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ADIABATIC PLASMA LENSES OF MOROZOV TYPE WITH PROFILED MAGNETIC FIELD THAT INCREASES ITS EFFICIENCY

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1. Introduction

In a plasma lens of Morozov type the magnetic surfaces are the equipotential ones of the electrical field [1]. The electrical potentials are inserted into the plasma by the suitable ring electrodes. It is supposed that in the applied strong magnetic field the transverse current is very small, and electrical field intensity and its spatial distribution in a plasma are determined completely by the magnetic field geometry and external potentials of the rings. On the whole, the experimental investigations confirm these statements [1-4]. The problem is in perfection of these lenses to increase its efficiency and force and decrease aberrations.

Unlike of the previous publications, in this work some variants of the long plasmaoptic focusing devices of Morozov type are studied. In this case the spherical aberrations reduce to minimum. The ring electrodes can be placed near-by the lens faces at the lateral surface, i.e. at the input and output of the magnetic force lines. For uniform long solenoid (see Part 2), the focusing distance not depends from ion injection radius, i.e. focusing of the wide aperture ion beams is possible. Besides, the force of these long plasma lenses more than of similar short ones. For non-uniform long solenoid, it is studied in Part 3 the problem of the external magnetic field distribution by such a manner that the radius of the boundary magnetic surface can coincide with the focused ion beam radius, on the whole lens length. In this way, the efficiency and force of the long lens are increased sufficiently. The analytical solution of this problem in the paraxial approximation is given.

The calculations of the corresponding solenoid parameters and ion trajectories are made on a personal computer using special programs (see Part 4). Forming of axially symmetric magnetic fields with specified dependence on longitudinal coordinate within the given accuracy is one of the incorrect (ill-posed) problems. Usually such fields are created by a solenoids consisted of coaxial sections (coils). In previous works related to the problem, the sections' current densities were used as some unknown linear parameters. In this work the more complex (non-linear) problem is solved using the regularization method of Tikhonov [5]. It was made the calculation of the adiabatic plasmaoptic focusing device of Morozov type for proton beam focusing with energy 1 MeV, injection radius 3 cm, at the boundary magnetic surface "charged" to the potential 5 keV.

2. Long plasmaoptic focusing devices of Morozov type in the uniform magnetic field

In the case of long plasmaoptic focusing device of Morozov type (if the solenoid lens is much more then its radius) the spherical aberrations reduce to minimum. The ring electrodes can be placed near by the lens faces at the lateral surface, i.e. at the input and output of the magnetic force lines. For compact localization of the ring electrodes, it is advisable to apply the inverse-current coils near-by the solenoid ends [3, 4].

In the Morozov lens the focusing electrostatic force F_e takes the expression: $F_e = qE_r = -q\dfrac{\partial\varphi(r,z)}{\partial r}$, (1)

where q is the ion charge, E_r is the radial electric field, j is the potential of the electric field.

On the main part of the considered uniform lens one can preset the suitable distribution of the potential along the radius $j = j_0 r^2 / a_0^2$ (where a_0 is the radius of the boundary magnetic surface, and φ_0 is its potential) by means of the ring electrodes placed near by the lens faces at the lateral surface. In this case

$$F_e(r) = -q\, 2\varphi_0 / a_0^2\, r \qquad (2)$$

The equation of the focusing ion motion takes the form: $\dfrac{d^2 r}{dz^2} + k_M^2 r = 0$, $k_M^2 = \dfrac{2q\varphi_0}{Mv^2 a_0^2}$, (3)

where M and v are the mass and velocity of ions.

The expressions for ion trajectories, and focusing distance in the lens are:

$$r = r_0 \cos(k_{Ì}\, z),\ L_f = \pi(2k_M)^{-1} = \frac{\pi v a_0}{2}\sqrt{\frac{M}{2q\varphi_0}}\ , \quad (4)$$

where r_0 is the radius of the ion injection.

If the lens length $l < L_f$, then the focusing distance $l_f = l + k_M^{-1}\mathbf{ctg}(k_M l)$. In the case of "thin" lens $k_M l << 1$ we have the focusing distance $l_f = (k_M^2 l)^{-1}$.

For uniform long solenoid, the focusing distance not depends from ion injection radius, i.e. focusing of the wide aperture ion beams is possible. The force of the long plasma lens is more than of the similar short one.

3. Long plasmaoptic focusing devices in the optimized non-uniform magnetic field

For non-uniform long solenoid, in this work it is studied the problem of the external magnetic field distribution by such a manner that the radius of the boundary magnetic surface can coincide with the focused ion beam radius, on the whole lens length (see [6], where the plasma magnetic lens was considered by the similar way). At the paraxial approximation the equation of the magnetic surfaces is as follows:

$$a^2(z) = a_0^2 B_z(0) / B_z(z), \quad (5)$$

where $a(z)$ is the magnetic surface radius, $B_z(z)$ is the longitudinal magnetic field on the axis, $B_z(0)$ and a_0 are determined by the boundary conditions at $z = 0$.

We assume that in the case of the strong magnetic field the electrons can move only along cylindrical magnetic surfaces enclosed one into another. Every magnetic surface "leans" upon the ring electrode that distributes its potential into the plasma. From Eq.(5) it is follows: if the equidistantness of the magnetic surfaces is set in some cross-section, so it conserves in any other one (but the distance can change adiabatically). As a result, if we preset the square-law potential distribution near the lens entrance (i.e., at z = 0), so it will be the same in any other focusing channel cross-section. It is necessary for focusing without spherical aberration because the electrostatic force focusing an ion toward the axis is proportional to its distance from the axis.

In this case, one can preset the suitable distribution of the potential along the radius $j = j_0 r^2 / a_0^2$ (here a_0 is the initial radius of the boundary magnetic surface at z = 0, and φ_0 is its potential) by means of the ring electrodes placed near by the lens faces at the lateral surface. With account of (5), we receive the equation for focused ion motion:

$$\frac{d^2 r}{dz^2} + \frac{B_z(z)}{B_z(0)} k_M^2 r = 0, \text{ where } k_M^2 = \frac{2q\varphi_0}{Mv^2 a_0^2} \quad (6)$$

To put together all ions in the focus, it is needed to search the form of the magnetic surface that limit the focusing channel. The limit magnetic surface is determined from the condition that its radius (a_0) coincides with the radius of the focused beam (R). Then functions $R(z)$ and $B_z(z)$ are determined from the equation:

$$\frac{d^2 R}{dz^2} + \frac{\kappa}{R} = 0, \text{ where } \kappa = \frac{2q\varphi_0}{Mv^2}. \quad (7)$$

The solution of the Eq.(7) (with initial conditions: $R = R_0, R' = R_0'$ at $z = 0$) has the form:

$$z = \sqrt{\pi/2\kappa} R_0 \,\mathbf{exp}\left(R_0'^2/2\kappa\right) \times \left[\Phi\left(\sqrt{R_0'^2/\kappa - 2\,\mathbf{ln}(R/R_0)}\right) - \Phi\left(R_0'/\sqrt{\kappa}\right)\right] \quad (8)$$

In the real experiments the focusing channel compression leads to the certain value R_g (not equal to zero) that corresponds to the coordinate z_g that is the end of the focusing channel. Later on the inertial ion focusing to the focal spot takes place. This point' coordinate is defined as follows:

$$z_f = \sqrt{\frac{\pi}{2\kappa}} R_0 \Phi_0 \left(\sqrt{2\,\mathbf{ln}\frac{R_0}{R_g}}\right) + \frac{R_g}{\sqrt{2\kappa\,\mathbf{ln}(R_0/R_g)}}. \quad (9)$$

Example. Let us consider the calculation of focusing the ion beam with 1 MeV energy, 3 cm initial radius, and at the boundary magnetic surface potential of 5 keV, i.e., $\kappa = 2q\varphi_0 / Mv^2 = \varphi_0/U = 0.005$.

Let us take $R_0/R_g = 2.72$ as a result of ion beam focusing and boundary magnetic surface compression. Using the formula (8), we determine the lens length $z_g = 45$ cm. By the formula (9) we determine the focusing length $z_f = 56$ cm which less than that for the uniform lens. Especially big gaining take place for the diverging beam focusing. The calculations of solenoid parameters and ion trajectories see in the next part.

In conclusion of this part, we add the following remark. As is noted in many works (e.g., [2]), for vacuum electrostatic lenses the focusing length $L_f \propto (U/\varphi_0)^2$, but for short electron or plasma electrostatic lenses $L_f \propto (U/j_0)$, i.e., it is much less. In this work it is shown that for long plasma (or electron) electrostatic lenses $L_f \propto (U/j_0)^{1/2}$, i.e., it is some more less. The suitable compression of the focusing channel gives additional gaining of several times over.

4. Calculation of non-uniform solenoids for charged particle focusing

4.1. For many problems of forming, acceleration and focusing of the charged particles it is necessary to create external non-uniform magnetic fields with specific configuration and within the framework of given accuracy. This problem is an incorrect inverse one (i.e., ill-posed) because it have no single-valued solution within the framework of given accuracy. Accordingly to the regularization method of A.N. Tikhonov [5], from the set of possible solutions the wanted one is chosen that optimized on specific criterion. In this work the magnetic fields are created with the help of the solenoids consisting of coaxial placed sections. The magnetic field intensity created by each section depends on its parameters generally nonlinearly. In previous works related to the problem above-mentioned, the sections' current densities were used as some unknown parameters. This is the simplest way to solve the problem of parameters selecting since current density is a linear parameter of the equation for a solenoid magnetic field intensity. In this work the more complex problem is solved by means of the regularization method [5] where the sections parameters of a specific kind are calculated. These can be the winding thickness, length or a section's internal radius etc., that is the values magnetic field intensity depends on, generally speaking, in the nonlinear way.

4.2. Let us consider a sectional solenoid of n coaxial sections with arbitrary cross-section. Let it be needed to create a magnetic field at the solenoid axis in an interval $[a, b]$ within the given accuracy δ, when the field intensity is prescribed by a function $f(z)$. Let us assume that field intensity, created by the i-th section at the point z, is described by the function $H_i(N_i, z)$, which in turn depends on a cross-section geometrical configuration, its size and location towards the measuring point. Then magnetic field created by the solenoid on its axis can be formulated as: $B(z) = \sum_{i=1}^{n} H_i(N_i, z)$. We shall estimate deviation of $B(z)$ from $f(z)$ in square metric, i.e. by the formula: $\rho(B, f) = \left\{\int_a^b [B(z) - f(z)]^2 dz\right\}^{1/2}$. This problem pertains to the class of incorrect ones in sense that within given accuracy it has not one-valued solution; more over – if a sections' number is large enough, its solution becomes unstable towards small variations of initial data. So, from a set of solutions matching $\rho(B, f) \le \delta$ where δ is a given number, one has to pick up the only solution optimized on a certain criteria (a solenoid volume, power consumption, etc). The problem is formulated as follows: to find an optimum set of parameters $(N_1, N_2,..., N_n)$ under which the functional

$$F(N_1,..., N_n, \beta) = \int_a^b \left[\sum_{i=1}^{n} H_i(N_i, z) - f(z) \right]^2 dz + \beta\Omega(N_1,..., N_n) \quad (10)$$

reaches its greatest lower bound [6]. Here $\Omega(N_1,...N_n)$ is a stabilizing functional determined by the optimization criteria, β is a parameter of regularization. The conditions, under which $\frac{\partial F(N_1,..., N_n, \beta)}{\partial N_k} = 0$, lead us to a set of nonlinear equations:

$$\int_a^b \left[\sum_{i=1}^{n} H_i(N_i, z) - f(z) \right] \frac{\partial H_k}{\partial N_k} dz + \beta \frac{\partial \Omega}{\partial N_k} = 0, \quad (11)$$

that, can be solved by one of the gradient methods.

4.3. In this section the method offered has been used in case of a magnetic field to be produced by a solenoid built of coils with rectangular cross-section. As a desirable parameter N_i the winding thickness $d_i = R_i - r_i$ has been accepted. Here R_i and r_i are the section's outer and inner radii respectively.

As a stabilizing functional the square of the solution Euclidean norm have been used:

$\Omega(d_1,..., d_n) = \sum_{j=1}^{n} \left(d_j - d_j^0\right)^2$, where d_i^0 is an initial value of the i-th section's parameter to be found. Such choice actually corresponds to the criteria of minimum volume of wire the solenoid is wound by.

4.4. As an example, let us consider the results of modeling the solenoid with prescribed magnetic field distribution on its length for the long adiabatic plasmaoptic focusing devices considered in Part 3. The diagram of such field is shown in Fig.1, and configuration of the solenoid creating it – in Fig.2. Its main parameters are as follows: the length is 65 cm, internal radius is 10 cm, length of each section is 6.5 cm, number of the sections is 5. The parameter to be calculated was thickness of winding: its value (in cm) for 1st to 5th section is 0.18, 0.12, 0.13, 0.95, 4.92. Average current density over cross-section of winding is 100 A/mm^2 (the pulsing supply). The relative error of the field approximation not exceeded $\Delta = 4 \cdot 10^{-2}$ over the length of 45 cm, i.e. 80 % of the solenoid length (it is sufficient for this case). After all, the proton trajectories in this lens with optimized solenoid have been calculated (Fig.3). As expected, all protons have

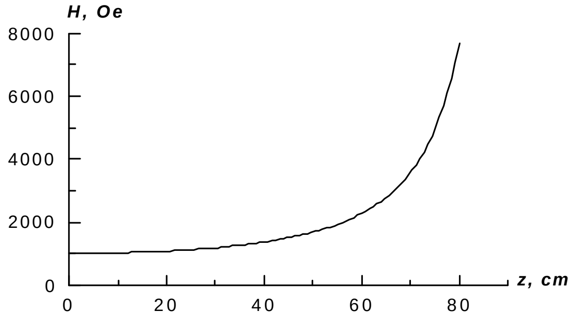

Fig. 1.

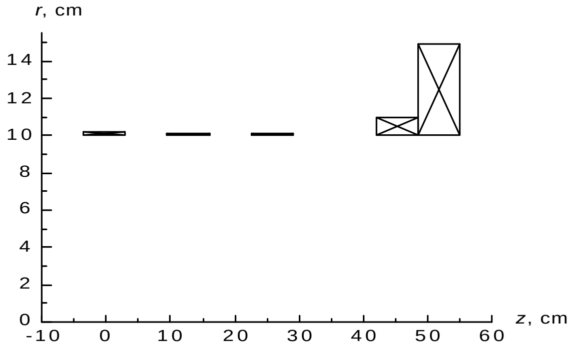

Fig. 2.

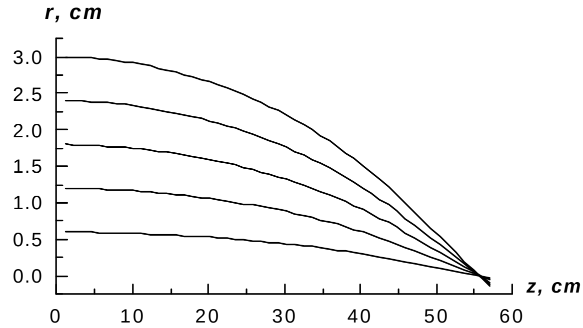

Fig. 3.

been focused in the same point because the focusing force is proportional to the distance of any proton from the axis (in any cross-section of the lens). In conclusion, it could be noted the exact coincidence of the focusing length value (L_f=56cm) determined analytically and by ion trajectories calculation on a computer.

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