\phi(<-\sigma_{11}>_+)}{E_1^0} - \left(\frac{\nu_{21}^0}{E_2^0} + \frac{\nu_{12}^0}{E_1^0}\right)\sigma_{11}\sigma_{22}\right] +$$

$$+\frac{1}{2}\left[\frac{<\sigma_{22}>_+^2}{(1-d)E_2^0} + \frac{<-\sigma_{22}>_+^2}{E_2^0} + \frac{\sigma_{12}^2}{(1-d)G_{12}^2}\right], \quad (1)$$

where E_D^{cp} is the strain energy associated with plane stress, "0" designates the initial quantities and $<X>_+$ is the positive part of X,

$$<X>_+ = X \quad \text{if} \quad X > 0,$$
$$<X>_+ = 0 \quad \text{if} \quad X < 0.$$

The behavior of fiber is linear elastic brittle in tension and nonlinear elastic in compression in the direction of the fibers [16]. A stiffness loss of 30% of the longitudinal modulus was observed near the rupture for a T300/914. The consideration of this phenomenon is determined from the function ϕ:

$$\phi(<-\sigma_{11}>_+) = \frac{2[\gamma<-\sigma_{11}>_+ + \ln(1-\gamma<-\sigma_{11}>_+)]}{\gamma^2}. \quad (2)$$

This function is chosen such that the expression of the modulus compression is a function of strain in the direction of fibers:

$$E_1^c = E_1^0(1-\gamma<-\sigma_{11}>_+), \quad (3)$$

where γ is a parameter identified as a four-points bending test.

The modeling layer allows to take into account the matrix degradation and deterioration of the fiber-matrix. A remarkable property is that the crack grows parallel to fibers [17]. This property may be taken into account at the microscale. A calculation by asymptotic homogenization shows that the only moduli that are affected by the transverse modulus E_2^0 and shear modulus G_{12}^0 [18].

The state laws are given by

$$\varepsilon_{ij}^e = \left.\frac{\partial E_D^e}{\partial \sigma_{ij}}\right|_{d,d',d_F}. \quad (4)$$

Therefore, the elastic orthotropic damageable laws can be written in the two-dimensional case as

$$\varepsilon_{11}^{e}=\frac{1}{1-dF}\left[\frac{<\sigma_{11}>_+}{E_1^0}+\frac{<-\sigma_{11}>_+}{E_1^0(1-\gamma<-\sigma_{11}>_+)}-\frac{\nu_{12}^0\sigma_{22}}{E_1^0}\right], \tag{5}$$

$$\varepsilon_{22}^{e}=\frac{<\sigma_{22}>_+}{E_2^0(1-d)}+\frac{<-\sigma_{22}>_+}{E_2^0}-\frac{\nu_{12}^0\sigma_{11}}{E_1^0(1-d_F)}, \tag{6}$$

$$\varepsilon_{12}^{e}=\frac{\sigma_{12}}{2G_{12}(1-d)}. \tag{7}$$

1.3. ***Evolution of Damage Variables***. The evolution of damage, which is governed by laws dependent on thermodynamic forces Y_d, $Y_{d'}$, and Y_F coupled respectively to internal scalar variables d, d', and d_F, has an expression:

$$Y_d=\frac{\partial}{\partial d}<<E_D^e>>|\sigma:cst=\frac{<<\sigma_{12}^2>>}{2G_{12}^0(1-d)^2}, \tag{8}$$

$$Y_{d'}=\frac{\partial}{\partial d}<<E_D^e>>|\sigma:cst=\frac{<<<\sigma_{22}>>_+^2>}{2E_2^0(1-d')^2}, \tag{9}$$

$$Y_F=\frac{\partial}{\partial d_F}<<E_D^e>>|\sigma:cst=$$

$$=\frac{1}{2(1-d_F)^2}<<\frac{<\sigma_{11}>_+^2}{2E_1^0}+\frac{<-\sigma_{11}>_+^2}{2E_1^c}-\left(\frac{\nu_{21}^0}{E_2^0}+\frac{\nu_{12}^0}{E_1^0}\right)\sigma_{11}\sigma_{22}>>. \tag{10}$$

In general, the laws of damage evolution can be written as a function of the history of loading time t: E_D^e is the energy of the monolayer and $<<X>>$ is the mean value of X along the thickness of the monolayer,

$$d|_t=A_d(Y_d|_\tau;\quad Y_{d'}|_{\tau'},\ \tau\le t);\quad d'|_t=A_{d'}(Y_d|_{\tau'};\quad Y_{d'}|_{\tau'},\ \tau\le t). \tag{11}$$

The laws of evolution A_d and $A_{d'}$ of quasi-static type, depending on the type of loading, are selected to meet the second law of thermodynamics.

2. **Damage Model with Delay Effect**. The laws of damage evolution time-delay that express the evolution of damage is not instant over loading. There exists in this type of model a characteristic time which, combined with a characteristic speed, allows one to define a characteristic length.

Value of $\dot{d}_F$, $\dot{d}$, and $\dot{d}'$ are scalar internal variables which represent the speeds of damage. These are constants in the thickness of the layer, the thermodynamic force that governs the damage evolution is

$$\sqrt{\underline{Y}_d} = \sqrt{Y_d + bY_{d'}}. \tag{12}$$

The models with delay effect used in this article are as follows:

Model in the direction of the fibers

$$\dot{d}_F = k_1 < F_F - d_F >_+^{n_1} \quad \text{if} \quad d_F < 1 \quad \text{otherwise} \quad d_F = 1 \tag{13}$$

with

$$F_F(\sqrt{Y_F}) = \frac{\sqrt{Y_F} - \sqrt{Y_{F0}}}{\sqrt{Y_{FC}}} < 1 \qquad \text{otherwise} \qquad F_F(\sqrt{Y_F}) = 1. \tag{14}$$

Model along the shear plane

$$\dot{d} = k_4 < F_d - d >_+^{n_4} \quad \text{if} \quad d < 1 \quad \text{and} \quad \sqrt{Y_{d'}} < \sqrt{Y_{s'}} \quad \text{otherwise} \quad d = 1 \tag{15}$$

with

$$F_d(\sqrt{\underline{Y}_d}) = \frac{\sqrt{\underline{Y}_d} - \sqrt{Y_0}}{\sqrt{Y_c}} < 1 \qquad \text{otherwise} \qquad F_d(\sqrt{\underline{Y}_d}) = 1. \tag{16}$$

Model in the transverse direction to fiber

$$\dot{d}' = k_2 < F_{d'} - d' >_+^{n_2} \quad \text{if} \quad d' < 1 \quad \text{and} \quad \sqrt{Y_{d'}} < \sqrt{Y_{s'}} \quad \text{otherwise} \quad d' = 1 \tag{17}$$

with

$$F_{d'}(\sqrt{\underline{Y}_d}) = \frac{\sqrt{\underline{Y}_d} - \sqrt{Y_{0'}}}{\sqrt{Y_{c'}}} < 1 \qquad \text{otherwise} \qquad F_{d'}(\sqrt{\underline{Y}_d}) = 1. \tag{18}$$

Here k_1, k_2, k_4, n_1, n_2, and n_4 are model parameters representing the material. The parameters $1/k_i$ are homogeneous in time characteristics. It should be noted that when the rate of damage is very low, we get the expressions of the change models without delay effect as statically identified.

2.1. ***Concept of the Localization Criterion for the Model with Delay Effect***. The idea of the method to develop a test of stability and localization is studied in the neighborhood of the solution X_0, the set of equations governing the mechanical problem under consideration. To study stability, we add a disturbance u to the solution X_0. Then the set of equations is linearized near the solution X_0, the system resulting from linearization can be written as

$$\frac{\partial u}{\partial t} = M(X_0)u,$$

where u represents the set of variables governing the problem considered in the neighborhood of the solution X_0, and $M(X_0)$ is a matrix depending on the solution of the undisturbed problem. The system admits a solution in the form $u(x, t) = \hat{u}(x)\exp(\lambda t)$. The criterion of stability and localization is derived from the eigenvalue problem as follows: $M(X_0)\hat{u} = \lambda \hat{u}$.

The solutions to this problem are likely to increase risk of instability from a given state, if the eigenvalues are positive $\lambda > 0$. The case of $\lambda = 0$ is the transition from stability to instability.

2.2. ***Development of Criteria Appropriate to the Model with Delay Effect***. The criterion is developed in the case of a one-dimensional elastic beam damageable-delay tensile loading subjected to forced displacement (Fig. 1). This criterion gives a necessary condition of localization of strain and damage.

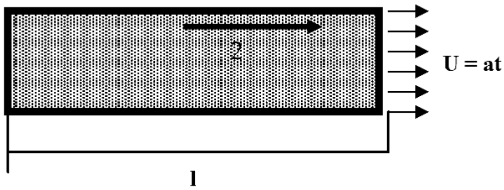

Fig. 1. Beam in tension-imposed displacement.

The equations governing the behavior of the beam in tension are given by

$$\frac{\partial \sigma}{\partial x} = 0, \tag{19}$$

$$\dot{\sigma} = E(1-d)\dot{\varepsilon}^e - \dot{d}E\varepsilon^e, \tag{20}$$

$$\dot{d} = f(\varepsilon^e, d), \tag{21}$$

where Eq. (19) is the equation that denotes the equilibrium, Eq. (20) is the relationship between velocity and strain elastic strain, and Eq. (21) represents the model with delay effect.

These equations are linearized around an equilibrium position noted "0"

$$\delta\dot{\sigma} = E(1-d_0)\delta\dot{\varepsilon}^e - E\dot{\varepsilon}_0^e\delta\varepsilon^e - E\varepsilon_0^e\delta\dot{d}, \tag{22}$$

$$\delta\dot{d} = \frac{\partial f}{\partial \varepsilon^e}\delta\varepsilon^e + \frac{\partial f}{\partial d}\delta d. \tag{23}$$

We search from an initial state noted "0" a perturbed solution of the above equations, which can be written as follows for any variable "X":

$$X(M, t) = X(M_0, t) + \delta X, \tag{24}$$

$$\delta X(M, t) = \hat{X}(M)\exp(\lambda t). \tag{25}$$

The variations are then written for all variables in this case:

$$\delta u(x, t) = \hat{u}(x)\exp(\lambda t), \tag{26}$$

$$\delta \sigma(x, t) = \hat{\sigma}(x)\exp(\lambda t), \tag{27}$$

$$\delta d(x, t) = \hat{d}(x)\exp(\lambda t). \tag{28}$$

By substituting these equations into the linearized equations of the beam problem yields when λ tends to zero:

$$\hat{\sigma} = \left\{ E(1 - d_0) + E\varepsilon_0^e \frac{\left|\dfrac{\partial f}{\partial \varepsilon^e}\right.}{\left|\dfrac{\partial f}{\partial d}\right.} \right\} \hat{\varepsilon}^e. \tag{29}$$

The case $\lambda = 0$ corresponds to the limiting case of transition to instability. This hypothesis can be reduced to the study of behavior independent of time. The instability criterion can be written as

$$d_0 = 1 + \varepsilon_0^e \frac{\left|\dfrac{\partial f}{\partial \varepsilon^e}\right.}{\left|\dfrac{\partial f}{\partial d}\right.}. \tag{30}$$

The following three equations determine the plot of effort vs. displacement and the point defined by the instability criterion

$$\sigma = E_2(1 - d)\varepsilon^e, \tag{31}$$

$$\dot{d} = k_2 \left\langle \sqrt{bE_2/2Y_c}\ \varepsilon^e - \sqrt{Y_0}/\sqrt{Y_c} - d \right\rangle_+^{n_2}, \tag{32}$$

$$d = 1 - \sqrt{\frac{bE_2}{2Y_c}}\,\varepsilon^e. \tag{33}$$

3. **Results and Discussion**. In Fig. 2, curves *1*, *2*, and *3* represent the stress in terms of strain respectively for $n_2 = 2$, 1, and 0.5. The criterion of instability (33) is the peak of each curve representing the stress as a function of strain. We then plot d vs. deformation ε^e. Value of d has been multiplied by a factor of 100 in Fig. 2. The intersection points represent the criterion values, respectively, for $n_2 = 2$, 1, and 0.5. Similar curves can be determined by varying the parameter k_2 or the loading speed.

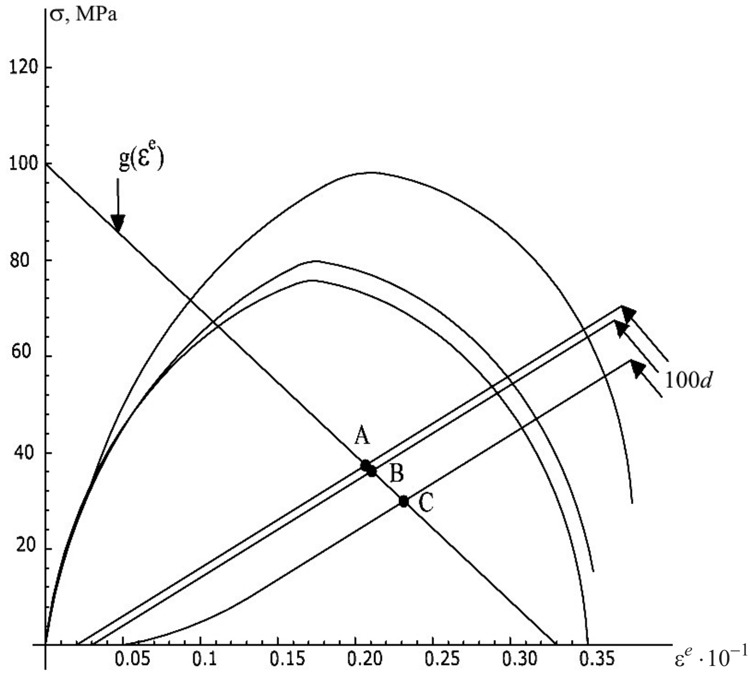

Fig. 2. Load vs. displacement.

It has been observed in this study that the model can also limit the speed of damage. Furthermore, the sensitivity of parameters k_2 and n_2 of model with delay effect has been studied in the case of a homogeneous beam in tension and length of unit section. We note that for a strain rate of 0.1 s^{-1} and a weighting factor $n_2 = 0.5$ the results of the model with delay effect does coincide with the model without delay effect only for high values of k_2.

Conclusions. In this work, we verified the criterion of instability and localization which implies that the beam is likely to break through localization of strain and damage. In the case of the homogeneous beam (flawless) in static tensile, the fracture occurs simultaneously throughout the structure. We will prove in the second part of this work that we can correctly predict the moment of fracture when the beam has a lack of rigidity, using a development that treats the provision regulating the use of mesomodeling.

Резюме

Запропоновано критерій нестабільності і локалізації пошкоджень у балці з однорідного ламіната Т300/914 для моделювання розвитку пошкоджень з урахуванням ефекту затримки. Результати, що отримані для одновимірного випадку, свідчать про те, що зона руйнування по всій конструкції виникає одночасно. Запропоновано розв’язок, що базується на мезомоделюванні композитів. Отримані за допомогою запропонованого підходу розрахункові дані дозволяють виконати точний прогноз втрати стійкості зразка при погіршанні параметрів його жорсткості.

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Received 09. 05. 2010