ON SINGLE-MODE EQUILIBRIUM AND SELF-ORGANIZATION OF REVERSAL FIELD PINCH

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The MGD self-stabilization of kink instability of paramagnetic z-pinch with strong current and some features of the RFP-like quasi-single-mode self-organization as a result of plasma azimuth and z-convection generated by MGD oscillations are studied in single mode approximation on basis of a quasi-linear model.

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1. INTRODUCTION

At present, it is established that toroidal discharges qualified as a reversal field pinches (RFP) with improved confinement are described by the "quasi-single mode" (QSM) nearly laminar kink oscillations with low-amplitudes [1]. This gives a basis to think that the single dominant mode in the MGD spectrum is inherent in the RFP nature and may be used to found the dynamic quasi-linear model of RFP discharges. In this paper, the magnetic configuration of cylindrical z-pinch is considered in the terms of general magneto-static equilibrium $rot\mathbf{B}=(\alpha\mathbf{B}+\beta\mathbf{B}\times\mathbf{e}_r)/B^2$ ($\mathbf{B}=(0,\ B_\theta(r),B_z(r))$ is the mean magnetic field). The single helical mode $m=1$, $n>>1$ is taken into consideration which belongs to the Alfven spectrum of unstable kinks of force-free paramagnetic configuration with parameters $\alpha>1$, $\beta<<1$ ($1/\alpha$ is the radius of paramagnetic pinching) to be close to conditions observed experimentally for high current pinches with low safety factor $q=arB_z/RB_\theta<<1$ (a and R are minor and major toroidal radiuses, the condition $r=1$ corresponds to the plasma column radius).

The frequency and increment of the kink as well as all radial amplitude distributions are determined by solution of the Hain-Lust linear diffusive pinch boundary problem in formulation [2] taking additionally into account arbitrary plasma convection. The mean components $(0,\ V_\theta(r),V_z(r))$, "poloidal" and "toroidal" rotational velocities, to be generated by oscillations, are determined as an the eigen-mode quadratic forms averaged on oscillations. Similarly, the parameter $\alpha(r)$ is determined by averaging of the **B**-projection of the Ohm's law for high conducting plasma. Substitution of the $V_\theta(r)$ and $V_z(r)$ into eigen-problem algorithm reveals the stabilization of the kink instability. At the same time the squared amplitude contribution of velocity and magnetic field oscillations into α (magnetic dynamo "**α-effect**") results in the appearance of a weak negative B_z in the edge plasma region. The dominant mode may be found from the condition of marginal stability under maximal amplitude. It is characteristic that not high amplitudes need for this: the velocity perturbations are measured in the "milli-Alfven" scale whereas magnetic perturbations are turn out about percents in comparison with B_z at the pinch axis.

We discuss some aspects of presented quasi-linear model of self-stabilization of a paramagnetic pinch to conclude that it may describe the initial fast phase of the RFP-equilibrium, whereas slow dissipative processes have to be taken into account to find an appropriate anomalous radial transport on the saturated amplitudes.

2. BASIC EQUATIONS

Basic equations are given by standard one-fluid MGD description and, at first, its linear version for ideal plasma implemented in the boundary problem for the plasma radial displacement ξ (r) in a helical wave:

$$\frac{d}{dr}\frac{D_A}{D}r\frac{d}{dr}(r\xi)+Q\xi=0, \qquad (1)$$

$$D_A=N\omega'^2-F^2,$$

$$D=N\omega'^2r^2-(k_z^2r^2+m^2)B^2,$$

$$Q=D_A-2B_\theta\frac{d}{dr}\frac{B_\theta}{r}-4k_zB_\theta W-2r\frac{d}{dr}GW,$$

$$W=(k_zB_\theta B^2+V_\theta B_zN\omega')/D,$$

$$F=\mathbf{kB}=k_zB_z-mB_\theta/r,$$

$$G=k_zrB_\theta+mB_z.$$

Eq. (1) presents the well-known Hain-Lust-Goedbloed equation in the form close to given in [2]. In contrast to [2] Eq. (1) takes also into account the plasma rotation introduced into ω' and, that is more essential, into W but neglects all β-effects. Eq. (1) implies the equilibrium equation that in a general case takes form $\beta=dP/dr-V_\theta^2/r$ where plasma pressure in ideal plasma obeys the equation $P=\beta_0N^{5/3}$ with $\beta_0\sim0.01$. Because of $\alpha>1$, and if only V_θ do not run up to supersonic values, to follow effects of plasma rotations on kink spectra, β may be put equal 0. (Although some results for the case $\beta\neq0$ without rotation will be shown below). The Alfven scale is used to reduce the description to the non-dimensional form: the commonly used Alfven velocity and frequency, $V_A=B(0)/(4\pi MN(0))^{1/2}$ (cm/s) and $\omega_A=V_A/a$ (s^{-1}), are assumed as an units for velocities and frequencies under calculations. At that the physical values $B(0)$ and $N(0)$ are chosen as units for magnetic field and plasma density at any r. The mean density profile $N(r)$ is assumed in the form $1-0.9r^2$ in definitions to Eq. (1).

The displacement ξ defines the plasma radial velocity disturbance δV_r through Doppler shifted frequency:

$$\delta V_r=-i\omega'\xi(r)e^{-i\omega t+ik_zz-im\theta},\ \omega'=\omega-\mathbf{kV}(r).$$

Boundary conditions are $\xi(1)=0$ and $\xi(0)\neq0$ which are admissible under additional condition $m^2=1$ as it is true for kinks. Some algebra gives all amplitude distributions $\delta N(r)$, $\delta\mathbf{B}(r)$, $\delta\mathbf{V}(r)$ in terms of functions $y_1(r)=\xi(r)$ and $y_2(r)=(d/dr)(r\xi(r))$ thereby the non-zero value $\xi(0)$ sets a zoom for the overall picture of oscillations.

In the network of standard one-liquid MGD description non-the linear balance equations may be obtained:

$$\langle \mathrm{div} N\mathbf{V}V_\theta \rangle + \frac{1}{r}\langle NV_rV_\theta\rangle =$$
$$-\nu_{ex}\langle NV_\theta\rangle + \left\langle \frac{B_r}{r}\frac{\partial}{\partial r} rB_\theta + \mathbf{kB}\frac{\partial}{\partial r}B_\theta - \frac{\partial}{r\partial\theta}B^2/2\right\rangle,$$
$$\langle \mathrm{div} N\mathbf{V}V_z\rangle =$$
$$-\nu_{ex}\langle NV_z\rangle + \left\langle B_r\frac{\partial}{\partial r}B_z + \mathbf{kB}\frac{\partial}{\partial r}B_\theta - \frac{\partial}{\partial z}B^2/2\right\rangle. \quad (2)$$

Eqs. (2) are the mean quasi-state momentum θ- and z-equilibrium where the fields N, $\mathbf{V}$, $\mathbf{B}$ contain a kink perturbation and angle bracket notes the averaging on its oscillation period (that is the same, angle averaging on the magnetic surface r=const). The momentum equilibriums are maintained with the friction force caused by ion-neutral collisions stopping the "spin-up" process caused with the disbalance of magnetic and inertial forces. The charge exchange collision frequency ν_{ex} is assumed as playing main role. Upon Alfven scaling, all dissipative frequencies are very small, the ν_{ex} can achieve maximal of the order of 10^{-3} that we mean below.

Saving contributions of main fields N, $\langle\mathbf{V}\rangle=(0,V_\theta(r), V_z(r))$, $\langle\mathbf{B}\rangle=(0,B_\theta(r),B_z(r))$ and field perturbations in Eqs. (2), one can obtain the explicit expressions for rotational velocities in the linear squared amplitude approximation:

$$V_\theta = -\frac{1}{N}2\,\mathrm{Re}[\delta N\delta V_\theta^*] +$$
$$\frac{1}{\nu_{ex}N}\frac{1}{r^2}\frac{d}{dr}r^2\, 2\,\mathrm{Re}[\delta B_r\delta B_\theta^* - N\delta V_r\delta V_\theta^*], \quad (3)$$
$$V_z = -\frac{1}{N}2\,\mathrm{Re}[\delta N\delta V_z^*] +$$
$$\frac{1}{\nu_{ex}N}\frac{1}{r}\frac{d}{dr}r2\,\mathrm{Re}[\delta B_r\delta B_z^* - N\delta V_r\delta V_z^*].$$

In the Eqs. (3), values V_θ, V_z are determined substantially by terms containing ν_{ex} in denominator. They ask very small amplitudes to have scale <<1 because rotational velocities of order 1 in the Alfven scaling are experimentally unobserved. It becomes also clear from the Ohm law for weakly non-ideal plasma averaged on oscillations:

$$E_0\mathbf{e}_z - \Phi'(r)\mathbf{e}_r + \mathbf{V}\times\mathbf{B} + \langle\delta\mathbf{V}\times\delta\mathbf{B}\rangle - \eta\,\mathrm{rot}\mathbf{B} = 0. \quad (4)$$

Pressure effects are omitted in Eq. (4), E_0 is the "toroidal" electric field driving the discharge (E_0 is constant in the straight cylinder case), Φ is an electrostatic potential, $\eta = (c^2/4\pi\sigma)/(aV_A)$, σ is a plasma conductivity. The "resistance" η is very small parameter (up to 10^{-7} accordingly to estimations; in calculations $\eta = 10^{-6}$ is implied). Then $\mathbf{B}$-projection of Eq. (4) gives

$$\alpha = \alpha_0 B_z + \frac{1}{\eta}\langle \delta V_r(\delta\mathbf{B}, \mathbf{B}\times\mathbf{e}_r) - \delta B_r(\delta\mathbf{V}, \mathbf{B}\times\mathbf{e}_r)\rangle, \quad (5)$$

where $\alpha_0=4\pi\sigma aE_0/cB(0)$ (all values are in absolute physical units) is the parameter of paramagnetic force-free model of diffusive pinch introduced as far back as before tokamak epoch. For the force-free model, $rot\mathbf{B} = \alpha\mathbf{B}/B^2$, which is true in the $\beta = 0$ approximation, the safety factor q equals $2\alpha_0 a/R$ at r=0 whereas $q(0)\approx 0.5a/R$ in RFP [1]. So the value α_0=4 is adopted in our calculations.

It is follows from Eq. (5) the amplitudes must be of the order of $\eta^{1/2}$ in order to the contribution of last terms on right side in (5) might be comparable with α_0. Thereby the parameter $\xi(0)$ must be specified with the gage 10^{-3} in alpha-effect modeling. Otherwise the velocity amplitude are being measured in "milli-Alfven" units.

On other hand, $\mathbf{B}\times\mathbf{e}_r$-projection of Eq. (4) gives the effect of oscillations on the mean radial velocity :

$$\langle V_r\rangle/\eta = -\alpha_0 B_\theta/B^2 + \langle\delta B_r(\delta\mathbf{V},\mathbf{B}) - \delta V_r(\delta\mathbf{B},\mathbf{B})\rangle/B^2. \quad (6)$$

I. e. the mean radial velocity turns out of order of "micro-Alfvens" in contrast with the mean V_θ, V_z which are about "milli-Alfvens" so far as $\nu_{ex}\approx\eta^{1/2}$. It proves the derivation of Eq. (3) in which a mean radial transport was neglected.

3. RESULTS

Fig. 1 illustrates effects plasma pressure on increments, $\gamma=\mathrm{Im}(\omega)$, of the right-hand kink (m=1) with different toroidal numbers $n=(R/a)k_z$ in plasma without any rotations. These results are obtained by solution of Goedbloed equation [2] by shooting method and include the case β_0=0 when Eq.(1) is sufficient and the increment turns out maximal. The spectrum envelops the region about k_z=2, thereby the particular toroidal number, $n = (R/a)k_z$= 8 (if the aspect ratio R/a=4) plays especial role. It is "neutral" mode in which $F(r_s)$=**kB**=0 at r_s=0. For chosen parameters boundary k_z =2 separates the zones of external modes, n<8, and internal ones, n>8, on the axis k_z. The cut line in Fig.1 shows the forbidden zone for finding stable modes in terms of formulations [2] or (1) because a singularity arrives within interval (0,1) if $0<r_s<1$.

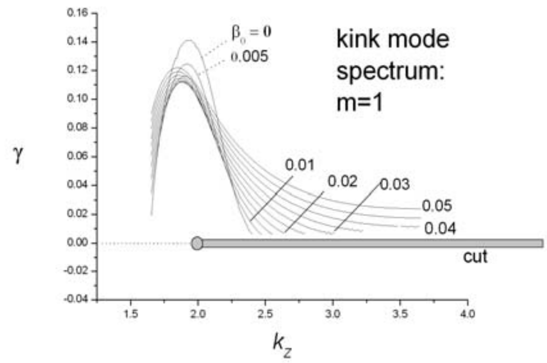

Fig.1. Dependence of the kink increments on k_z and plasma pressure in motionless plasma

Fig. 2 illustrates the process of stabilization of kink modes under the influence of plasma rotations which are generated by all theirs own field oscillations inside plasma in accordance with expression (3) for the radial distributions $V_\theta(r)$, $V_z(r)$ under condition β_0=0. The growth rates are plotted for some set of internal modes including the neutral mode against the amplitude parameter $\xi(0)$ with gage $\eta^{1/2}$. The most instable neutral mode is saturated under most amplitude $\eta^{1/2}$ and can be claimed as a "dominant" mode in the β=0 model.

The complete stabilization of internal modes is not achievable in the shooting algorithm because an approach to the cut line breaks the iteration process. Note, the modes under considerations are non-local ones when the radial profile $\xi(r)$ has no nodes inside plasma, within interval $(0<r<1)$. Why internal modes demonstrate so abrupt relaxation with growing of small amplitude $\xi(0)$ is understandable if to look the Fig. 3 where full set of amplitude functions, $y_1=\xi(r)$ and $y_2=(d/dr)(r\xi(r))$, are shown for internal mode $k_z= 2.2$ on an approach to saturation. The stabilization process extremely sharpens the amplitude profiles near $r=r_s$ for internal modes.

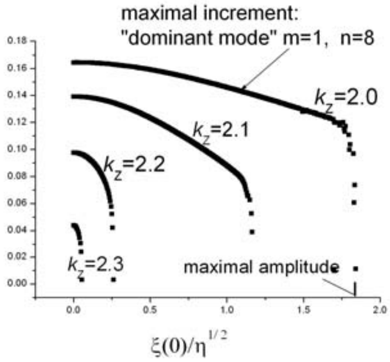

Fig. 2. Stabilization of different modes in the case $\beta_0=0$

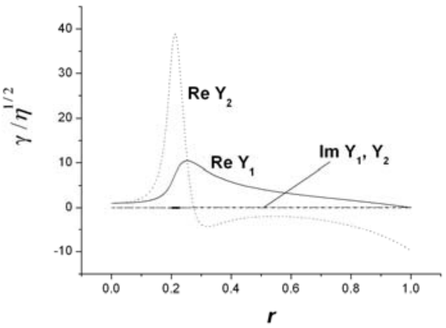

Fig. 3. Shapes of amplitude function y_1, y_2 under parameters k_z=2.2, γ=0.05

So, even if $\xi(0)$ is very small the local values at r_s become sufficient to show considerable effect on solution of the problem (1) in this case. So the results may be considered as of physical interest so far the local rotational velocities nearby resonant surfaces do not become supersonic with diminishing of γ. Fig. 4 demonstrate the effect of rotation on the paramagnetic radial distributions arriving under parameters of Fig. 3. Some reversed z-field appears near plasma edge and configuration becomes more close to the RFP pattern than the initial paramagnetic model.

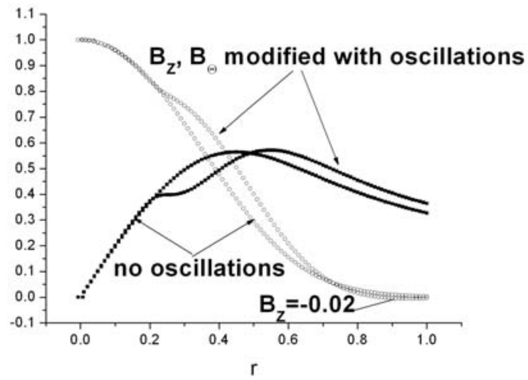

Fig. 4. Deformation of equilibrium by oscillations

CONCLUSIONS

We made sure that the non-local kink-modes, to be unstable in the weakly non-ideal plasma of paramagnetic pinches, can experience the relaxation of growth rates due to the plasma rotation generated by own field kink perturbations. Some mode may be dominant in the spectrum of discrete toroidal numbers n. Theoretical quasi-linear model determining the rotational velocities and the algorithm taking into account the plasma rotation are developed. The process of the mode saturation is characterized by arising of a reversed z-field nearby plasma surface.

However the presented model don't able to describe a static maintenance of amplitudes as well to find the value of real frequency of resonant modes in the limit γ=0. More dissipative processes must be involved into the model as well as all β-effects. Some closing in the theory of tearing modes ought to take place. Without such a development the model cannot to explain also haw a dominant mode can provide the positivity of mean radial velocity in terms of the approach (6).

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ОБ ОДНОМОДОВОМ РАВНОВЕСИИ И САМООРГАНИЗАЦИИ ПИНЧА С ОБРАЩЕННЫМ ПОЛЕМ

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Рассмотрены стабилизация неустойчивости кинков парамагнитного z-пинча с сильным током и некоторые особенности самоорганизации пинчей с обращенным полем под влиянием полоидального и тороидального вращения, генерированного колебаниями в одномодовом приближении на основе квазилинейной модели.

ПРО ОДНОМОДОВУ РІВНОВАГУ ТА САМООРГАНІЗАЦІЮ ПІНЧА З ОБЕРНЕНИМ ПОЛЕМ

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Розглянуто стабілізацію нестійкості кінків парамагнітного пінча з сильним струмом та деякі особливості самоорганізації пінчів з оберненим полем під впливом полоїдального й тороїдального обертання, генерованого коливаннями в одномодовому наближенні на основі квазілінійної моделі.