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V.V. ULANSKY, I.A. MACHALIN

MATHEMATICAL MODELS FOR EVALUATION OF OPERATIONAL READINESS OF PERIODICALLY INSPECTED ELECTRONIC SYSTEMS

Анотація. *Розроблено математичні моделі та доведено теореми, що дозволяють визначити стаціонарний і нестаціонарний коефіцієнти оперативної готовності при довільному і експоненційному законах розподілу напрацювання системи до відмови. Доведені вирази, які, на відміну від відомих, враховують характеристики достовірності багаторазового контролю працездатності. Показано, що середній час напрацювання на незапланований ремонт системи багато в чому визначається ймовірністю помилкової відмови.*
Ключові слова: *системи авіоніки, технічне обслуговування, надійність, періодичний контроль працездатності, функція оперативної готовності, коефіцієнт оперативної готовності, помилки контролю.*

Аннотация. *Разработаны математические модели и доказаны теоремы, позволяющие определить стационарный и нестационарный коэффициенты оперативной готовности при произвольном и экспоненциальном законах распределения наработки системы до отказа. Доказанные выражения, в отличие от известных, учитывают характеристики достоверности многократного контроля работоспособности. Показано, что среднее время наработки на незапланированный ремонт системы во многом определяется вероятностью ложного отказа.*
Ключевые слова: *системы авионики, техническое обслуживание, надежность, периодический контроль работоспособности, функция оперативной готовности, коэффициент оперативной готовности, ошибки контроля.*

Abstract. *The mathematical models and theorems that identify stationary and non-stationary coefficients of operational readiness are developed for an arbitrary and exponential distribution of time to failure of avionics systems. Proven expressions, in contrast to all other known, take into account the trustworthiness characteristics of multiple inspections. It is shown that the mean time between unscheduled repairs of the system is largely determined by the probability of a false failure.*
Keywords: *avionics systems, maintenance operations, reliability, periodic control of working ability, function of operational readiness, coefficient of operational readiness, inspection errors.*

1. Introduction

Statement of the problem. The effectiveness of some aircraft electronic systems is largely determined by the operational readiness of the systems to perform their functions within a specified time at a certain range of flight operations. In this paper, the operational readiness is defined as the property of a given system to be available when it demanded and to operate within the tolerances for a specified period of time. The measures of the operational readiness of aircraft electronic systems determine the level of safety and regularity of the aircraft operations. One of the major challenges currently facing the airlines is the development of optimal maintenance programs of operated aircraft fleet. To optimize the maintenance program it is necessary to develop mathematical models for evaluating the measures of the operational readiness of electronic systems. The operational readiness can be characterized by different measures. The most widely used measures are the non-stationary and stationary operational readiness coefficients. The operational readiness coefficient is defined as the probability that the system will be in the operable state at any time, except for the scheduled periods during which the use of the system on the appointment is not required, and from that moment will work reliably within a specified interval of time. The non-stationary and stationary coefficients of operational readiness are considered, respectively, on a finite and an infinite interval of service planning.

Analysis of the recent research and publications. A number of papers, for example

[1–4], have been devoted to performance evaluation of operational readiness of technical systems. In these studies as an integral index of reliability and availability of systems the coefficient of operational readiness is proposed to use. For avionics systems, the values of this coefficient depend not only on the faultlessness and maintainability characteristics but also on the trustworthiness characteristics of the onboard built-in test equipment (BITE), since due to an error checking the adoption of erroneous decisions is possible. However, the well-known expressions do not include the trustworthiness indexes of multiple inspections, so do not provide an adequate assessment of the operational readiness of onboard electronic systems.

Thus, the aim of the article is to develop the mathematical models for determining the non-stationary and stationary coefficients of operational readiness taking into account the characteristics of trustworthiness of multiple inspections, faultlessness, maintainability, and spare part system sufficiency.

Problem statement and description of maintenance strategy. Currently, aircraft avionics meets the requirements of ARINC 700 [5]. Each avionics system is represented by a set of redundant and easily replaced blocks called the line replaceable units (LRUs). Each LRU is a single-block system consisting of several modules and having the BITE. The modular design of LRUs provides easy access to circuits and components for testing and replacement in case of failures. Each LRU operates till safety failure, which is recorded during the flight or at the base airport after landing the aircraft. Rejected LRUs are replaced in the base airport by the spare LRUs from the warehouse. Since all avionic systems are redundant, the failure of any LRU does not lead to the failure of the corresponding system. Therefore, this strategy is called the strategy till safety failure or breakdown maintenance strategy. It is necessary to construct a mathematical model that takes into account the main parameters of the exploitation process to evaluate the operational readiness measures.

For determining the operational readiness measures we will use the well-known property of the regenerative processes [6, 7], which consists in the fact that the fraction of time during which the system was in the state $E_\mu \,(\mu = \overline{1,r})$ equals the ratio of average time spent in the state E_μ per regeneration cycle to the average duration of the cycle.

Let us consider the maintenance model of an avionics system consisting of a single LRU, i.e., the system structure in terms of reliability is not considered. This is due to the fact that the LRU is the smallest removable onboard part of any system of avionics. The LRU operation process can be considered on a finite or an infinite time interval as a sequence of changing various LRU states. For definiteness, we first consider a finite interval of the planning service. Therefore, the LRU behavior in the range of service planning $(0,T)$ can be described by a stochastic process $L(t)$ with a finite space of states $E = \bigcup_i E_i$. The process $L(t)$ changes only stepwise, with each jump due to the transition of the LRU in one of possible states. It is assumed that a regenerative stochastic process, having the property will always return to the point of regeneration, from which further development of the process does not depend on its past behavior and is a copy of the probabilistic process that began at the moment t .

Suppose that at time $t = 0$ the operation of a new LRU begins and the periodic LRU inspections are planned in the moments $\tau, 2\tau, 3\tau, ... N\tau$, where $N = \frac{T}{\tau} - 1$ is the number of the LRU inspections on the $(0,T)$ range. Assume that the checking of the LRU is run before each aircraft departure. By the LRU checking results at the moment $k\tau \,(k = \overline{1,N})$ the following decisions are possible:

– to use the LRU until the next inspection, if it is recognized as operable;

– to repair the LRU, if it is recognized inoperable, and then allow its further use;

– to nominate next inspection moment of the LRU in the moment $(k+1)\tau$.

Let us define the random process $L(t)$. At any given time t the LRU can be in one of the following states [8–12]: E_1, if at the moment t the LRU is used as intended and is in the operable state; E_2, if at the moment t the LRU is used as intended and is in an inoperable state (latent failure); E_3, if at the moment t the LRU is not used for its intended purpose because of operability checking; E_4, if at the moment t the LRU is not used as intended and its removal is held from the board of an aircraft; E_5, if at the moment t defective LRU is in the state of unscheduled idle on the board of an aircraft at the base airport because of dissatisfaction with the application to a spare LRU from the warehouse; E_6, if at the moment t the LRU is not used for its intended purpose because the "false recovery" is performed; E_7, if at the moment t the LRU is not used for its intended purpose because of the "proper recovery"; E_8, if at the moment t the LRU is not used as intended and performed its installation on the board of an aircraft.

Let S_i be the time in the state $E_i (i=\overline{1,8})$. Obviously, S_i is a random variable with expected mean time $M[E_i]=MS_i$. Define Ξ as the time to failure. The uncertainty in the values that Ξ can take is described through a failure distribution function $F(\xi)$ which characterizes the probability of $P\{\Xi \le \xi\}$. The equations for $\overline{MS_1, MS_8}$ were published in [8, 12]. The average regeneration cycle of the LRU is determined by the formula:

$$MS_0 = \sum_{i=1}^{8} MS_i .$$

2. Non-stationary operational readiness coefficient

Let $P(k\tau,\theta)$ be the probability that the LRU will be operable at time $k\tau+\chi$ $\left(k=\overline{0,N}\right)$ and it will work faultlessly within a specified time θ starting from the moment $k\tau+\chi$, where $0<\chi\le\tau-\theta$, and θ is the time of the task. Suppose χ is a random variable with a uniform distribution on the interval from $k\tau$ to $(k+1)\tau-\theta$, and its distribution is described by the probability density function

$$f(x)=\frac{1}{\tau-\theta}. \tag{1}$$

Theorem 1. The following formula holds for the non-stationary function of operational readiness:

$$\begin{aligned} P(k\tau,\theta) == {} & \frac{1}{\tau-\theta}\int_{0}^{\tau-\theta}\int_{k\tau+x+\theta}^{\infty} P_{PO}\left(\overline{\tau,(k-1)\tau};k\tau|\vartheta\right)\omega(\vartheta)d\vartheta dx + \\ & + \frac{1}{\tau-\theta}\sum_{j=1}^{k} P_R(j\tau)\int_{0}^{\tau-\theta}\int_{(k-j)\tau+x+\theta}^{\infty} P_{PO}\left(\overline{\tau,(k-j-1)\tau};(k-\mathrm{j})\tau|\vartheta\right)\omega(\vartheta)d\vartheta dx , \end{aligned} \tag{2}$$

where $P_{PO}\left(\overline{\tau,(k-1)\tau};k\tau|\xi\right)$ is the conditional probability of the event “properly operable”, defined as the probability of co-occurrence of the following events: in the operability checking at the instants $\overline{\tau,k\tau}$ the LRU was recognized operable, on condition that $\Xi=\xi$ and $k\tau<\xi\le(k+1)\tau$; Ξ is the LRU random operation time to failure; $P_R(j\tau)$ is the probability of the LRU recovery at instant $j\tau$; $\omega(\xi)$ is the probability density function of the random variable Ξ.

Proof. The non-stationary function of operational readiness can be defined as the probability that the interval of a trouble-free LRU operation $[\chi, \chi+\theta]$ entirely falls within one of the intervals between inspections $(k\tau, (k+1)\tau)$, $k=\overline{0, N}$. It should be taken into account that at any of the moments $k\tau$ the LRU can be recovered (properly or falsely).

The LRU will work faultlessly in the interval $[k\tau+\chi, k\tau+\chi+\theta]$, if one of the following events occurs:

– the time to failure of the system is more than $k\tau+\chi+\theta$ and by inspection results at instants $\overline{\tau, k\tau}$ the LRU has been acknowledged as operational, i.e.,

$$\Delta_1 = (\Xi > k\tau+\chi+\theta) \bigcap \left(\bigcap_{i=1}^{k} \Xi_i^* > i\tau \right), \tag{3}$$

where Ξ_i^* is a random assessment of the operating time to failure Ξ by the checking results of the LRU at the time $i\tau$ [13];

– the last LRU recovery occurred at the moment $j\tau\left(j=\overline{1,k}\right)$, and then the LRU no longer failed. When checking the operability at the moments $\overline{(j+1)\tau, k\tau}$ the LRU was recognized operable, i.e.

$$\Delta_2 = \bigcup_{j=1}^{k} \left\{ B(j\tau) \bigcap (\Xi > (k-j)\tau+\chi+\theta) \bigcap \left(\bigcap_{i=j+1}^{k} \Xi_i^* > (i-j)\tau \right) \right\}, \tag{4}$$

where $B(j\tau)$ is the event consisting in the LRU recovery at the instant $j\tau$.

The probability of the LRU recovery is calculated as

$$P_R(j\tau) = P\{B(j\tau)\} = P_{FR}(j\tau) + P_{PR}(j\tau), \tag{5}$$

where $P_{FR}(j\tau)$ and $P_{PR}(j\tau)$ are, respectively, the probability of a false and proper LRU recovery [9, 10].

The probability of the event (3) we find by integrating the probability density $\omega_0\left(\xi; \overline{\xi_1^*, \xi_k^*}\right)$ of scalar random variables $\overline{\Xi_1^*, \Xi_k^*}, \Xi$ [13] within appropriate limits:

$$P(k\tau+\theta) = P(\Delta_1) = \frac{1}{\tau-\theta} \int_0^{\tau-\theta} \int_{k\tau+x+\theta}^{\infty} P_{PO}\left(\overline{\tau, (k-1)\tau}; k\tau | \vartheta\right) \omega(\vartheta) d\vartheta dx. \tag{6}$$

Using the addition theorem of probability and the probability density functions $\omega_0\left(\xi; \overline{\xi_1^*, \xi_k^*}\right)$ and (1), we find the probability of the event (4):

$$P(\Delta_2) = \sum_{j=1}^{k} P_R(j\tau) P(k\tau - j\tau + \theta), \tag{7}$$

where

$$P(k\tau - j\tau + \theta) = \frac{1}{\tau-\theta} \int_0^{\tau-\theta} \int_{(k-j)\tau+x+\theta}^{\infty} P_{PO}\left(\overline{\tau, (k-j-1)\tau}; (k-j)\tau | \vartheta\right) \omega(\vartheta) d\vartheta dx. \tag{8}$$

On the basis of the addition theorem probability for mutually exclusive events we can write:

$$P(k\tau, \theta) = P(\Delta_1) + P(\Delta_2). \tag{9}$$

Substituting expressions (6) and (7) into (9) gives (2). This proves the theorem.

Equation (2) can be simplified by setting $P_R(0)=1$. Then

$$P(k\tau,\theta)=\frac{1}{\tau-\theta}\sum_{j=0}^{k}P_R(j\tau)\int_0^{\tau-\theta}\int_{(k-j)\tau+x+\theta}^{\infty}P_{PO}\left(\overline{\tau,(k-j-1)\tau};(k-j)\tau|\vartheta\right)\omega(\vartheta). \quad (10)$$

Corollary 1. If the LRU has an exponential distribution of time to failure, then the following relations hold:

$$P(k\tau,\theta)=\frac{e^{-\lambda\theta}\left(1-e^{-\lambda(\tau-\theta)}\right)}{\lambda(\tau-\theta)}\sum_{j=0}^{k}P_R(j\tau)(1-\alpha)^{k-j}e^{-(k-j)\lambda\tau}\;, \quad (11)$$

$$P_{FR}(j\tau)=\alpha\sum_{\nu=0}^{j-1}P_R(\nu\tau)e^{-\lambda(j-\nu)\tau}(1-\alpha)^{j-\nu-1}, \quad (12)$$

$$P_{PR}(j\tau)=(1-\beta)\sum_{\nu=0}^{j-1}P_R(\nu\tau)\left[\sum_{\mu=\nu}^{j-1}\left(e^{-(\mu-\nu)\lambda\tau}-e^{-(\mu+1-\nu)\lambda\tau}\right)(1-\alpha)^{\mu-\nu}\beta^{j-1-\mu}\right], \quad (13)$$

where α and β are, respectively, the conditional probabilities of false rejection and undetected failure for the LRU operability checking. The proof of corollary is omitted because of its bulkiness.

Example 1. Calculate the non-stationary function of operational readiness, if the LRU has $MTBF=M[\Xi]=1/\lambda=10000\,\text{h}$, warranty service life $T=5000\,\text{h}$, average duration between the LRU operability checking $\tau=10\,\text{h}$, time of the task $\theta=1\,\text{h}$, conditional probability of “false rejection” and “undetected failure” during operability checking by the built-in test equipment $\alpha=\beta=0{,}005$.

Using (5) and (10)-(12), we calculate the values of the probabilities P_R, P_{FR}, P_{PR} and $P(k\tau,\theta)$. They are listed in Table 1.

As can be seen from Table 1, starting from the sixth checkout all probabilities reached stationary values. It should be noted that the probability of false recovery in five times as much the probability of proper recovery.

Table 1. Values of the calculated probabilities, depending on the number of checking operability

j	P_R	P_{FR}	P_{PR}	$P(k\tau,\theta)$
0	1,0	0	1,0	0,999400221605680
1	0,005989505164959	0,004995002499167	0,000994502665792	0,999395227102240
2	0,005994447745731	0,004994977536638	0,000999470209093	0,999395202154683
3	0,005994472433934	0,004994977411327	0,000999495021984	0,999395202030070
4	0,005994472557252	0,004994977411327	0,000999495145925	0,999395202029444
5	0,005994472557868	0,004994977411324	0,000999495146544	0,999395202029444
6	0,005994472557871	0,004994977411324	0,000999495146547	0,999395202029444
7	0,005994472557871	0,004994977411324	0,000999495146547	0,999395202029444
500	0,005994472557871	0,004994977411324	0,000999495146547	0,999395202029444

In general, the non-stationary coefficient of operational readiness is determined by averaging expression (10) on the interval $(0,T)$

$$K_{OR}(T,\theta)=\frac{1}{(N+1)(\tau-\theta)}\sum_{k=0}^{N}\sum_{j=0}^{k}P_R(j\tau)\int_0^{\tau-\theta}\int_{(k-j)\tau+x+\theta}^{\infty}P_{PO}\left(\overline{\tau,(k-j-1)\tau};(k-j)\tau|\vartheta\right)\omega(\vartheta)d\vartheta dx. \quad (14)$$

With an exponential distribution of time to failure we get from formula (14)

$$K_{OR}(T,\theta)=\frac{e^{-\lambda\theta}\left(1-e^{-\lambda(\tau-\theta)}\right)}{(N+1)\lambda(\tau-\theta)}\sum_{k=0}^{N}\sum_{j=0}^{k}P_R(j\tau)(1-\alpha)^{k-j}e^{-(k-j)\lambda\tau}\ . \qquad (15)$$

Example 2. Calculate the non-stationary coefficient of operational readiness for the initial data of Example 1.

By substituting the initial data in (15), we obtain $K_{OR}(T,\theta)=0{,}999395$.

3. Stationary operational readiness coefficient

The stationary coefficient of operational readiness is used in the case of infinite time of service planning $(0,\infty)$. The following theorem determines the stationary coefficient of operational readiness.

Theorem 2. For the stationary coefficient of operational readiness the following formula holds:

$$K_{OR}(\theta)=\frac{\tau/(\tau-\theta)}{MS_0-MS_3}\sum_{k=0}^{\infty}\int_{0}^{\tau-\theta}\int_{k\tau+x+\theta}^{\infty}P_{PO}\left(\overline{\tau,(k-1)\tau};k\tau|\vartheta\right)\omega(\vartheta)d\vartheta dx\,. \qquad (16)$$

Proof. Suppose as before that χ is a random variable with the uniform distribution in the interval between $k\tau$ and $(k+1)\tau$ and the probability density function determined by (1). To prove (16) we express the probability $P(k\tau,\theta)$ through the renewal density function, and then proceed to the limit

$$K_{OR}(\theta)=\lim_{k\to\infty}P(k\tau,\theta).$$

The probability $P(k\tau+\theta)$ is given by (6).

Since the LRU recovery is only possible at discrete moments of time $\tau,2\tau,\ldots$, the renewal density function can be expressed through the δ-function:

$$h(\eta)=\sum_{j=1}^{k}P_R(j\tau)\delta(\eta-j\tau). \qquad (17)$$

Using (17), expression (7) can be represented in the integral form:

$$P(\Delta_2)=\int_{0}^{k\tau}P(k\tau-\eta+\theta)h(\eta)d\eta.$$

On the basis of the addition theorem of probability for the mutually exclusive events we can write:

$$P(k\tau,\theta)=P(\Delta_1)+P(\Delta_2)=P(k\tau+\theta)+\int_{0}^{k\tau}P(k\tau-\eta+\theta)h(\eta)d\eta.$$

Since $P_{PO}\left(\overline{\tau,(k-1)\tau};k\tau|\xi\right)\le 1$, then $P(k\tau+\theta)\le 1-F(k\tau+\theta)$, so $\lim_{k\to\infty}P(k\tau+\theta)=0$.

Further, since the function $P(k\tau+\theta)$ is not negative, it is of limited variation on the semi-axis $(0,\infty)$ and it satisfies the inequality

$$\int_{0}^{\infty}P(y)dy\le\int_{0}^{\infty}[1-F(y)]dy<\infty,$$

then according to Smith's theorem in the case of lattice random variable [7] we have:

$$\lim_{k\to\infty}\int_0^{k\tau} P(k\tau-\eta+\theta)h(\eta)d\eta=\frac{\tau}{\mu}\sum_{k=0}^{\infty}P(k\tau+\theta) \qquad (18)$$

where μ is the average time between the LRU recovery.

Since for the calculation of the stationary coefficient of operational readiness the scheduled maintenance is not taken into account, then

$$\mu = MS_0 - MS_3. \qquad (19)$$

Substituting expressions (6) and (19) into (18) gives (16). The theorem is proved.

Corollary 2. If there are no errors at the operability checking, then

$$K_{OR}(\theta)=\frac{\tau/(\tau-\theta)\sum_{k=0}^{\infty}\int_0^{\tau-\theta}\int_{k\tau+x+\theta}^{\infty}\omega(\vartheta)d\vartheta dx}{\tau\sum_{k=0}^{\infty}(k+1)\{F[(k+1)\tau]-F(k\tau)\}+t_D+MS_5+t_{PR}+t_{FR}+t_M}, \qquad (20)$$

where MS_5 is given by

$$MS_5=\Psi(\Delta t_{SP}+t_M+t_D-t_C),\ \Psi=\begin{cases}0, & if \quad t_C\ge(\Delta t_{SP}+t_D+t_M);\\ 1, & if \quad t_C<(\Delta t_{SP}+t_D+t_M);\end{cases}$$

Δt_{SP} is the average delay time of an application for a spare LRU; t_M is the average installation time of the LRU on the board of an aircraft; t_D is average time of dismantling the LRU from the board of an aircraft; t_{PR} and t_{FR} is the average time of the LRU proper and false recovery respectively; Ψ – indicator function; t_C is the average scheduled time of technical parking of an aircraft.

Proof. In the absence of errors in the checks of operability the probability $P_{PO}\left(\overline{\tau,(k-1)\tau};k\tau|\xi\right)=1$, therefore

$$\sum_{k=0}^{\infty}\int_0^{\tau-\theta}\int_{k\tau+x+\theta}^{\infty}P_{PO}\left(\overline{\tau,(k-1)\tau};k\tau|\vartheta\right)\omega(\vartheta)d\vartheta dx=\sum_{k=0}^{\infty}\int_0^{\tau-\theta}\int_{k\tau+x+\theta}^{\infty}\omega(\vartheta)d\vartheta dx.$$

Furthermore, from [8, 12] follows that in the case of ideal operability checking

$$MS_1+MS_2=\tau\sum_{k=0}^{\infty}(k+1)\{F[(k+1)\tau]-F(k\tau)\}; \qquad (21)$$

$$\begin{aligned}MS_0-MS_3&=MS_1+MS_2+MS_4+MS_5+MS_6+MS_7+MS_8=\\&=\tau\sum_{k=0}^{\infty}(k+1)\{F[(k+1)\tau]-F(k\tau)\}+t_D+MS_5+t_{PR}+t_{FR}+t_M.\end{aligned} \qquad (22)$$

Substitution of (21) and (22) into (16) gives (20). The corollary is proved.

Corollary 3. If the LRU has an exponential distribution of time to failure, then the following formula holds:

$$K_{OR}(\theta)=\frac{\tau e^{-\lambda\theta}\left(1-e^{-\lambda(\tau-\theta)}\right)}{\lambda(\tau-\theta)\left[1-(1-\alpha)e^{-\lambda\tau}\right](MS_0-MS_3)}. \qquad (23)$$

Proof. With the exponential distribution law of time to failure we can get

$$\sum_{k=0}^{\infty}\int_{0}^{\tau-\theta}\int_{k\tau+x+\theta}^{\infty}P_{PO}\left(\overline{\tau,(k-1)\tau};k\tau|\vartheta\right)\omega(\vartheta)d\vartheta dx=\sum_{k=0}^{\infty}\int_{0}^{\tau-\theta}\int_{k\tau+x+\theta}^{\infty}(1-\alpha)^{k}\lambda e^{-\lambda\vartheta}d\vartheta dx=$$

$$=\frac{e^{-\lambda\theta}\left(1-e^{-\lambda(\tau-\theta)}\right)}{\lambda}\sum_{k=0}^{\infty}\left[(1-\alpha)e^{-\lambda\tau}\right]^{k}=\frac{e^{-\lambda\theta}\left(1-e^{-\lambda(\tau-\theta)}\right)}{\lambda\left[1-(1-\alpha)e^{-\lambda\tau}\right]}. \qquad (24)$$

With (24) formula (16) reduces to (23). The corollary is proved.

Corollary 4. If the conditions of corollary 3 are satisfied and $\theta << \tau$, then

$$K_{OR}(\theta)\approx Ae^{-\lambda\theta}, \qquad (25)$$

where A is the LRU availability determined as [12]

$$A=\frac{MS_1}{MS_0-MS_3}. \qquad (26)$$

Proof. When $\theta << \tau$ expression (24) simplifies and takes the following form:

$$\sum_{k=0}^{\infty}\int_{0}^{\tau-\theta}\int_{k\tau+x+\theta}^{\infty}P_{PO}\left(\overline{\tau,(k-1)\tau};k\tau|\vartheta\right)\omega(\vartheta)d\vartheta dx\approx\frac{e^{-\lambda\theta}\left(1-e^{-\lambda\tau}\right)}{\lambda\left[1-(1-\alpha)e^{-\lambda\tau}\right]}=MS_1e^{-\lambda\theta}. \qquad (27)$$

Further, in equation (12) it is easy to observe that

$$\tau/(\tau-\theta)\approx 1. \qquad (28)$$

Substituting (27) and (28) in (16), we obtain

$$K_{OR}(\theta)\approx\frac{MS_1e^{-\lambda\theta}}{MS_0-MS_3}. \qquad (29)$$

In view of (26), the expression (29) becomes (25). The corollary is proved.

It should be noted that formula (25) is widely used in reliability theory. However, as can be seen from the comparison of expressions (16) and (25), the coefficient of operational readiness representation, as a product of the availability and the probability of failure-free operation during the operating time θ, is valid only for the exponential distribution of time to failure and $\theta << \tau$.

Example 3. Calculate the coefficient of operational readiness, if $MTBF=1/\lambda=10000$ h, $\tau=2{,}5$ h, $\theta=0{,}5$ h, $\alpha=\beta=0{,}005$, $t_M=t_D=0{,}25$ h, $t_{PR}=8$ h, $t_{FR}=4$ h and $MS_5=0{,}5$ h.

Substituting the initial data in the following formulas obtained in [12]:

$$MS_1=\frac{1-e^{-\lambda\tau}}{\lambda\left[1-(1-\alpha)e^{-\lambda\tau}\right]}; \qquad (30)$$

$$MS_2=\frac{1}{1-(1-\alpha)e^{-\lambda\tau}}\left[\frac{\tau\left(1-\beta e^{-\lambda\tau}\right)}{1-\beta}-\frac{1-e^{-\lambda\tau}}{\lambda}\right];\quad MS_6=\frac{t_{FR}\alpha e^{-\lambda\tau}}{1-(1-\alpha)e^{-\lambda\tau}};\quad MS_7=\frac{t_{PR}(1-e^{-\lambda\tau})}{1-(1-\alpha)e^{-\lambda\tau}},$$

we calculate the values $MS_1=476{,}2$ h, $MS_2=0{,}06$ h, $MS_6=3{,}81$ h and $MS_7=0{,}38$ h.

Next, using (25), we find $K_{OR}(\theta)=0{,}990$. It can be seen from Example 3 that the mean time MS_1 is much smaller than $MTBF$. However, in case of ideal LRU checking, i.e. when $\alpha=\beta=0$, from (30) follows that $MS_1=MTBF=1/\lambda$.Thus, the false failures of operability checks significantly reduce the mean time between unscheduled repairs (MTBUR). Airborne electronic systems are used in the interrupted time regime, which is caused by alternating areas of aircraft operations and flight waiting in the parking lot. The finding of the LRU in states of E_3, E_4, E_6, E_7 and E_8 is associated with the value costs because the relevant works can be carried out during the stay of the aircraft in the parking lot or after removing the LRU from the aircraft board. Therefore, in determining the probabilistic parameters of maintenance we can exclude the intervals corresponding to these states from the regeneration cycle.

Let us to introduce a new time axis which is associated only with the use of LRU for performing its intended functions. At any time on the new axis the LRU may be in one of the states E_1, E_2 and E_5. The average regeneration cycle MS_0 is now determined as follows

$$MS_0 = MS_1 + MS_2 + MS_5,$$

and the operational readiness coefficient is given by

$$K_{OR}(\theta) \approx \frac{MS_1 e^{-\lambda\theta}}{MS_0}. \quad (31)$$

Example 4. Calculate the operational readiness coefficient, if $MTBF = 1/\lambda = 10000$ h, $\tau = 10$ h, $\theta = 1$ h, $\alpha = \beta = 0{,}005$ and $MS_5 = 0$.

Substituting the initial data in (31) gives $K_{OR}(\theta) \approx 0{,}999395$. The calculated value is exactly equal to the result of Example 3, which indicates the correctness of the derived formulas.

4. Conclusions

The exploitation states have been determined in which a single block system of avionics may be found when it is in use. The theorem that has been proved determines the non-stationary function of the operational readiness for an arbitrary failure distribution with taking into account the trustworthiness characteristics of multiple operability checks.

The proved corollary allows determining the non-stationary function and coefficient of operational readiness for an exponential failure distribution.

The proved theorem determines the expression for the stationary operational readiness coefficient for an arbitrary distribution of time to failure, as well as the corollaries, determining the stationary coefficient for an exponential distribution of failures under different assumptions. It has been shown that well-known expressions for the stationary coefficient of operational readiness are special cases of the derived formulas, which give a more adequate value of this coefficient. In a specific example of the exponential failure distribution it has been shown that the mean time between unscheduled repairs is largely determined by the probability of the false failures registered by the built-in test equipment.

These results allow assessing the effectiveness of maintenance strategies of avionics systems till safe failure and justify the requirements for the built-in test equipment. The obtained results are advisable to use as in the design phase of avionics systems as well as in the phase of avionics exploitation. Further development of these results can be conducted for optimizing the maintenance of modern avionics.

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