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Graph Analysis of Underground Transport Networks

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Анализ графов транспортных подземных сетей

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Аналіз графів транспортних підземних мереж

The methodic of city subway networks analysis on the basis of graph characteristics (centrality, connectivity and shape) is proposed. The subways characteristics were calculated from subways indexes (number of lines, number of stations, length of lines in kilometers, ridership per year) and from indicators of cities urbanization (area and population). The interrelation between graph (road) structures and weights of their edges, and between π -index describing the shape of the graph and the number of passengers is demonstrated. It is shown on a practical example that the analysis of structure of proposed road network graphs can be useful in determining the sequence of new roads construction. Clustering of underground transport networks based on characteristics of network graph structure was performed for the first time.
Keywords: graph analysis, transport network, underground, GIS

Предложена методика анализа городских сетей метрополитенов на основе характеристик графов (центральность, связность и форма). Значения характеристик рассчитаны на основе индексов метрополитенов (количество линий, количество станций, протяженность линий в километрах, пассажиропоток в год) и показателей урбанизации городов (площадь и численности населения). Показана взаимосвязь между структурой графов транспортных сетей, весом их дуг и π -индексом для описания формы графа и количества пассажиров. На практическом примере показано, что анализ структуры представленных графов транспортных сетей может использоваться для определения последовательности этапов строительства новых линий. Впервые выполнена кластеризация транспортных подземных сетей на основе характеристик структуры графа сети.
Ключевые слова: анализ графов, транспортные сети, метрополитен, ГИС

Запропоновано методику аналізу міських мереж метрополітенів на основі характеристик графів (центральність, зв'язність і форма). Значення характеристик розраховані на основі індексів метрополітенів (кількість ліній, кількість станцій, протяжність ліній в кілометрах, пасажиропотік на рік) і показників урбанізації міст (площа та чисельність населення). Продемонстровано взаємозв'язок між структурою графів транспортних мереж, вагою їх дуг і π -індексом для опису форми графа і кількості пасажирів. На практичному прикладі показано, що аналіз структури представлених графів транспортних мереж може використовуватися для визначення послідовності етапів будівництва нових ліній. Вперше виконано кластеризацію транспортних підземних мереж на основі характеристик структури графа мережі.
Ключові слова: аналіз графів, транспортні мережі, метрополітен, ГІС

Introduction

Researches on applying graph theory to analyze transportation networks have been carrying out since the 1960-s till the present days. The most famous works were published by David Levinson, Mike Batty, Paul Longley etc.

Analysis of transport networks graphs involves solving problems:

– forecasting and evaluating transport network growth [1, 2];

– studying the influence of transport network structure and topology on quantitative indicators of traffic flows [3];

– investigation of dependence of network structure from the size of cities ground transportation and urban structure [4];

– construction of dynamic models of urban systems using GIS technologies based on cellular automata, agent-based modeling and fractal analysis [5, 6, 7];

– investigation relationships between quantitative indicators of transport network structure and its performance, density and urban spatial pattern, and the trips distance for early solution of transport problems etc.

The up-to-date researches don't pay enough attention to network bandwidth analysis depending on the parameters of structure of the network graph. This aspect is investigated in the paper.

Basic Definitions

Graph $G(V, E)$ is a pair of two sets – non-empty set of nodes V and an assemblage E of unordered pairs of distinct elements of V set [8]: $G(V, E) = \langle V; E \rangle, v \neq 0, E \in V \times V$.

Pairs of the E assemblage are called as ribs. The number of nodes of a graph G is denoted as n and the number of edges – m $(n = n(G), m = m(G)) : n(G) = |V|, m(G) = |E|$, where $|V|, |E|$ – V, E cardinal number. Assume that v_1, v_2 – nodes, $e = (v_1, v_2)$ – edge between them.

Maximum distance for a given graph G is called as diameter:

$$D(G) = \max_{u,v \in G} d(u, v).$$

The set of nodes at the same distance g from the node v is called as tier: $D(v, g) = \{u \in V | d(v, u) = g\}$. The distance matrix $(d_{i,j}), i, j = 1,2,...,n$ of the G graph is defined as: $d_{i,j} = d(v_i, v_j)$.

Characteristics of Graph Structure

The principle characteristics of graph structure are its centrality, connectivity and shape. Centrality characterizes the positionality of graph nodes. Absolute index S_i of the v_i node accessibility is the sum of distances from this node to other nodes [9, 10]: $S_i = \sum_j d_{ij}$.

The v_{i^*} node is called a central if it possesses the smallest absolute value of reachability index $S_{i^*} = \min_{1 \le i \le n} S_i$.

The König's number K_j is also the absolute index of the node v_i reachability:

$K_i = \max_{1 \le j \le n} d_{ij}$.

The central v_{i^*} node possesses the least König's number: $K_{j^*} = \min_{1 \le i \le n} K_i$.

The degree of deviation of the i-th node from the central one: $\eta_i = \frac{S_i}{S_{i*}}$.

The index of hierarchy $y_i = d(v_i, v_{i^*})$ shows the topological distance from the central node. Nodes with the same values of index of hierarchy form a tier. Centrality is useful for analysis of centers location of local or regional entities, transportation nodes. A measure of centrality on the set of centralities $\{S_i\}$ of graph nodes is integration of:

$$S = \frac{1}{2}\sum_{i,j} d_{ij} = \frac{1}{2}\sum_i S_i .$$

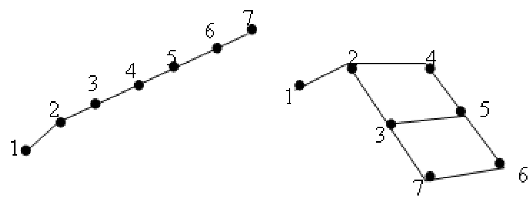
The S_{i^*} index of centrality of the center node defines the graph unipolarity:

$$S_{i^*} = \min_{1 \le i \le n} S_i .$$

The variance on a set of central nodes describes the graph centralization:

$$H = \sum_i (S_i - S_{i*}) = 2S - nS_{i*} .$$

The territory of the city consists of finite sets of objects: the set of enterprises, buildings, roads, etc. The essence of the territorial community consists in existing mathematical relationships between these objects. The configuration of line-node structure of territory objects placement and relationship is modeled using graphs [11]. For example, structures simulated by graphs in Fig. 1 describe three stages of territory development: initial (a), medium (b), ripe (c) on a qualitative level.



a) b) c)

Figure 1 – Graphs model for three stages of the territory development: initial (a), medium (b) and ripe (c)

Connected graph is modeling the structure of urban subway and road network. The graph is connected if there is a path from any node to any other.

The graph connectivity parameters characterize it intenseness with ribs and degree of triangulation. The best known of them are three connectivity parameters – α, β, φ -indexes:

$$\alpha = \frac{m - n + k}{2n - 5}; \qquad 0 < \alpha < 1; \tag{1}$$

$$\beta = \frac{m}{n}; \qquad 0 \le \beta \le 3; \tag{2}$$

$$\varphi = \frac{m}{3(n-2)}; \qquad 0 \le \varphi \le 1, \tag{3}$$

where m, n, k are the number of edges, nodes and graph connected components respectively.

To calculate the graph shape parameters the matrix $M = (\mu_{ij}), i = 1,2,...,n, j = 1,2,...,n-1$ of ordinal vicinity can be used:

$$\mu_{ij} = \{\text{the number of vicinal vertices of the j-th order for the node } v_i\},$$

where vicinal nodes of the j-th order for node v_i are nodes of the $D(v_{i,j})$ tier (neighbors of the 1-st order – adjacent nodes). For example, the sequence neighborhood matrixes for graphs from Fig. 1(a), Fig. 1(b), Fig. 1(c) are represented in Fig. 2.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 & 0 \\ 2 & 2 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 & 0 \\ 2 & 2 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 & 0 \\ 3 & 3 & 0 & 0 & 0 & 0 \\ 2 & 3 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 & 0 \\ 2 & 2 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 3 & 3 & 1 & 0 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 & 0 \end{bmatrix}$$

a) b) c)

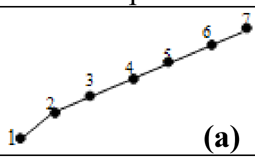
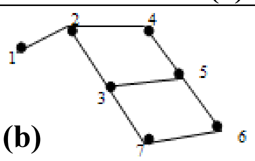
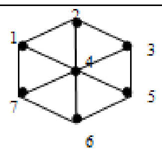
Figure 2 – Sequence neighborhood matrix for graphs from Fig. 1

If two graphs possess same number of nodes the more compact of them is the one having more zero columns in the M matrix. The graph on Fig.1(c) is more compact than the graphs in Fig.1(a) and Fig.1(b) (2 times and 4/3 times, respectively).

The ratio of the graph edges total length P to its diameter D determines the shape of the graph described by π-index: $\pi = P / D$.

Table 1 present values of the observed parameters for 3 graph model shown in Fig. 1.

Table 1 – Parameters of graphs structures shown in Fig. 1.

Graph	Centrality					
 (a)	$S_1=S_7=21$ $S_2=S_6=16$ $S_3=S_5=13$ $S_4=12$ $i^*=4$	$K_1=K_7=6$ $K_2=K_6=5$ $K_3=K_5=4$ $K_4=3$	$\eta_1=\eta_7=1,8$ $\eta_2=\eta_6=1,3$ $\eta_3=\eta_5=1,0$ $\eta_4=1$	$\gamma_1=\gamma_7=3$ $\gamma_2=\gamma_6=2$ $\gamma_3=\gamma_5=1$ $\gamma_4=0$	$S=56$ $S_{i^*}=12$	$H=28$
 (b)	$S_1=15$ $S_2=S6=13$ $S_3=9$, $S_4=14$ $S_5=10$ $S_7=12$ $i^*=3$	$K_1=K_2=4$ $K_4=K_6=4$ $K_5=K_7=3$ $K_2=2$	$\eta_1=1,7$ $\eta_2=\eta_6=1,4$ $\eta_3=1$ $\eta_4=1,6$ $\eta_5=1,1$ $\eta_7=1,3$	$\gamma_1=\gamma_4=2$ $\gamma_6=2$ $\gamma_2=\gamma_5=1$ $\gamma_7=1$ $\gamma_4=0$	$S=43$ $S_{i^*}=9$	$H=23$
 (c)	$S_1=S_2=9$ $S_3=S_5=9$ $S_6=S_7=9$ $S_4=6$ $i^*=4$	$K_1=K_2=2$ $K_3=K_5=2$ $K6=K_7=2$ $K_4=1$	$\eta_1=\eta_2=1,5$ $\eta_3=\eta_5=1,5$ $\eta_6=\eta_7=1,5$ $\eta_4=1,6$	$\gamma_1=\gamma_2=1$ $\gamma_3=\gamma_5=1$ $\gamma_6=\gamma_7=1$ $\gamma_4=0$	$S=30$ $S_{i^*}=6$	$H=18$
Connectivity						
(a)	$\alpha = 0$			$\beta = 0,9$	$\varphi = 0,4$	
(b)	$\alpha = 0,2$			$\beta = 1,1$	$\varphi = 0,5$	
(c)	$\alpha = 0,7$			$\beta = 1,7$	$\varphi = 0,8$	
Shape						
(a)	$Q = 0$			$\pi = 1$		
(b)	$Q = 2$			$\pi = 2$		
(c)	$Q = 3$			$\pi = 6$		

For attributed graph with weighted edges the diameter of the graph and the length of its edges can be expressed not only topologically (as the number of edges), but also metrically (through the attributes of edges).

The Examples of Analysis of Subway Graphs Network Characteristics

Consider an example demonstrating the usefulness of analysis of the graph structure for spatially referenced objects [10]. Given network of roads is represented as graph in Fig. 3. Each edge of the graph is the road, each node – the station (point) or a node of roads intersection. External road (indicated by arrows) were not take into account because of their secondary importance. Assume that 7 new roads indicated by dashed lines in Fig. 3 were designed.

Which of these roads will have the greatest impact on the availability of one point with respect to another within the territory? How will the appearance of each new road impact on the relative availability of individual points within the network? To answer these questions, the parameters of centrality of the original graph (with no new roads) and graphs (a) – (f) in Fig. 3 were calculated. The calculated parameters are presented in Table 2.

It is evident from the table that each new road improves communication between settlements (reduces average path length in the network of roads), but their effectiveness varies.

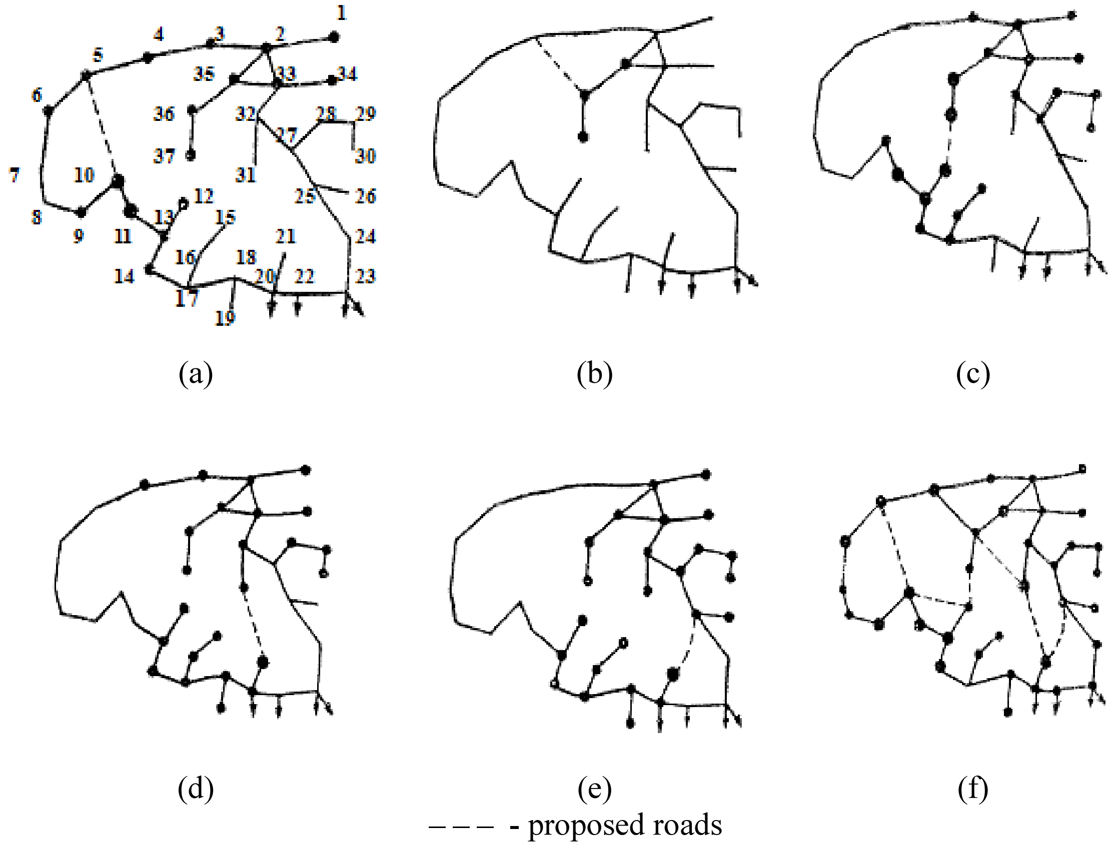


Figure 3 – The impact of new roads in the network on the relative availability of settlements (dots highlights items that will benefit from the construction of these roads, the size of dots indicates the degree of winning)

Table 2 – The parameters of graphs modeling road networks

Paramerets	Roads network graphs (according to Fig. 3)						
	Initial	a	b	c	d	e	f
The average length of the path in the network	242	220	238	216	217	221	174
S_{i*}	190	178	190	168	156	164	122
S	4359	3961	4283	3895	3920	3982	3125
The place-preference (according to S)	9	4	6	2	3	5	1
H	1688	1336	1536	1574	2068	1896	1736
% of decrease of the average path length in the network	0	9,1	1,7	10,6	10,0	8,6	28,3
Nodes v_i beneficial to the new road with the degree Δ_i	-	Δ_{10}=70 Δ_{11}=58 Δ_5=52 Δ_4=46 Δ_{13}=46	Δ_{36}=33 Δ_{37}=33 Δ_{35}=10	Δ_{37}=105 Δ_{12}=104 Δ_{36}=74 Δ_{13}=69 Δ_{11}=52	Δ_{21}=80 Δ_{31}=65 Δ_{20}=43 Δ_{18}=40 Δ_{19}=40	Δ_{21}=74 Δ_{20}=36 Δ_{19}=34 Δ_{18}=34	Δ_{37}=146 Δ_{12}=141 Δ_{36}=126 Δ_{21}=122 Δ_{31}=106

The most effective road is (f) as it reduces the average length of a path in the network on 10.6%, the least effective – (b). Simultaneous creation of new roads (a) - (f), shown by the graph in Fig. 3 (f), reduces the average path length of the network on 28.3%. The benefit taken for settlements from building the new roads is determined by changes in the average path length for each locality. For the (a) road construction the most interested are the 10-th, 11-th and 5-th settlements, in road (b) - the 36-th, 37-th, in road (c) – the 37-th, 12-th, 36-th, in road (d) – the 21-th, 31-th, 20-th, for the (e) road – the 21-th, 20-th settlements. The costs given by settlements for building of new roads can be distributed in proportion to the winning, defined by parameter Δ_i ($\Delta_i = S_i \text{ node}_{\text{initial graph}} - S_i$) in Table 2.

Calculation of Subway Indexes

In the other example the subway network graphs structure for some European cities is analyzed (Table 3). All subway schemes and cities subway statistics are available through [12, 13] (Table 4, 5, 6). The number of stations X_4 includes transfer stations taken from [14]. The number of edges and nodes m, n of subway network graphs are calculated by the scheme taking into account the fact that several transfer stations (from various subway lines) create one node of the graph.

Correlation coefficients for graph connectivity characteristics (α, β, π indexes – alfa, beta, P_i, PID) and input indexes $X_1, X_2, ..., X_6$ is the highest for the π-indexes and the lowest for φ-indexes – f_i. This means that the π - indices are informative characteristic for analysis of subways networks graphs (Table 7).

The Paris, Moscow, Prague, Kiev and Saint Petersburg are characterized by the largest values of passenger-to-kilometers (kilometers) ratio, which is especially evident from fig.4(b) in auxiliary scale. Only Prague and Kiev have in common the largest values of passenger / population ratio (this fact may be caused by the large number of tourists) (fig.4(a)).

Table 3 – Subway schemes of European cities

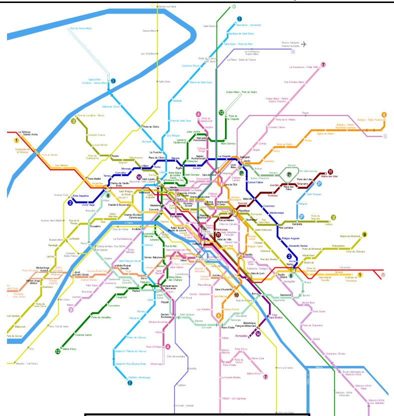
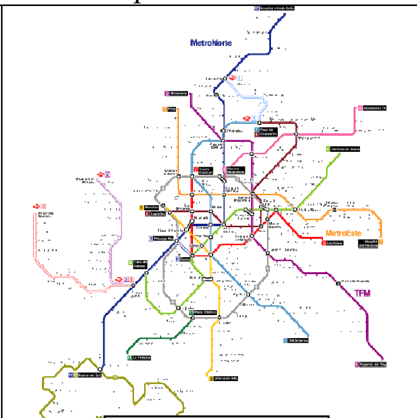
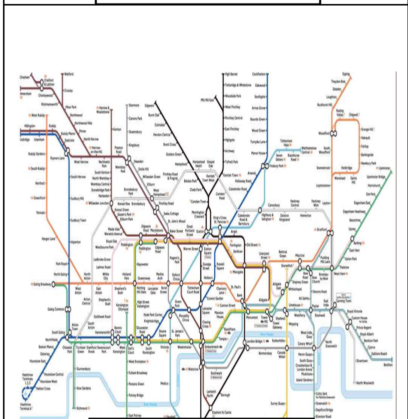
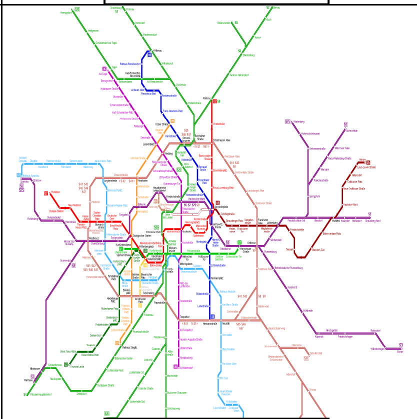
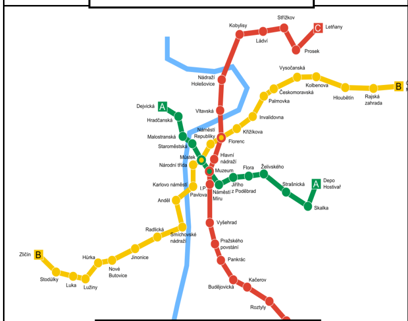
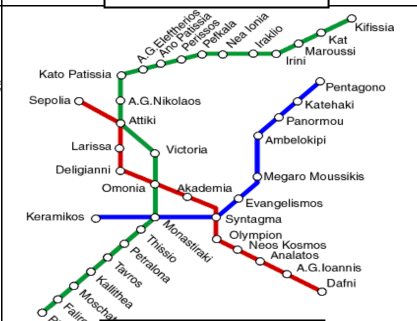
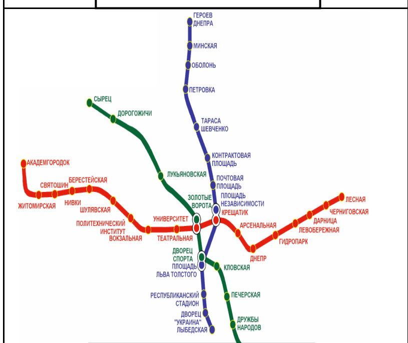
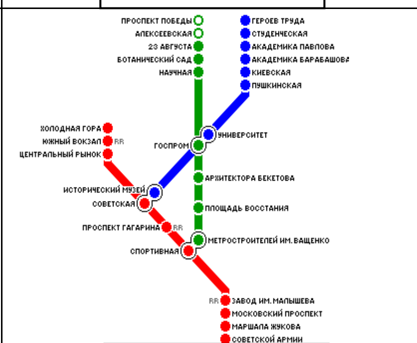
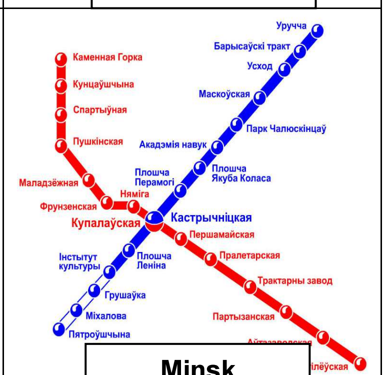
 Paris	 Madrid	 Moscow
 London	 Berlin	 Saint Petersburg
 Prague	 Athens	 Budapest
 Kiev	 Kharkov	 Minsk

Table 4 – Input data for subways

№	City (subway location)	Length P, km	Population (million)	Ridership per year (million)	Number of stations	Urban area, km^2	Number of lines
		X_1	X_2	X_3	X_4	X_5	X_6
1	**Athens**	**78,3**	**3,50**	**407**	**64**	**583**	**3**
2	**Berlin**	**146,11**	**3,95**	**507**	**196**	**1347**	**9**
3	**Budapest**	**32,3**	**1,73**	**171**	**42**	**894**	**3**
4	**Warsaw**	**21,7**	**1,71**	**139**	**21**	**544**	**1**
5	**Hamburg**	**112,9**	**1,79**	**209**	**108**	**712**	**4**
6	**Dnepropetrovsk**	**7,1**	**1,00**	**8**	**6**	**324**	**1**
7	**Kiev**	**65,9**	**2,81**	**527**	**51**	**544**	**3**
8	**London**	**470**	**9,40**	**1171**	**381**	**1623**	**11**
9	**Madrid**	**224,5**	**5,90**	**602**	**288**	**1321**	**13**
10	**Minsk**	**35,4**	**1,85**	**281**	**28**	**324**	**2**
11	**Moscow**	**313,1**	**15,50**	**2464**	**188**	**4403**	**12**
12	**Paris**	**217,3**	**10,36**	**1524**	**383**	**2845**	**16**
13	**Prague**	**59,4**	**1,27**	**589**	**57**	**285**	**3**
14	**Saint Petersburg**	**113**	**4,84**	**784**	**67**	**1191**	**5**
15	**Kharkov**	**39,3**	**1,45**	**239**	**29**	**466**	**3**

Table 5 – Subway characteristics

№	City (subway location)	Edges weights	Node weights	Frequency of travels	Areal weights	Areal density of lines	Linear density of stations	Areal density of stations
		X_3/X_1	X_3/X_4	X_3/X_2	X_3/X_5*10^3	X_5/X_1	X_1/X_4	X_5/X_4
1	Athens	5,2	6,4	116,3	698,1	7,4	1,2	9,1
2	Berlin	3,5	2,6	128,4	376,6	9,2	0,7	6,9
3	Budapest	5,3	4,1	98,6	190,7	27,7	0,8	21,3
4	Warsaw	6,4	6,6	81,4	255,9	25,1	1,0	25,9
5	Hamburg	1,9	1,9	116,8	293,5	6,3	1,0	6,6
6	Dnepropetrovsk	1,1	1,4	8,1	25,0	45,6	1,2	54,0
7	Kiev	8,0	10,3	187,8	968,2	8,3	1,3	10,7
8	London	2,5	3,1	124,6	721,5	3,5	1,2	4,3
9	Madrid	2,7	2,1	101,9	455,3	5,9	0,8	4,6
10	Minsk	7,9	10,0	152,3	868,2	9,2	1,3	11,6
11	Moscow	7,9	13,1	159,0	559,6	14,1	1,7	23,4
12	Paris	7,0	4,0	147,2	535,7	13,1	0,6	7,4
13	Prague	9,9	10,3	465,8	2067,4	4,8	1,0	5,0
14	Saint Petersburg	6,9	11,7	162,0	658,2	10,5	1,7	17,8
15	Kharkov	6,1	8,3	165,0	513,5	11,9	1,4	16,1

Table 6 – Subway graphs characteristics

№	City (subway location)	m	n	P	D	DM (D, km)	alfa	beta	fi	Pi (P/D)	PID (X_1/DM)
1	Athens	61	60	61	23	35,0	0,02	1,02	0,35	2,65	2,24
2	Berlin	187	177	187	39	31,8	0,03	1,06	0,36	4,79	4,60
3	Budapest	39	40	39	19	17,3	0,00	0,98	0,34	2,05	1,87
4	Warsaw	20	21	20	20	21,7	0,00	0,95	0,35	1,00	1,00
5	Hamburg	104	97	104	45	55,8	0,04	1,07	0,36	2,31	2,02
6	Dnepropetrovsk	5	6	5	5	7,1	0,00	0,83	0,42	1,00	1,00
7	Kiev	48	48	48	17	23,9	0,01	1,00	0,35	2,82	2,76
8	London	370	316	370	59	74,0	0,09	1,17	0,39	6,27	6,35
9	Madrid	276	237	276	32	40,6	0,09	1,16	0,39	8,63	5,53
10	Minsk	26	27	26	13	18,1	0,00	0,96	0,35	2,00	1,96
11	Moscow	176	152	176	24	45,1	0,08	1,16	0,39	7,33	6,94
12	Paris	367	307	367	37	24,3	0,10	1,20	0,40	9,92	8,94
13	Prague	54	54	54	23	25,6	0,01	1,00	0,35	2,35	2,32
14	Saint Petersburg	62	59	62	18	30,1	0,04	1,05	0,36	3,44	3,75
15	Kharkov	26	26	26	12	17,3	0,02	1,00	0,36	2,17	2,27

Table 7 – Correlation coefficients for subway graphs characteristics

	X1	X2	X3	X4	X5	X6	alfa	beta	f_i	P_i	PID
X_1	1										
X_2	0,8	1									
X_3	0,7	1,0	1								
X_4	0,8	0,8	0,6	1							
X_5	0,7	1,0	0,9	0,7	1						
X_6	0,8	0,8	0,8	1,0	0,8	1					
alfa	0,9	0,8	0,8	0,9	0,8	0,9	1				
beta	0,8	0,8	0,7	0,9	0,7	0,9	0,9	1			
f_i	0,5	0,5	0,4	0,6	0,5	0,6	0,6	0,3	1		
P_i	0,8	0,8	0,8	1,0	0,8	1,0	0,9	0,9	0,6	1	
PID	0,8	0,9	0,8	0,9	0,8	1,0	0,9	0,9	0,5	1,0	1

Does the ridership per year depend on parameters of the graph? According to Fig. 5, the P_i parameter of the Paris and Madrid subway network graphs allows to suggest about the possibility to increase ridership in these cities in comparison with the observed situation. Ridership per year in Moscow, Saint Petersburg and Prague are optimal with respect to P_i parameter. At the same time the difference (Fig. 6) of $P_i = \frac{P}{D}$ (calculated from the graph) and $PID = \frac{X_1}{DM}$ (calculated from the length of subway lines in km) parameters is the largest for Madrid and Paris. Paris and Madrid subway networks may have the largest passenger traffic (larger than in Moscow).

If we compare the subway networks to indicate how the total length of subway lines (X_1) matches to π-index (Fig. 7), we will conclude that the London network is the longest and possesses the lower π-index value than the Paris network (whose length is two times shorter).

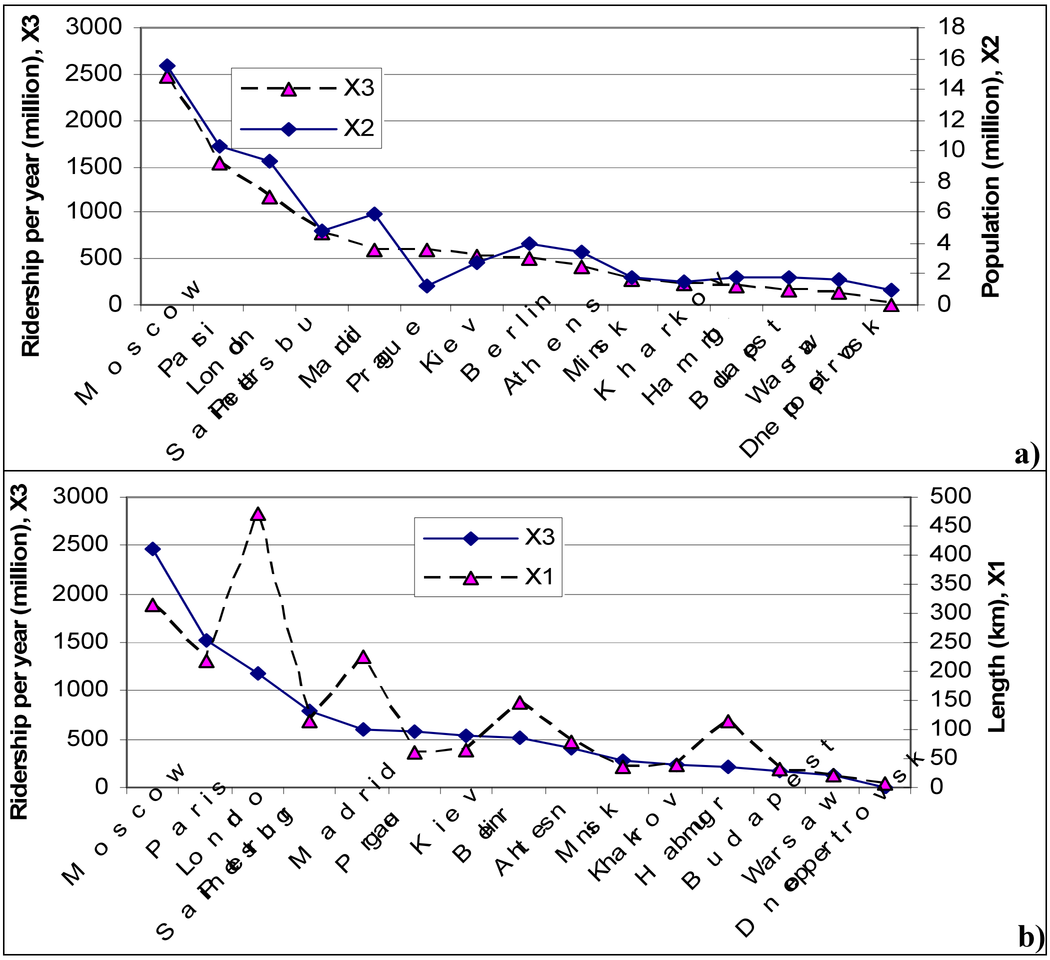


Figure 4 – Plots for subway characteristics of European cities: a) ridership per year (millions) and population (millions); b) ridership per year (millions) and length (km)

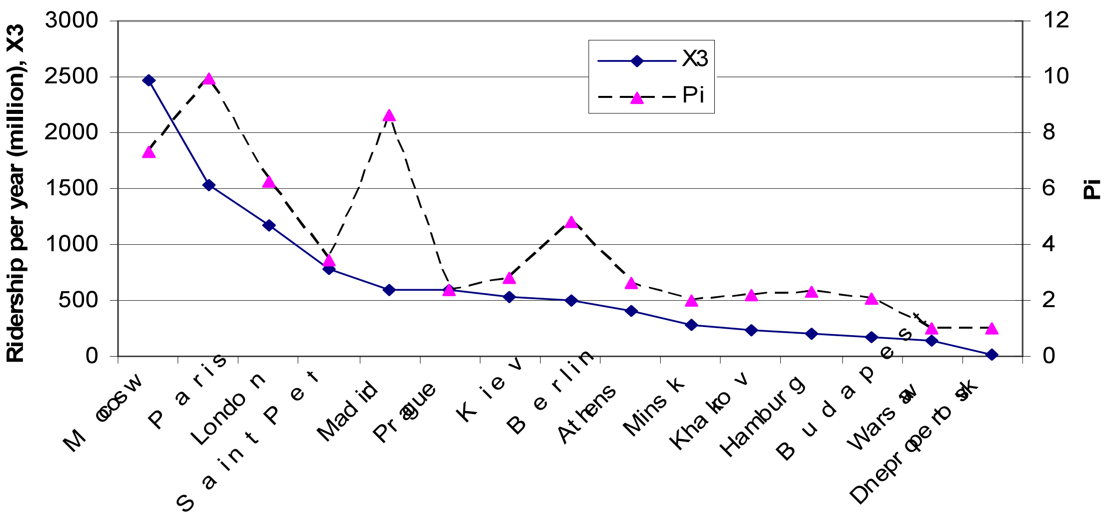


Figure 5 – Plots of ridership and π-index values

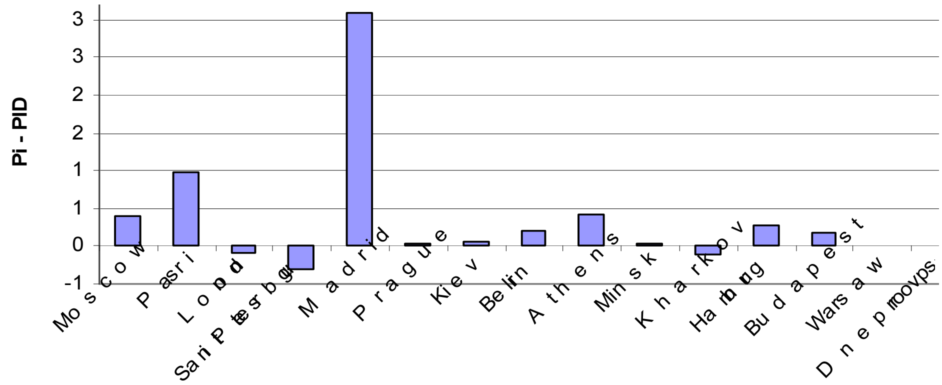


Figure 6 – Differences of P_i and PID indexes

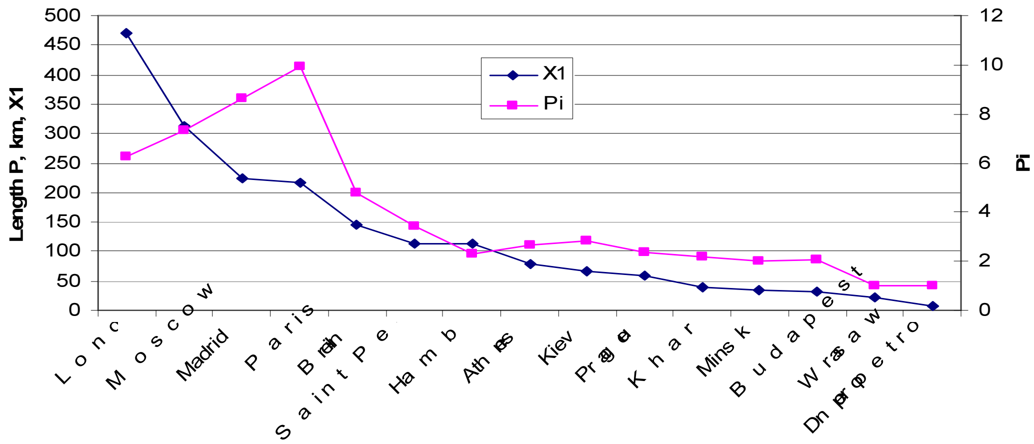


Figure 7 – Plots of Length P (km) and π-index values

Clustering of examined networks based on graph characteristics (α, β, π – indexes) into K clusters ($K = 2, 3, 4$) using the k-means method highlights the next clusters (Table 8).

Table 8 – Clustering of subway networks based on graph characteristics

City (subway location)	Clustering on the basis of α, β, π – indexes			Clustering on the basis of X_1, X_2, X_3, X_4, X_5 characteristics		
	K=2	K=3	K=4	K=2	K=3	K=4
Athens	2	2	2	2	3	2
Berlin	2	2	4	2	3	3
Budapest	2	2	2	2	2	2
Warsaw	2	3	3	2	2	2
Hamburg	2	2	2	2	2	2
Dnepropetrovsk	2	3	3	2	2	2
Kiev	2	2	2	2	2	2
London	1	1	1	1	1	1
Madrid	1	1	1	2	3	3
Minsk	2	2	2	2	2	2
Moscow	1	1	1	1	1	4
Paris	1	1	1	1	1	1
Prague	2	2	2	2	2	2
Saint Petersburg	2	2	4	2	3	2
Kharkov	2	2	2	2	2	2

The peculiarities of obtained clusters:

- (London, Madrid, Moscow, Paris) are characterized by the highest values of the α, β, π parameters;
- for (Berlin, Saint Petersburg) the values of parameters are above average;
- for (Athens, Budapest, Hamburg, Kiev, Minsk, Prague, Kharkov) the values of α, β, π parameters are close to average;
- (Warsaw, Dnepropetrovsk) are characterized by the lowest values of α, β, π parameters.

Cities clustering on the basis of input characteristics (X_1, X_2, X_4, X_5) does not include Madrid, Moscow or Warsaw, Dnepropetrovsk into the same cluster. So the structure of cluster of subway network graphs based on α, β, π indexes is not the same as the structure of clusters based on input data X_1, X_2, X_3, X_4, X_5.

Conclusions

The methodic for city subway networks analysis on the basis of graph characteristics (centrality, connectivity and shape) is proposed. The subways characteristics are calculated from values of subways indexes (number of lines, number of stations, length in kilometers, ridership per year) and from indicators of cities urbanization (area and population). A relationship between graphs (roads) structure and weights of their edges, between π-index describing the shape of the graph and the number of passengers is demonstrated. It is shown on a practical example that the analysis of structure of proposed road network graphs can be useful in determining the sequence of new roads construction. Clustering of underground transport networks based on characteristics of network graph structure was performed for the first time.

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RESUME

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Graph Analysis of Underground Transport Networks

Background: Researches on applying graph theory to analyze transportation networks have been carrying out since the 1960-s till the present days. The most famous works were published by David Levinson, Mike Batty, Paul Longley etc. Analysis of transport networks graphs involves solving problems: forecasting and evaluating transport network growth; studying the influence of transport network structure and topology on quantitative indicators of traffic flows; investigation of dependence of network structure from the size of cities ground transportation and urban structure; construction of dynamic models of urban systems using GIS technologies based on cellular automata, agent-based modeling and fractal analysis; investigation relationships between quantitative indicators of transport network structure and its performance, density and urban spatial pattern, and the trips distance for early solution of transport problems etc. The up-to-date researches don't pay enough attention to network bandwidth analysis depending on the parameters of structure of the network graph. This aspect is investigated in the paper.

Materials and methods: The subway network graphs structure for some European cities is analyzed using the methods of graph theory and clustering. The number of edges and nodes of subway network graphs were calculated by the scheme taking into account the fact that several transfer stations (from various subway lines) create one node of the graph. All subway schemes and cities subway statistics are available through Internet.

Results: It is observer that the ridership per year depends on parameters of the graph. In Paris and Madrid subway network graphs structure indexes allow to suggest about the possibility to increase ridership in these cities in comparison with the observed situation. Ridership per year in Moscow, Saint Petersburg and Prague are optimal. At the same time Paris and Madrid subway networks may have the largest passenger traffic (larger than in Moscow). Clustering of examined networks based on graph characteristics) into clusters highlights the next clusters: London, Madrid, Moscow, Paris are characterized by the highest values of the graph connectivity parameters; for Berlin, Saint Petersburg the values of parameters are above average; for Athens, Budapest, Hamburg, Kiev, Minsk, Prague, Kharkov the values of graph connectivity parameters are close to average; Warsaw, Dnepropetrovsk are characterized by the lowest values of parameters. The structure of cluster of subway network graphs based on indexes is not the same as the structure of clusters based on input data.

Conclusion: The methodic for city subway networks analysis on the basis of graph characteristics (centrality, connectivity and shape) is proposed. The subways characteristics are calculated from values of subways indexes (number of lines, number of stations, length in kilometers, ridership per year) and from indicators of cities urbanization (area and population). The relationship between graphs (roads) structure and weights of their edges, between π -index describing the shape of the graph and the number of passengers is demonstrated. It is shown on a practical example that the analysis of structure of proposed road network graphs can be useful in determining the sequence of new roads construction. Clustering of underground transport networks based on characteristics of network graph structure was performed for the first time.

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