ADIABATIC ION-SOUND WAVES

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A low frequency potential electric field in plasma is considered. Solutions in the form of the ion-sound waves in supposition, that the considered process is an adiabatic one for rapid particles (electrons) and isothermal one for slow particles (ions), are obtained. Coming from the Boltsmann distribution adiabatic Debye radius is obtained and similarly adiabatic Jeans wave-length for the gravitating system is obtained. Due to kinetic description dispersion equation for adiabatic sound-waves is defined more precisely.

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1. ADIABATIC LINEARIZATION OF THE BOLTSMANN DISTRIBUTION

Considering subsystems as ideal gases we obtain for characteristic motion velocities of ions and electrons $u_e \sim \sqrt{T_e/m}$ such a relation toward the rate of the examined movement (ionic sound)

$$u_i \ll u \ll u_e . \quad (1)$$

That is why electrons in a sound-wave have relaxed and behave as equilibrium gas in the external field with the Boltsmann distribution [1]

$$n_e = n_0 \exp(e\varphi/T_e), \quad (2)$$

where φ is scalar potential of the electric field of charged particles of plasma with the density n_0 of charges of every sign. As is generally known [1], in a linear theory it is possible to separate the potential and vortical field, and the last has high-frequency optical branches only, that is why here it will not be studied.

According to (1), in the first order one can consider ions "cold" (and ignore influence of thermal motion of ions on spreading waves), i.e. proceed from motion equation

$$\frac{d\mathbf{v_i}}{dt} = -e\nabla\varphi/M . \quad (3)$$

Also we use a linearized continuity equation

$$\frac{\partial \delta n_i}{\partial t} + n_0 div\mathbf{v}_i = 0 \quad (4)$$

and the Poisson equation

$$\Delta\delta\varphi = -4\pi e(\delta n_i - \delta n_e) . \quad (5)$$

If one supposes electronic components to be isothermal, then linearization of equation (2) gives

$$\delta n_e = n_0 e\delta\varphi/T_e \quad (6)$$

and we will get for the wave solution of the system (3)-(6) the known [1] sound-waves $u_{ST} = \sqrt{\frac{T_e}{M}}$.

As is generally known [2], the ordinary (isothermal) theory of ion-sound oscillations gives the incorrect value of spread velocity. Using of adiabatic state equation in a theory with hydrodynamic description of electronic component [3] corrects the situation with the value of spread velocity

$$u_S = \sqrt{\frac{\gamma T_e}{M}} , \quad (7)$$

where γ is the adiabatic index, T_e is the temperature of electronic gas, M is the ion mass.

We will show how we obtained this result proceeding from equilibrium and kinetic approaches to description of electronic components of plasma. The state equation in the case of adiabatic motion of electrons, as is generally known, has the form

$$TV^{\gamma-1} = const. \quad (8)$$

Linearization of (8) gives

$$\delta T_e = (\gamma-1)\frac{T_e}{n_e}\delta n_e . \quad (9)$$

Linearization of equation (2) with the use of bond (9) brings us over to expression

$$\delta n_e\left(1+\frac{n_0 e\varphi}{T_e^2}\frac{T_e}{n_0}(\gamma-1)\right) = n_0 e\frac{\delta\varphi_e}{T_e} . \quad (10)$$

And also by the virial theorem average potential energy of charge in the electric field (which has a coulomb form for the main approximation of retired potentials) is expressed in terms of kinetic energy in such a way [4]

$$U = -2K .$$

Obviously [1], transversal part of both velocity and electromagnetic field must be cast aside, as not relating to the longitudinal ion-sound waves in the main approximation. That is why

$$K = T/2$$

and $U = -T$. And average potential energy of electrons is

$$U = -e\varphi, \quad (11)$$

therefore from (10) we have

$$\delta n_e = n_0 e\frac{\delta\varphi_e}{\gamma T_e} . \quad (12)$$

This equation differs from (6) only by the change of temperature in γ times. It allows to express deviation of scalar potential in terms of deviation of density. For the long-wave sound oscillations in the Poisson equation it is possible to neglect laplasian, that gives equality of density deviations

$$\delta n_i = \delta n_e . \quad (13)$$

Then we have instead of (3)

$$\frac{d\mathbf{v_i}}{dt} = -\frac{\gamma T_e}{n_0 M}\nabla\delta n_e . \quad (14)$$

that jointly with (4) after the calculations similar to the isothermal case being conducted gives velocity of adiabatic ion-sound waves, which coincides with (7).

In addition, by a standard method [1] in supposition of mobility only of electronic components ($\delta n_i = 0$) after substitution of relation (12) in the Poisson equation (5) the electronic radius of (adiabatic) screening turns out to be

$$r_{DSe} = \sqrt{\frac{\gamma T_e}{4\pi e^2}} . \qquad (15)$$

Absolutely similarly it is possible to calculate a Jeans wave-length at the system of gravitating particles of mass m and temperature T . Note that a result corresponds to replacement in (15) $e^2 \to Gm^2$, where G is gravitation constant, i.e.

$$\lambda_{DS} = \sqrt{\frac{\gamma T}{4\pi G m^2}}. \qquad (16)$$

2. KINETIC DESCRIPTION

An alternative to hydrodynamic consideration is kinetic one. Thus we must solve the Vlasov equation

$$\frac{\partial f_{a0}}{\partial t} + \vec{\upsilon}\nabla f_{a0} + e_a \vec{E}\frac{\partial f_{a0}}{\partial \vec{p}} = 0 \cdot \qquad (17)$$

with the potential field

$$\vec{E} = -\nabla\varphi \qquad (18)$$

for the one-particle distribution functions of both electronic and ionic components of plasma. It is easily to verify, that the Maxwell-Boltsmann distribution is the equilibrium solution [1]

$$f_{a0} = \exp\left(\frac{F(T,\tilde{V}) - \varepsilon(x)}{T}\right). \qquad (19)$$

We will consider small deviation from an equilibrium using the Chepmen-Enskog method [5].

For the high-frequency field the temperature dependence is not important (it is possible even to deal with «cold» particles), a temperature does not have time to change - isothermal state. Then small deviation from the Boltsmann distribution will be

$$\delta f_a = \exp\left(\frac{F(T,\tilde{V}) - \varepsilon(x)}{T}\right)\left(-\frac{P}{T}\delta\tilde{V}\right). \qquad (20)$$

Here $\tilde{V} = \frac{V}{N}$ is specific volume per a particle. At the constant amount of particles in the system we have for deviation of density

$$\delta n = -\frac{N}{V^2}\delta V \ \ (n = \frac{N}{V}), \qquad (21)$$

i.e. after integration by velocities we have for ionic component

$$\int \delta f_i d^3\upsilon = n\left(-\frac{N}{V}\frac{\delta V}{N}\right) = \delta n_i . \qquad (22)$$

However, if the field frequencies are considerably less than characteristic one for the subsystem of this sort of particles, particles will have time to tune their temperatures adiabatically. This for small deviation from the Boltsmann distribution will give

$$\delta f_a = \exp\left(\frac{F(T,\tilde{V}) - \varepsilon(x)}{T}\right)\times \\ \times\left(-\frac{F(T,\tilde{V}) - \varepsilon(x)}{T^2}\delta T - \frac{S}{T}\delta T - \frac{P}{T}\delta\tilde{V}\right). \qquad (23)$$

We remember that $F + TS = E$. Integrating by velocity, we have

$$\int \delta f_a d^3\upsilon = n\left(-\frac{e\varphi}{T^2}\delta T - \frac{P}{T}\delta\tilde{V}\right). \qquad (24)$$

Equation of state (8) gives relationship between density and temperature (9). I.e.

$$\int \delta f_a d^3\upsilon = n\left(-\frac{-e\varphi}{T}(\gamma - 1)\frac{1}{n}\delta n + \tilde{V}\delta n\right), \qquad (25)$$

that is why from (11) and (25) we have for electrons

$$\int \delta f_e d^3\upsilon = \gamma\delta n_e . \qquad (26)$$

Here we used potentiality (longitudinal) of motion, that can be interpreted as presence of only one degree of freedom $l = 1$, i.e. on a standard formula for ideal gas

$$\gamma = \frac{c_P}{c_V} = \frac{l+2}{l} = 3 . \qquad (27)$$

On the other hand, carrying out standard linearization of the Vlasov equation we have small deviation of distribution function from the equilibrium form due to the change of the field

$$\frac{\partial \delta f_a}{\partial t} + \vec{\upsilon}\nabla\delta f_a + e_a\vec{E}\frac{\partial f_{a0}}{\partial \vec{p}} = 0 , \qquad (28)$$

whence after the Fourier transformations we will get

$$\delta f_{a\omega\vec{k}} = ie_a \frac{\vec{E}_{\omega\vec{k}}\frac{\partial f_{a0}}{\partial \vec{p}}}{\omega - \vec{k}\vec{\upsilon}} . \qquad (29)$$

For frequencies of ionic sound, small for electronic and large for ionic component of plasma, we will put in the Poisson equation (5) expressions (22) for ionic and (26) for electronic densities accordingly

$$\Delta\delta\varphi = -4\pi e(\int \delta f_i d^3\upsilon - \frac{1}{\gamma}\int \delta f_e d^3\upsilon) . \qquad (30)$$

We will define dielectric function ε by standard relation appearance

$$div\varepsilon\vec{E} = 0 . \qquad (31)$$

We will write down dielectric function of plasma for frequencies of ionic sound proceeding from (30)

$$\varepsilon\left(\vec{k},\omega\right) = 1 + \frac{4\pi}{k^2}\int d^3\upsilon(e_i^2\frac{\vec{k}\frac{\partial f_{i0}}{\partial \vec{p}}}{\omega - \vec{k}\vec{\upsilon}} + \frac{1}{\gamma}e_e^2\frac{\vec{k}\frac{\partial f_{e0}}{\partial \vec{p}}}{\omega - \vec{k}\vec{\upsilon}}) . \qquad (32)$$

The wave solutions appear at $\varepsilon = 0$, then neglecting decreasing, we have

$$1 + \frac{\Omega_e^2}{k^2 u_{Se}^2} - \frac{\Omega_i^2}{\omega^2}\left(1 + \frac{3k^2 u_{Ti}^2}{\omega^2}\right) = 0, \qquad (33)$$

where $u_{Se}^2 = \frac{\gamma T_e}{m_e}$, that in main approximation gives

$$\omega \approx k u_{Se}\frac{\Omega_i}{\sqrt{k^2 u_{Se}^2 + \Omega_e^2}}\left(1 + \frac{3T_i}{T_e}\right) \approx k\sqrt{\frac{\gamma T_e + 3T_i}{M}}. \qquad (34)$$

This result coincides with the formal result of calculation of velocity in a hydrodynamic theory [3], because $\gamma = 3$, however we did not suppose the adiabatic state equation for heavy particles (ions) explained by nothing.

In addition, ignoring low temperature ionic component, we can consider the Debye screening for $\omega \to 0$. From (32) we have

$$\varepsilon\left(\vec{k},\omega=0\right)=1+\frac{4\pi}{k^2}\int d^3\upsilon\left(\frac{1}{\gamma}e_e^2\frac{\vec{k}\frac{\partial f_{e0}}{\partial\vec{p}}}{-\vec{k}\vec{\upsilon}}\right)=1+\frac{1}{k^2r_{DSe}^2}. \quad (35)$$

Whence the radius of screening of the potential field of external charge coincides with (15).

3. CONCLUSIONS

Processes in plasma (low-frequency (adiabatic) ones for rapid particles - electrons and high-frequency ones for slow – ions) are studied . The solutions in the form of ion-sound waves in supposition, that it is an adiabatic process for rapid particles (electrons) and isothermal one for slow particles (ions) are obtained. Proceeding from the Boltsmann distribution the solutions in the form of adiabatic ion-sound waves, the adiabatic radius of screening of the electric field and, similarly, adiabatic Jeans wave-length for the gravitating system are obtained. Due to kinetic description dispersion equation for adiabatic ion-sound waves is defined more precisely.

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REFERENCES

1. *Electrodynamika plasmy*. /Ed. by A.I. Akhiezer. Moscow: "Nauka", 1974, (in Russian).
2. A.Y. Wong, N.D' Angelo, R.W. Motley // *Phys. Rev. Lett.* 1962, v. 9, p. 415.
3. B.A. Trubnikov. *Theoriya plasmy*. Moscow: "Nauka", 1986, (in Russian).
4. L.D. Landau, E.M. Lifshyts. *Teoreticheskaya fizika*. Moscow: "Nauka", 1976, v. 5 (in Russian).
5. E.M. Lifshyts, L.P. Pitaevskiy. *Fizicheskaya kinetika*. Moscow: "Nauka", 1979 (in Russian).

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АДИАБАТИЧЕСКИЕ ИОННО-ЗВУКОВЫЕ ВОЛНЫ

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Рассмотрено низкочастотное потенциальное электрическое поле в плазме. Получены решения в виде ионно-звуковых волн в предположении, что это адиабатический процесс для быстрых частиц (электронов) и изотермический для медленных (ионов). Исходя из распределения Больцмана получен адиабатический радиус Дебая и, аналогично ему, адиабатическая длина волны Джинса для гравитирующей системы. Благодаря кинетическому описанию уточнено дисперсионное уравнение для адиабатических ионно-звуковых волн.

АДІАБАТИЧНІ ІОННО-ЗВУКОВІ ХВИЛІ

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Розглянуто низькочастотне потенційне електричне поле у плазмі. Одержані розв'язки у вигляді іонно-звукових хвиль за припущенням, що це адіабатичний процес для швидких частинок (електронів) й ізотермічний для повільних (іонів). Виходячи з розподілу Больцмана отримано адіабатичний радіус Дебая і, аналогічно йому, адіабатичну довжину хвилі Джинса для гравітуючої системи. Завдяки кінетичному опису уточнено дисперсійне рівняння для адіабатичних іонно-звукових хвиль.