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MASS ION SEPARATION IN THE PLASMA FLOW MOVED ALONG THE TOROIDAL MAGNETIC FIELD

E.N. Sokolenko, N.A. Khizhnyak

National Scientific Center KPTI, Ukraine Kharkov 61108, Akademicheskaja 1

E-mail: khizh@khizh.kharkov.ua

The motion of rarefied (one particle approximation) and dense (a drift approximation) plasma flows in the toroidal solenoid magnetic field with the intensity of $\vec{H}=\left(0, H_0\dfrac{r_0}{r}, 0\right)$ is considered. It is shown that the base parameter characterizing separate particle paths is $a=\dfrac{Mv_0c}{eH_0r_0}$, where v_0 is the particle velocity parallel to the magnetic field on the input of a toroidal part of the magnetocircuit, r_0 is the radius of magnetic field line curvature of this part. The role of short circuit currents in moving dense plasma flows and their effect on particle paths taking into account, self-consistent electrical polarized field is discussed. The possibility of using such fields for mass ion plasma flow separation both in the rarefied plasma and dense ones is revealed.

It is shown, that the presence of the short circuits influences essentially on the radial particle movement, but the axial displacement even in the dense plasma and irrespective of conductance of the short circuit is determined by particle masses.

This gives the possibility to expect ion mass separation in the toroidal solenoid magnetic field would be preserved also in powerful ion flows. The effect of electron temperature and electron-ion collisions on separation properties of the curved magnetic field is discussed. The analysis of the equation of motion of particles has been done in this case in the frame of the drift theory taking into account collisions in different approximations for the collision parameter $q=w_{He}t_{ei}$.

INTRODUCTION

Movement of a separate charged particle of mass m and charge e in the toroidal magnetic field $\vec{H}(\bar{r})=\left(0, H_0\dfrac{r_0}{r}, 0\right)$ under initial conditions $r=r_0,\quad \dot{r}=0,\quad \dot{j}_0=\dfrac{v_0}{r_0},\quad z_0=\dot{z}_0=0$ (the origin of the cylindrical coordinate system, r, j, z are in the center of culvature of magnetic force lines) has the well known [1,2] first integral of movement

$$\dot{y}^2-\dot{y}_0^2+a^2\left(\frac{1}{y^2}-1\right)+(\ln y)^2=0 \qquad (1)$$

where $y=\dfrac{r}{r_0}$, $\dot{y}_0=\dfrac{Me\dot{r}_0}{eH_0r_0}$ and $a=\dfrac{Mv_0c}{eH_0r_0}$ are the base parameters of movement. Here v_0 - is the velocity of particle forward movement on the solenoid input (the initial velocity of movement along the tangent with respect to the force line). Cross movement along z determined by the radial relation

$$\frac{dz}{dt}=\frac{eH_0r_0}{Mc}\ln\frac{r}{r_0}, \text{ or } \dot{x}=\ln y, \qquad (2)$$

where $x=\dfrac{z}{r_0}$. From (1) it follows that the radial particle movement can be considered as the movement in some potential well described by the effective potential energy

$$U(y)=a^2\left(\frac{1}{y^2}-1\right)+(\ln y)^2 \qquad (3)$$

Zeros of potential energy define classical points of particle returning having zero radial velocity $\dot{y}_0=0$. The first root equals to the unity $y_1=1$, and the second one can be determined approximately. For $a^2>1$ this root is satisfactorily defined by the relation

$$y_2=e^a, \qquad (4)$$

and for $a^2<<1$ the equation of radial movement is integrated to the end and then one has

$$y(t)=1+\frac{a^2}{1+3a^2}\left(1-\cos\sqrt{1+3a^2}\,t\right) \qquad (5)$$

Consequently, the radial movement of a particle is oscillatory one between the limit points $y_1=1$ and $y_2=1+\dfrac{2a^2}{1+3a^2}$. The parameter a proportional to mass of a particle, then the toroidal magnetic field may be used for mass particle separation that is of the definite practical interest. So, for $a^2=0{,}11$,

$\Delta y = y_2 - y_1 \approx 0{,}2$; for $a^2 = 0{,}25$, $\Delta y = 0{,}4$, and for $a^2 = 0{,}44$ $\Delta y \approx 0{,}7$, that can be used for mass ion separation.

In detail the character of particle movement was investigated by means of numerical methods of computations integrating the equation of particle movement in the magnetic field of the toroidal solenoid $H_j = H_0 \frac{r_0}{r}$.

The above considerations remain valid ones so far as the charged particle flow is rarefied i.e. the magnetic turning force

$w_{Hi} \approx \frac{eH_0}{Mc}$ exceeds electrical polarization forces proportional to $\Omega_i = \sqrt{\frac{4pne^2}{M}}$, or

$$\left(\frac{\Omega_i}{w_{Hi}}\right)^2 = \frac{4p\,nMc^2}{H_0^2} = a^2\left(\frac{\Omega_i r_0}{v_0}\right)^2 << 1 . \quad (6)$$

In case of dense plasma flows the mechanism of passing of charged particles through the curved part of the magnetic field becomes considerably complicated one that can be shown by means of this problem considered in the drift approximation.

MOVEMENT OF DENSE PLASMA FLOW

Movement of plasma flow in the toroidal solenoid magnetic field for sufficiently small a^2 for basic its charge components can be done in the frame of the drift approximation. The equation of velocity of the guiding center of particles $\vec{w}$ is as follows:

$$\vec{w} = c\frac{\left[\vec{E}\times\vec{H}\right]}{H^2} + \frac{Mc}{eH^4}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\times$$
$$\times\left[\vec{H}\times\left(\vec{H}\nabla\right)\vec{H}\right] + \frac{Mc^2}{eH^2}\left[\vec{H}\times\frac{d}{dt}\frac{\left[\vec{E}\times\vec{H}\right]}{H^2}\right] \quad (7)$$

where $v_\parallel$ and $v_\perp$ - are the longitudinal and cross with respect to magnetic force lines components of particle velocity in cylindrical coordinates r, j, z take the following form:

$$w_z = c\frac{E_r}{H_j} + \frac{Mc}{eH_j r}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right) +$$
$$+ \frac{Mc^2}{eH_j}\frac{d}{dt}\left(\frac{E_z}{H_j}\right), \quad (8)$$
$$w_r = -c\frac{E_z}{H_j} + \frac{Mc^2}{eH_j}\frac{d}{dt}\left(\frac{E_r}{H_j}\right), w_{11} = \frac{r_0 v_0}{r}$$

Further, drift velocities of ions we shall denote as w^+, and electrons as w^-.

Equations (8) have been investigated by Schmidt [3], who has shown that for high plasma density the latter was not already deflected by the solenoid magnetic field and moved along the straight line. Numerous experiments carried out in particular by Kharkov researchers [4] have shown that in case of long plasma flows (the length of which exceeds the length of curvature of plasma tube) this conclusion does not correspond to real conditions. Even dense flows for which there is

$$\frac{4pnMc^2}{H^2} >> 1$$

pass easily the curved part of the plasma tube. Passing of plasma through the solenoid field can be explained only by that shorting currents have arisen removing the polarization field in plasma.

According to the drift theory the electron and ion drift parallel to the axis z on opposite sides of plasma. Plasma polarization in the direction of z due to this drift gives rise to appearing non-compensated electrical charges on the edges of a plasma bunch and as the consequence to arising of the electrical polarization field E_z. These charges excite some electrical field with respect to the plasma flow part being yet in the straight-line magnetic field. But plasma conductivity along force lines of the magnetic field is high and under the influence of arising field excess electrons leave and lacking ones on the opposite side of plasma bunch move to non-compensated charges decreasing polarization field. As a result of this in plasma moving in the curved part of plasma tube the z-th component of current interacting with the solenoid magnetic field and turning plasma flow along magnetic field force lines is generated.

To determine the electrical polarized field we use the following system of equations

$$\frac{\P E_z}{\P t} = -4pe\left(n^+w_z^+ - n^-w_z^-\right) - 4p\, j_0$$
$$\frac{\P E_r}{\P t} = -4pe\left(n^+w_r^+ - n^-w_r^-\right) \quad (9)$$
$$div\vec{E} = 4pe\left(n^+ - n^-\right)$$

where n^+ and n^- - are the densities of ion and electron plasma components, j_0 - is the density of shorting electrical currents,

$$j_0 = hE_z, \quad (10)$$

h - is the conductivity of the shorting circuit.

Thus, the solenoid magnetic field is stationary and any induction processes are not excited in moving plasma we have

$$\frac{d}{dt}\left(\frac{E_x}{H_j}\right) = \frac{1}{H_j}\frac{dE_x}{dt} \quad (11)$$

and from the continuity equation with the Lagrange variables $ndv = n_0 dv_0$ we find

$$n^{+} = n_0 \frac{r_0}{r_i}, \quad n^{-} = n_0 \frac{r_0}{r_e}$$

(r_i, r_e -are the average radii of movement of ion and electron components of plasma), thus, from the field equation (9) we obtain:

$$\frac{d}{dt}\left(\frac{E_z}{H_\varphi}\right)_i = \frac{r_i}{r_e}\frac{4\pi e n_0}{H_0}\left(w_z^- - w_z^+\right) - \frac{4\pi r_i}{H_0 r_0} j_0 ,$$

$$\frac{d}{dt}\left(\frac{E_z}{H_\varphi}\right)_e = \frac{r_e}{r_i}\frac{4\pi e n_0}{H_0}\left(w_z^- - w_z^+\right) - \frac{4\pi r_e}{H_0 r_0} j_0$$

$$\frac{d}{dt}\left(\frac{E_r}{H_\varphi}\right)_i = \frac{r_i}{r_e}\frac{4\pi e n_0}{H_0}\left(w_r^- - w_r^+\right) \quad (12)$$

$$\frac{d}{dt}\left(\frac{E_r}{H_\varphi}\right)_e = \frac{r_e}{r_i}\frac{4\pi e n_0}{H_0}\left(w_r^- - w_r^+\right)$$

Found relations with respect to the equation of movement in the drift approximation (8), permit to obtain the system of linear algebraic equations determine velocities of guiding centers $w_r^{\pm}$, $w_z^{\pm}$ for ions and electrons of plasma:

$$\left(1 + \chi_0 \frac{r_i^2}{r_e}\right) w_z^+ - \chi_0 \frac{r_i^2}{r_e} w_z^- = c\frac{E_r}{H_\varphi} + R_i ,$$

$$-\chi_0 \mu \frac{r_e^2}{r_i} w_z^+ + \left(1 + \chi_0 \mu \frac{r_e^2}{r_i}\right) w_z^- = c\frac{E_r}{H_\varphi} - R_e ,$$

$$\left(1 + \chi_0 \frac{r_i^2}{r_e}\right) w_r^+ - \chi_0 \frac{r_i^2}{r_e} w_r^- = -c\frac{E_z}{H_\varphi}, \quad (13)$$

$$-\chi_0 \mu \frac{r_e^2}{r_i} w_r^+ + \left(1 + \chi_0 \mu \frac{r_e^2}{r_i}\right) w_r^- = -c\frac{E_z}{H_\varphi}$$

Here designations have been taken

$$R_i = \frac{Mc}{eH_0 r_0}\left[\left(\frac{v_0^2 r_0^2}{r_i^2} + \frac{v_\perp^2}{2}\right) - \frac{4\pi c r_i^2}{H_0 r_0} j_0\right],$$

$$R_e = \frac{mc}{eH_0 r_0}\left[\left(\frac{v_0^2 r_0^2}{r_e^2} + \frac{v_\perp^2}{2}\right) - \frac{4\pi c r_e^2}{H_0 r_0} j_0\right], \quad (14)$$

$$\mu = \frac{m}{M}, \chi_0 = \frac{\varepsilon - 1}{r_0} = \frac{4\pi n_0 M c^2}{r_0 H_0^2}$$

The obtained system of equations is the initial one for describing plasma movement in the magnetic field of the toroidal solenoid in the drift approximation.

ANALYSIS OF PARTICLES MOVEMENT WITH DIFFERENT MASSES

From the system of equations (13) for $\chi_0 r_0$ >>1 we find that

$$w_r^+ = w_r^- = \frac{dr}{dt} = -\frac{cr}{H_0 r_0} E_z \quad (15)$$

so that $r_i = r_e$ (for guiding centers) if only $r_i = r_e$ for $t = t_0$, on the input of the curved magnetic field therefore the mean radial polarized field is absent $E_r = 0$, and that is

$$w_z^+ = \frac{1}{1 + \chi_0 r(1+\mu)} R_i ,$$

$$w_z^- = -\frac{1}{\chi_0 r} R_e \quad (16)$$

and furthermore,

$$\frac{m}{M} R_i = R_e \quad (17)$$

and the equation of radial movement takes the final form

$$\frac{d^2 r}{dt^2} = \frac{1}{r}\left(\frac{v_0^2 r_0^2}{r^2} + \frac{v_\perp^2}{2}\right) + \frac{4\pi c}{\chi_0 r_0 H_0} j_0 \quad (18)$$

where according to (10) and (15) we have

$$j_0 = -\eta \frac{H_0 r_0}{cr}\frac{dr}{dt} \quad (19)$$

The equation (18) determines the movement of dense plasma of different ion compositions in the toroidal solenoid magnetic field and it turns out that in dense plasma all the charge components move practically in the same way.

However, the movement of particles along the $0Z$ axis (16) of such a solenoid is determined by their masses and therefore to investigate separating properties of dense plasma flows it is necessary to research the Z-th movement of different ion components.

CONCLUSIONS

1. In case of movement of rarefied plasma in the toroidal magnetic field radial deflections of particles from a force line of the guiding magnetic field take place, these deflections are essentially defined by particle mass and can be used in plasma separators of isotopes.

2. For increasing of plasma density electrical forces begin to play the important role in particle movement. For $\chi_0 r_0$ >>1 radial movement of guiding centers of particles with various masses are not practically differed between one another but now the Z-th ion movement in the self-consistent electrical field of plasma are determine by ion masses.

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