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DYNAMIC POLLUTION MODEL

1. Introduction

The goal of the work is a macroeconomics modeling of ecological problems, more exactly, problem of industrial pollution and cleaning with special technologies. This problem is actual one especially if to take into consideration current caloric pollution.

2. Ecology and natural capital

In many works, see [1], discussed problems of economy and ecology and environments. In classical works the resources of Nature are considered as land resources. The flows of goods and finance in charts of macroeconomists usually do not suggests the underlying usage of natural resources and do not take into the account the possibility of pollution and resource depletion. As the author argued in [2], "society is sailing by the wrong compass at expense of environment, the belief in continuous exponential growth is the hart of environmental problems ". The active policy of economical growth is the source of pollution and scarcity of natural resource. In classical model which review we make later, does not include this important detail.

3. Short overview of main macroeconomic dynamic models

Solow-Ramcey model [3, 4].

Macroeconomics studies economics in general, so it uses generalized functions, which describe the system behavior as whole. In his pioneering paper [2] Solow proposed to use for description of aggregated output using the following production function:

$$Y(t) = F(K(t), A(t)L(t))$$

where

K(t) is capital resource at time *t*.

L(t) is labor input in production at time *t*.

A(t) is technological efficiency at time *t*.

We assume that this production function has constant returns to scale concerning capital and labor. This possibly implies that economics more or less large and there are no possibility to have advantages of specialization expressed in increasing production at more scale than input was increased.

The specific and well known example of such function is the function of the following kind:

$$F(K, AL) = K^{\alpha}(AL)^{1-\alpha}$$

This is a Cobb-Douglas function. This function is easy to use and as the first approximation to the reality this is also acceptable.

To write the dynamical equation for production the usual techniques is used [4]. In this model we do not take into the account the possibility of technological change, so we postulate that

$$A \equiv 1$$

Using usual method reducing such function to the type depending only on one scaled variable *k*:

if we define

$$k = K / L, y = Y / L$$

where *y* is effective output per labor unit, and *k* is effective capital per labor unite we obtain the following simple expression:

$$y = f(k) = k^{\alpha}$$

4. Construction of the model

We use classical way asserting that we have the simple economy with one product. This product can be or invested or consumed or depreciated. New level of the product in the *t+1* moment of time composed of the old level in the previous moment of time *t* plus production at this moment less depreciation of capital in this moment.

Symbolically:

$$k(t+1) = k(t) + f(k(t)) - bk(t)$$

where *b* is the rate of capital depreciation (we suppose that $b > 0$)

The previous equation maybe rewritten in the form

$$\Delta k = f(k) - bk$$

or taking the limit when time step becomes infinitesimally small, we get:

$$\dot{k} = f(k) - bk$$

As usually the point over *k* defines a time derivative.

So we have evolved the first dynamical equation.

The second equation is got following the next considerations [5]. The stock of goods is capital - such definition is accepted by many economists. These goods are used for consumption or for using in production together with natural resources. The use of such resources always accompanies by more or less undesirable effects (“land impoverishment”, pollution and so on). Sometimes these effects are recovered by the Nature (at least we believe that such recovery occurred), but sometimes such type of recovery is absent. Such “negative good” we take into the account to create the second equation for “negative production” (pollution).

To construct the equation we use the same idea as in the previous equation construction where we have described the process of production of the capital, supposing that the used production function does not generate any “negative goods”.

Let us suppose that, on the contrary, our model economy produces such undesirable goods and for simplicity we assume that such production is the fixed share of produced capital, that is a share of the whole yield for “negative goods” is *a* and for “usual goods” is *1-a*.

In this case we can rewrite the first equation as:

$$\dot{k} = (1-a)f(k) - bk$$

Let us denominate “negative good” by *p*. As it is stated above, only a share of production is “negative good”, so the first approximation for the second equation is:

$$\dot{p} = af(k)$$

In this equation, we did not take in our consideration neither natural rate of purification or recovering, produced by Nature, nor human efforts for correcting pollution.

This may be reflected by introducing a negative member which reduces the velocity of pollution. We represent this member as production of “negative good” quantity and a new production function which reflects the human technologies that correct the negative influence on the Nature. This function uses as input capital *k*.

Now we have the second equation:

$$\dot{p} = af(k) - pg(k)$$

where *g(k)* is the “anti pollution” production function.

Finally we have such system of two differential equations that describes our simple economy with pollution:

$$\dot{k} = (1-a)f(k) - bk$$
$$\dot{p} = af(k) - pg(k)$$

5. Investigation the system dynamics

In our investigation we assume that

$$f(k) = k^{\alpha}$$
$$g(k) = k^{\beta}$$

$$\alpha > 0, \beta > 0$$

Taking into the account such supposition our system has one nontrivial point of equilibrium (except trivial when *k=0* and *p=0*):

As it is easy to see that condition where speed of changes *p* and *k* equals zero, is

$$k = (\frac{b}{1-a})^{\frac{1}{1-\alpha}}$$

$$p = a(\frac{b}{1-a})^{\frac{\alpha-\beta}{\alpha-1}}$$

Using the first order conditions for stability of this point on the phase diagram we can find the following values of Eigen values for these equations: they are always real and the condition of their negativity are:

$$\lambda_1 = b(\alpha - 1) < 0$$
$$\lambda_2 = -(\frac{b}{1-a})^{\frac{\beta}{\alpha-1}} < 0$$

The first condition gives us constraint

$$\alpha < 1$$

That is if we want to have stability than the growth of production function must not be very fast to give the opportunity for the Nature to recover damage.

The second condition shows:

$$a < 1$$

This means that the pollution rate have to be not very much and our economy have to produce not only “negative goods”, positive goods we need to use in anti pollution technology. So the efforts of human society are vitally necessary to correct the negative influence (pollution) of economic activity.

6. Conclusion

The proposed model is based on the classical assumptions and gives us clear and simple conditions for some important economical variables.

References

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