UDC 533.9

SIMULATIONS OF WIDE-APERTURE ION BEAM FOCUSING BY THE PLASMA LENS FORMED BY A MAGNETIC COIL AND SYSTEM OF ELECTRODES

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In the plasma physics and problems of controlled thermonuclear fusion a noticeable role are played the trends connected with focusing of intense ion beams of middle and high energies. In particular, such trends are the inertial thermonuclear fusion on light and heavy ions, researches of radiating resistance of the first wall materials, generation of the high power neutral particle beams by the charge exchange of intense ion beams, etc. The problems of intense ion beam focusing are important also for the nuclear physics, physics of high energies, physics and engineering of accelerators, beam technologies. The essential feature of intense ion beams is that they should be charge compensated during the focusing to prevent their destruction. In this case, the application of plasmaoptic focusing systems is expedient which development is initiated by A.I. Morozov and co-workers [1, 2], and recently successfully developed by A.A. Goncharov group (e.g., [3, 4]). Now the problem consists in optimization of such lenses, mainly, in reduction of aberrations and focusing force increasing.

So, this work is devoted to calculations of ion beam focusing by the lens of Morozov type formed by a current-carrying coil in a plasma, and a system of ring electrodes.

In the plasma electrostatic lens of Morozov type the magnetic surfaces are the equipotentials of the electrical field [1]. It is supposed, that the current across a magnetic field is absent, and intensity and spatial distribution of electrical field in a plasma are completely determined by magnetic field geometry and boundary condition. The last one is given as a continuous function $\Phi(R,z)$, where Φ is the potential (that is set from the outside), and R is the cylindrical surface radius. In practice the electrical potentials are entered in plasma by a discrete manner, using of ring electrodes, due to the system of the "charged" magnetic surfaces can be formed in the plasma. The experimental researches [2-4] basically confirm the theoretical model [1], but some problems remain, in particular, the reasons of rather significant spherical aberrations and methods of their elimination. On the basis of the experimental experience, it is possible to consider that the probable corrections to the theory can be taken into account as additional aberrations.

In the large work of A.I. Morozov and S.V. Lebedev [1] various problems of plasmaoptics are investigated theoretically including consideration of the axial electrostatic plasma lenses. In particular, the estimation of the focal length for the elementary plasma lens formed by the circular current is given. Meaning importance of this problem for practical calculations of electrostatic plasma lenses, we will consider it in more details, with account of non-paraxial (wide-aperture) focused beams and exact expression for a magnetic field.

The magnetic field of the circular current J (with the radius of the coil $a_ñ$ and its coordinate l on the axis z) is described by the azimuthal component of the vector-potential (e.g., see [5]):

$$A_\varphi = \frac{4J}{ck}\sqrt{\frac{a_c}{r}}\left[(1-\frac{k^2}{2})K(k)-E(k)\right], k^2 = \frac{4a_c r}{(a_c+r)^2+(z-l)^2} \quad (1)$$

where c is the light velocity, K and E are the complete elliptic integrals of the 1-st and 2-nd kind.

Following [1], we enter the function of the magnetic flow $\psi = rA_\varphi$. The expression $\psi(r,z) = const$ is the equation of the magnetic surface and also the equation of the magnetic force line on the plane (r, z). (the set of such lines is calculated and given in the corresponding figures, see below). In this lens the equipotential property of magnetic surfaces is determined by relation $\Phi = \Phi(\psi)$, where Φ is the potential of the electrical field.

Let's express the components of the electrical and magnetic field through ψ and A_φ:

$$E_r = -\frac{d\Phi}{d\psi}\frac{d\psi}{dr}; \; E_z = -\frac{d\Phi}{d\psi}\frac{d\psi}{dz} = -\frac{d\Phi}{d\psi} r \frac{dA_\varphi}{dz},$$

$$H_r = -\frac{dA_\varphi}{dz}; \; H_z = \frac{1}{r}\frac{d}{dr} rA_\varphi \quad (2)$$

Hence it follows:

$$E_z = \frac{d\Phi}{d\psi} H_r r, \; E_r = -\frac{d\Phi}{d\psi} H_z r \quad (3)$$

We will consider two cases of dependence Φ versus ψ having practical meaning.

Case 1. In the work [1] the plasma lens formed by the circular current is very shortly considered at the electrical potential distribution according to the condition

$$\Phi = b\psi = brA_\varphi, \text{ where } b = const, \quad (4)$$

and the estimation of its focusing length is given: $F = a_c W / 2q\Phi_0\theta$, where W is the kinetic energy of ions, Φ_0 is the potential of the coil, $\theta \approx 1$ is the dimensionless parameter depended on the geometry of the system.

Let's consider this problem more in detail, with application of computer modeling. For performance of the relation (4) we will set the boundary condition as the distribution of electrical potential on the cylindrical surface with the radius R_{e2} (in practice it is set by system of ring electrodes [2-4]):

$$\Phi(R_{e2}, z) = bR_{e2}A_{\varphi}(R_{e2}, z) \quad (5)$$

The electrical and magnetic fields are connected by the relations which follow from (3) and (4):

$$E_z = brH_r\,,\ E_r = -brH_z \quad (6)$$

The constant b, on which the force of the lens depends, is determined by setting (with help of the electrodes) appropriate value of the electrical field intensity E_{r2} in the point (r_2, z_0):

$$b = -E_{r2}\,/\,r_2H_z(r_2, z_0) \quad (7)$$

At focusing of ions with the mass Ì and charge q, the equations of motion look like:

$$M\ddot{r} = -qE_r\,,\ M\ddot{z} = -qE_z \quad (8)$$

(Here the magnetic component of the Lorentz force was neglected that in this case, for $H_r \sim H_z$, is allowable at the energy ~10 keV/nuclon, and for paraxial ions is allowable at energy ~ 1 MeV/nuclon).

The initial conditions will set as:

at $t = 0$ $z = z_0$, $v_r = 0$, $v_z = v_0$, $r = r_0$, (9)

where the radius of ion injection r_0 is set from 0 up to size smaller the radius of electrodes, z_0 = -10 cm, v_r and v_z are the radial and longitudinal velocity of ions.

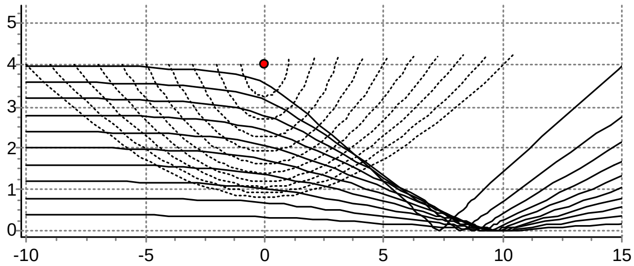

Fig. 1. The results of the calculations of ion trajectories for the case 1 (all values in cm).

The initial parameters of the beam are as follows: the proton beam with energy 20 keV, current 1 A, radius 4 cm; the beam is uniform along the radius and charge compensated by electrons. These conditions are close to the experimental ones in Refs. [3, 4].

The results of calculations of ion trajectories for the case 1 are given in the Fig. 1, whence it is visible, that only the paraxial particles are well focused. On this base it is found the distribution of the ion current density versus the radius in the cross-section of the paraxial ion focusing at the coordinate of z_s=9.9 cm: the maximal current density is j = 135 A/cì^2, the half-width of the focal spot is δr = 0.02 cm, and the relative amount of ions within the limits of the half-width is about 10 %. The non-paraxial ions (which is much more since their amount in a layer is proportional to the radius of injection) are «overfocused», moreover, the larger an injection radius, the earlier an ion intersects the axial line. The explanation is connected with the fact that rA_{φ} and Φ increase too rapidly versus r near by the coil surface. In the Fig. 2 it is presented the distribution of the ion current density versus the radius in the cross-section near the minimum beam radius that demonstrates bad focusing of non-paraxial ions: at the cross-section coordinate of z_s=8.2 cm there is the maximal current density j = 11 A/cì^2, the half-width of the focal spot is δr = 0.15 cm, and the relative amount of ions within the limits of the half-width is about 50 %.

Optimization of the case 1. The condition of ideal focusing is the requirement, that in any cross-section of a lens the focusing force can be proportional to a deviation of an ion from the axis, that is E_r µ r. As it is visible from (6), it is reduced to the condition $H_z(r)$ = const, that is realized in long solenoids (see Ref.[6]).

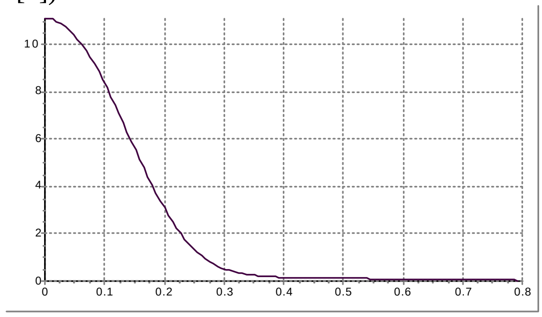

Fig. 2. Distribution of ion current density (A/cm^2) versus radius (cm) near the minimum beam radius (case 1).

Case 2. Let's consider the variant, when in the plane of the coil $z = z_0$ the linear normalized distribution of the radial electrical field is set:

$$E_r(r, z_0) = E_r(r_1, z_0)\frac{r}{r_1} = E_{r1}\frac{r}{r_1} \quad (9)$$

In practice such statement of the problem can be carried out by setting potentials on the electrodes that insert into the plasma, and measuring the distribution of the electrical field intensity in the plasma. (The method of local non-contact measurements of electrical field intensity was proposed and experimentally grounded in Ref.[7]).

In this work the electrical field in the plasma is determined by the calculation way. Thus the functions $\Phi = \Phi(\psi)$ and $d\Phi/d\psi$ are set parametrically:

$$\begin{cases} \Phi(\psi(r, z_0)) = -\dfrac{1}{2}\dfrac{E_{r1}}{r_1}r^2 \\ \psi(r, z_0) = rA_{\varphi}(r, z_0) \end{cases}$$

$$\begin{cases} \dfrac{d\Phi(\psi(r, z_0))}{d\psi} = -\dfrac{E_{r1}}{r_1H_z(r, z_0)} \\ \psi(r, z_0) = rA_{\varphi}(r, z_0) \end{cases} \quad (10)$$

On the cylindrical surface with radius R_{e1} we will set the boundary condition as the distribution of electrical potential:

$$\Phi(\psi(R_{e1}, z)) = \Phi(R_{e1}A_{\varphi}(R_{e1}, z)) \quad (11)$$

Using the equations (9-11) for determination of electrical fields from the formulas (3), and then the motion equations (7) and initial conditions (8), it is possible to calculate the ion trajectories.

The results of calculations of ion trajectories for the case 2 are given in the Fig. 3. On this base it is found the distribution of the ion current density versus the radius in the cross-section of the paraxial ion focusing at

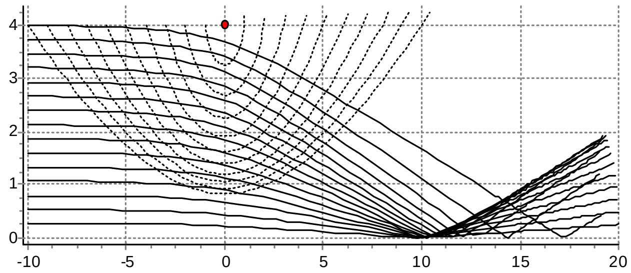

Fig.3. The results of the calculations of ion trajectories for the case 2 (all values in cm).

the coordinate z_s=10.3 cm: the maximal current density is j = 66 A/cì^2, the half-width of the focal spot is δr = 0.03 cm, and the relative amount of ions within the limits of the half-width is about 13 %. From the distribution of ion radii at z_s=10.3 cm versus its injection radii; it is evident that well focused paraxial ions have initial radii 0–1.5 cm. The non-paraxial ions this time are underfocused, moreover, the larger an injection radius, the later an ion intersects the axial line. Now the explanation is connected with the fact that, due to magnetic surfaces curvature, the non-paraxial ions have not sufficient time-of-flight in the region of the high focusing fields. In the Fig. 4 it is presented for the case 2 the distribution of the ion current density versus the radius in the cross-section near the minimum beam radius that demonstrates bad focusing of non-paraxial ions: at the cross-section coordinate of z_s=11.5 cm there is the maximal current density j = 4.3 A/cì^2, the half-width of the focal spot is δr = 0.25 cm, and the relative amount of ions within the half-width limits is about 50 %. (As we have studied, in the case of long solenoids, and at the $\Phi \propto r^2$ distribution, the good focusing takes place because the focusing force is proportional to the ion deflection from the axis [6]).

Optimization of the case 2. In the formula (9) for the distribution of the radial electrical field on the radius the terms of a high degree on r were added. The coefficients at them were selected by testing of variants. The essential improvement of focusing is received at the distribution E_r=390r+0.285r^7 (where E_r in *V/cm*, and r in *cm*). For this case the results of calculation of ion trajectories are given in the Fig. 5. In the Fig. 6 it is presented the distribution of the ion current density versus the radius in the cross-section near the minimum beam radius: at the cross-section coordinate z_s=12.7 cm there is the maximum current density j = 230 A/cì^2, the half-width of the focal spot is δr = 0.025 cm, and the relative amount of ions within the half-width limits is about 50 %.

In principle, the problem of the optimum electrical field distribution can be solved by the special algorithm developing. Furthermore, it is expedient to proceed from this simplest lens to the lens with an arbitrary solenoid, using the field superposition principle. Thus for each experimental sample of the lens it is possible to create its computational model intended for determination of the optimum parameters and modes of operation.

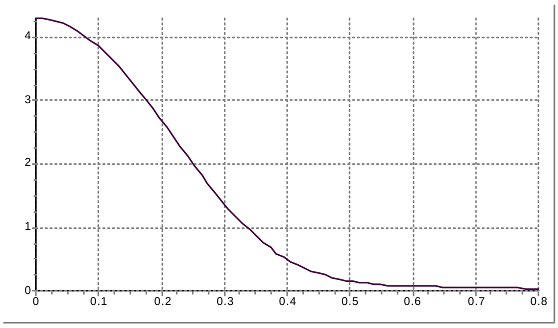

Fig.4. Distribution of ion current density (A/cm^2) versus radius (cm) near the minimum beam radius (case 2).

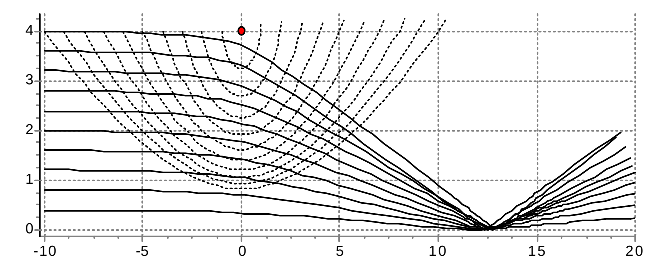

Fig. 5. The results of the calculations of ion trajectories for the case 2, after optimization (all values in *cm*).

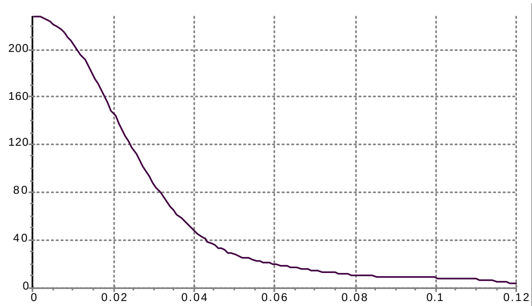

Fig. 6. Distribution of ion current density (A/cm^2) versus radius (*cm*) in the cross-section near the minimum beam radius (case 2, after optimization).

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