EXCITATION OF HARMONICS BY FLOWS OF THE CHARGED PARTICLES IN THE MAGNETIC FIELD

V.A. Buts, A.M. Yegorov, A.P. Tolstoluzhsky
NSC "Kharkov Institute of Physics and Technology", Kharkov, Ukraine,
E-mail: vbuts@kipt.kharkov.ua

Results of research of the electromagnetic waves excitation by ensemble of relativistic oscillators in a magnetic field are presented. The basic attention is given to studying of generation efficiency of electromagnetic oscillations at harmonics of cyclotron frequency. It is shown, that presence of medium can essentially change efficiency of excitation of high numbers harmonics and all picture of development of oscillators instability. Increments of instability are received. Influence of thermal spread of oscillators is investigated

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1. INTRODUCTION

In the previous researches (see, for example, [1-2]) it has been shown, that charged oscillators which move in periodically non-uniform media or in periodically non-uniform potentials, can effectively radiate high numbers of harmonics. Effectively radiate high numbers of harmonics even nonrelativistic oscillators. The physical reason of such radiation is existence in periodically non-uniform media of not own virtual slow waves. When phase velocity of these virtual waves appears close to the velocity of the charged particles, efficiency of radiation on high harmonics sharply increases. In the present paper results of research of radiation, both the single charged particles, and flows of the charged particles which move in a constant external magnetic field are stated. It is supposed that, medium in which particles move, can slow down the electromagnetic waves excited by particles. Without going into physical reason of such slowing, this property is described by presence of constant dielectric permeability. Let's note, that recently the big attention is given to research of mechanisms of electromagnetic waves generation in a submillimetric and infra-red range of frequencies. One of such mechanisms is radiation relativistic electrons, moving in a magnetic field, on harmonics of cyclotron frequency. Below we for simplicity shall be limited to consideration of ensemble of relativistic particles in the medium with dielectric permeability ε_0. Particles are in a constant external magnetic field $\vec{H}_0$. Let's direct axis Z along the field $\vec{H}_0$. Axis X we shall choose so, that Y component of a wave vector $\vec{k}$ is equal to zero, that is $\vec{k}=(k_\perp,0,k_z)$, and in space of impulses it is used cylindrical system of coordinates $(p_\perp,\varphi,p_z)$. We shall study excitation of electromagnetic waves which extend across the magnetic field. Distribution functions of ensemble oscillators we shall choose in the form of:

$$f_0(p_z,p_\perp)=\frac{n_b}{2\pi p_\perp}\delta(p_z)\frac{1}{p_T\sqrt{2\pi}}e^{-\frac{(p_\perp-p_{\perp0})^2}{2p_T^2}} \qquad (1)$$

2. DISPERSION EQUATION. INCREMENT

Instability of electromagnetic waves, propagating in the medium with dielectric permeability ε_0, is described with dispersion equation [3,4]):

$$\det\left[\frac{k^2c^2}{\omega^2}\left(\frac{k_ik_j}{k^2}-\delta_{ij}\right)+\varepsilon_{ij}\right]=0\,, \qquad (2)$$

where $\varepsilon_{ij}=\varepsilon_0\delta_{ij}+\varepsilon_{ij}^b$., ε_{ij}^b -- tensor of dielectric permeability of oscillators.

We will investigate instability ensemble of oscillators with respect to excitation of electromagnetic waves at the harmonics of electron cyclotron frequency. For the considered type of wave equation (2) can be reduce to the type:

$$\varepsilon_0+\varepsilon_{22}^b-k_\perp{}^2c^2/\omega^2=0\,. \qquad (3)$$

2.1. INCREMENT OF OSCILLATIONS, EXCITED BY COLD OSCILLATORS

For cold oscillators in the case of transversal propagation of waves $(k_z=0)$ component $\varepsilon_{2,2}^b$ of tensor $\varepsilon_{i,j}^b$ looks like:

$$\varepsilon_{22}^b=\sum_{s=-\infty}^{\infty}\left[\frac{1}{\lambda_0}\left(\lambda_0{}^2J_s'^2\right)'P+\lambda_0{}^2J'^2_sQ\right], \qquad (4)$$

where

$$P=-\frac{\omega_b^2\sqrt{1-\beta_{\perp0}^2}}{\omega\left(\omega-s\tilde{\omega}_{H0}\right)},\ Q=\frac{\omega_b^2\beta_{\perp0}^2\sqrt{1-\beta_{\perp0}^2}}{\lambda_0{}^2\left(\omega-s\tilde{\omega}_{H0}\right)^2},\ \beta_{\perp0}^2=\frac{v_{\perp0}^2}{c^2},$$

$$\tilde{\omega}_{H0}=\omega_H\sqrt{1-\beta_{\perp0}^2},\ J_s\equiv J_s(\lambda_0),\ J_s'\equiv\frac{dJ_s(\lambda_0)}{d\lambda_0},\ \lambda_0=\frac{k_\perp v_{\perp0}}{\tilde{\omega}_{H0}}.$$

We will consider frequency ω of the excited wave near to frequency $n\tilde{\omega}_{H0}$. Then remaining in right part (4) only main terms will get:

$$\varepsilon_0-\frac{k_\perp{}^2c^2}{\omega^2}+\frac{\omega_b^2\sqrt{1-\beta_{\perp0}^2}}{\left(\omega-n\tilde{\omega}_{H0}\right)}\left(\frac{\beta_{\perp0}^2J_n'^2}{\left(\omega-n\tilde{\omega}_{H0}\right)}-\frac{\left(\lambda_0{}^2J_n'^2\right)'}{\lambda_0\omega}\right)=0\,. \quad (5)$$

Maximal increment of oscillations is arrived at equality of eigen frequency $\omega_0=k_\perp c.\sqrt{\varepsilon_0}$ to frequency of the excited wave $n\tilde{\omega}_{H0}$, therefore, considering in (5) $\omega=n\tilde{\omega}_{H0}+\delta,\ \omega_0\approx n\tilde{\omega}_{H0},\ \delta<<\omega$, will get cube equaltion for complex addition δ to frequency ω_0

$$2\varepsilon_0\frac{\delta}{n\tilde{\omega}_{H0}}+\frac{\omega_b^2\beta_{\perp0}^2\sqrt{1-\beta_{\perp0}^2}}{\delta^2}J_n'^2-\frac{\left(\lambda_0{}^2J_n'^2\right)'}{\lambda_0}\frac{\omega_b^2\sqrt{1-\beta_{\perp0}^2}}{n\tilde{\omega}_{H0}\delta}=0\,, \quad (6)$$

where $\lambda_0=n\beta_{\perp0}\sqrt{\varepsilon_0}$.

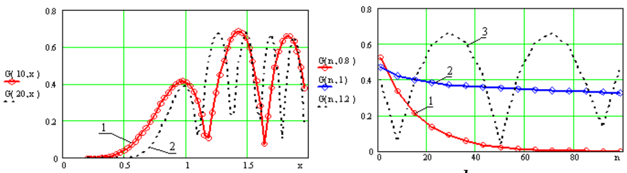
Remainig major members on the value δ^{-1}, find increment of instability

$$\operatorname{Im}\delta = \omega_H \frac{\sqrt{3}}{2}\left[\frac{\omega_b^2}{\omega_H^2}\frac{\left(1-\beta_{\perp 0}^2\right)}{2\varepsilon_0} n\beta_{\perp 0}^2 J_n'^2(n\beta_{\perp 0}\sqrt{\varepsilon_0})\right]^{\frac{1}{3}}. \quad (7)$$

From (7) evidently, that dependence of increment on the number n of the excited harmonic is concluded in expression

$$G(n,x) = \left[nJ_n'^2(nx)\right]^{\frac{1}{3}}. \quad (8)$$

Graphs of dependence $G(n,x)$ on x ($x \equiv \beta_{\perp 0}\sqrt{\varepsilon_0}$) at different values n and dependence $G(n,x)$ on n at different values x are resulted on a Fig. 1.



a *b*

Fig.1. Dependences $G(n,x)$:

a) on x at: 1- $n=10$,2- $n=20$;

b) on n at: 1- x =0.8, 2- x =1, 3- x =1.2

As be obvious from these pictures, increment, proportiona $G(n,x) = \left[nJ_n'^2(nx)\right]^{\frac{1}{3}}$, quickly decreases at an increase n, if $\beta_{\perp 0}\sqrt{\varepsilon_0} < 1$. At $\beta_{\perp 0}\sqrt{\varepsilon_0} = 1$ increment very slowly falls with growth n. In addition, in this case, it is possible to show that at $n > 10$ $G(n,1) \approx 0.55n^{-1/9}$. In case $\beta_{\perp 0}\sqrt{\varepsilon_0} > 1$ dependence of increment on the number n is an almost periodic function. Thus, maximal increment is arrived at at $\beta_{\perp 0}\sqrt{\varepsilon_0} \geq 1$.

2.2. INCREMENT OF OSCILLATIONS EXCITED BY OSCILLATORS WITH THERMAL SPREAD

As in case of cold oscillators, including, that

$$\omega = n\tilde{\omega}_{H0} + \delta,\ \omega_0 \equiv \frac{k_\perp c}{\sqrt{\varepsilon_0}} \approx n\tilde{\omega}_{H0},\ \delta << \omega\ \ (\omega - n\tilde{\omega}_H)^{-1},$$

receive the equation for finding of the complex additive δ to the frequency ω_0:

$$2\varepsilon_0\Delta + \frac{\Omega_b^2}{1+\Delta}\frac{1}{\sigma\sqrt{2\pi}} \times$$
$$\times\int_0^\infty \frac{J_s'^2(\lambda)}{\gamma_0 - (1+\Delta)\gamma}\left(1+\frac{x^2-x}{\sigma^2}\right)\exp\left[-\frac{(x-1)^2}{2\sigma^2}\right]dx = 0, \quad (9)$$

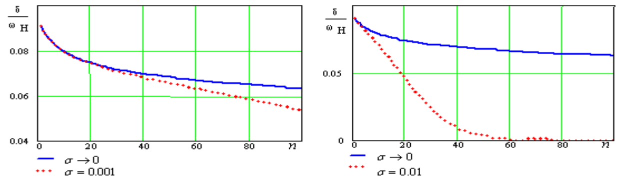
in which following dimensionless variables and parameters are used:

$$\Delta = \frac{\delta}{n\tilde{\omega}_{H0}},\ \Omega_b = \frac{\omega_b}{n\tilde{\omega}_{H0}},\ \sigma = \frac{p_T}{p_{\perp 0}},\ x = \frac{p_\perp}{p_{\perp 0}},$$

$$\gamma = \sqrt{1+\frac{p_{\perp 0}^2}{m^2c^2}x^2},\ \gamma_0 = \sqrt{1+\frac{p_{\perp 0}^2}{m^2c^2}},\ \lambda = n\beta_{\perp 0}\sqrt{\varepsilon_0}x.$$

The equation (9) has been solved numerically for $\gamma_0 = 3$, $\omega_b/\omega_H = 0.5$ and various values $\sigma = 0.01$, $\sigma = 0.001$, $\beta_{\perp 0}\sqrt{\varepsilon_0} = 1.2$.

In Fig. 2a dependence of increment from number of excited harmonic for cold oscillators and oscillators with thermal spread is shown at $\sigma = 0.001$ and $\beta_{\perp 0}\sqrt{\varepsilon_0} = 1$. One can see, that dependences practically coincide down to $n \approx 40$. At increase of thermal spread ($\sigma = 0.01$, Fig.2b) increment quickly falls down with growth of number n.



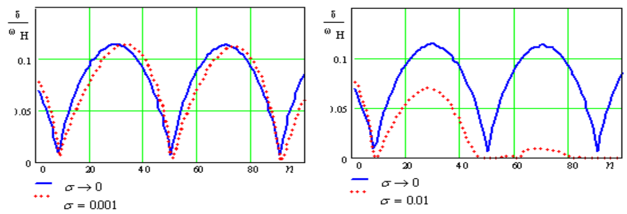
a *b*

Fig.2. Dependences of increment on n *at:*

a) σ =0 and σ =0.001;

b) σ =0 and σ =0.01

On Fig. 3a increments for oscillators with small thermal spread $\sigma = 0.001$ and $\beta_{\perp 0}\sqrt{\varepsilon_0} = 1.2$ are resulted. Increments for cold oscillators and oscillators with thermal spread differ a little. In the case of $\sigma = 0.01$; $\beta_{\perp 0}\sqrt{\varepsilon_0} = 1.2$ (Fig. 3b) increment for oscillators with thermal spread falls to zero for harmonics with numbers more than 50, but remains to enough greater in the region of $20 < n < 40$.



a *b*

Fig.3. Dependences of increment on n *at:*

a) σ =0 and σ =0.001;

b) σ =0 and σ =0.01

3. THE ELEMENTARY MECHANISM OF OSCILLATOR'S RADIATION

Radiating oscillator losses осциллятора in the magnetic field are described by the formula [5]

$$P_n = (e^2\omega_H^2 / v_\perp)w(n),$$

$$w(n) = \frac{n}{\varepsilon_0}\left[2\varepsilon_0\beta_\perp^2 J_{2n}'(2n\beta_{\perp 0}\sqrt{\varepsilon_0}) - (1-\beta_{\perp 0}^2\varepsilon_0)\int_0^{2n\beta_{\perp 0}\sqrt{\varepsilon_0}} J_{2n}(x)dx\right],$$

n - number of cyclotron harmonic.

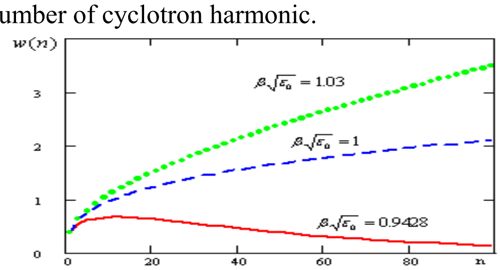


Fig.4. Dependences $w(n)$ on n at: $\beta_{\perp 0}\sqrt{\varepsilon_0}$ =0.9428;

$\beta_{\perp 0}\sqrt{\varepsilon_0}$ =1; $\beta_{\perp 0}\sqrt{\varepsilon_0}$ =1.03

In Fig. 4 graphics of dependence $w(n)$ on number of the harmonic for γ_0 =3 and various values of dielectric permeability ε_0 are resulted. It is visible, that efficiency of radiation improves with increase of the harmonic's number in conditions, when parameter $\beta_{\perp 0}\sqrt{\varepsilon_0} \geq 1$.

4. BASIC RESULTS

Increments of waves amplitudes growth are received. It is shown, that presence of medium with refraction parameter essentially changes the picture of oscillators instability development. So, increment quickly falls at increasing of harmonic's number , if $\beta_{\perp 0}\sqrt{\varepsilon_0} < 1$. In the case $\beta_{\perp 0}\sqrt{\varepsilon_0} = 1$ increment very slowly falls down with growth of harmonic's number as $n^{-1/9}$. In the case $\beta_{\perp 0}\sqrt{\varepsilon_0} > 1$ dependence of increment due to harmonic's number n is almost periodic function, that is in this case there are zones of effective excitation of electromagnetic radiation. Thus, maximal increment is reached at $\beta_{\perp 0}\sqrt{\varepsilon_0} \geq 1$, it is possible to explain it as follows. With increase of frequency of oscillations which are generated, there is not only reduction of number of coherent particles in a bunch, but also a simultaneous increase in radiating losses of particles at radiation in this region of frequencies. It is shown, that there are conditions in which it is possible to excite effectively harmonics of cyclotron frequency by oscillators with thermal disorder. However the thermal disorder should not exceed 5 % from cross-section speed of the beam. Above we were limited to model of medium in which slowing-down of excited electromagnetic waves has been caused by presence of medium with dielectric permeability ε_0.

It is necessary to tell, that it is already enough for a long time such media are used as nonlinear elements for excitation of high numbers harmonic's (see, for example, [6-9]). Thus it is possible to reach significant values of susceptibilities even for high numbers of harmonics. For obtaining of such results vapors of metal atoms are used. On this way it is possible to excite due to nonlinear effects ultra-violet, and even radiation in a soft X-ray range (down to 141-harmonic). Efficiency of such nonlinear transformation, however, is very insignificant. Other opportunity consists in using periodically non-uniform media (lattices of ideal crystals and superlattices). At that the electromagnetic fields, waves propagating in such media, contain spatial harmonics which phase speed can be enough small.

Such possibility has been investigated in a series of works (look [1-2] and the literature quoted there). For the case considered in the present work it means, that oscillators cooperate not with the basic mode, and with the slow spatial harmonic.

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ВОЗБУЖДЕНИЕ ГАРМОНИК ПОТОКАМИ ЗАРЯЖЕННЫХ ЧАСТИЦ В МАГНИТНОМ ПОЛЕ

В.А. Буц, А.М. Егоров, А.П. Толстолужский

Изложены результаты исследования возбуждения электромагнитных волн ансамблем релятивистских осцилляторов в магнитном поле. Основное внимание уделено изучению эффективности генерации электромагнитных колебаний на гармониках циклотронной частоты. Показано, что наличие среды может существенно изменить эффективность возбуждения высоких номеров гармоник и всю картину развития неустойчивости осцилляторов. Получены инкременты неустойчивости. Исследовано влияние теплового разброса осцилляторов.

ЗБУДЖЕННЯ ГАРМОНІК ПОТОКАМИ ЗАРЯДЖЕНИХ ЧАСТИНОК У МАГНІТНОМУ ПОЛІ

В.О. Буц, О.М. Єгоров, О.П. Толстолужський

Викладено результати дослідження збудження електромагнітних хвиль ансамблем релятивістських осциляторів в магнітному полі. Основна увага приділена вивченню ефективності генерації електромагнітних коливань на гармоніках циклотронної частоти. Показано, що наявність середовища може істотно змінити ефективність збудження високих номерів гармонік і всю картину розвитку нестійкості осциляторів. Отримано інкременти нестійкості. Досліджено вплив теплового розкиду осциляторів.