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FEATURES OF COAXIAL WAVES IN MAGNETIZED PLASMA CYLINDER

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The resonance properties of plasma cylinder are carried out for nonzero axial wave vector. Waves with frequency greater then ion cyclotron frequency but less then electron frequency are considered. From Maxwell equations we get the system of two coupling differential equations of second order for these waves. There exist two types of coaxial waves in the vacuum gap. One of them has zero axial electric field (TE-mode). This is the analog of azimuthal surface waves. Another one has zero axial magnetic field (TB-mode). As it is shown, in the broad range of plasma and device parameters the resonance frequencies of TB-mode are close to resonance frequencies of TE-mode. This leads to effective coupling of these modes and energy transfer from one mode to another. The time of transfer is much shorter, then collisional absorption time. As an consequence, the wave fields change significantly in plasma and vacuum.

The surface waves are applied in plasma sources and plasma electronics. Also they are excited at RF plasma in magnetic traps (so called coaxial modes) and effect plasma periphery. These stimulate theoretical investigations of surface waves. They often use the model of homogeneous circular plasma cylinder with axial magnetic field, separated from metal wall by vacuum for surface waves studies. The axially symmetric $m = 0$ surface waves with $k_\| \neq 0$ ($k_\|$ is a wave vector along confining magnetic field) were analyzed in details e.g. in [1]. Since [2] axially nonsymmetric ($m \neq 0$, $k_\| = 0$, $E_z = 0$) surface waves are intensively studied. Such waves is named as azimuth surface waves. As for fast magnetosonic waves the azimuth surface waves with $m > 0$ and with $m < 0$ propagates in different ways due to plasma gyrotropy. Both for $m = 0$ case and for $k_\| = 0$ case one can extract from Maxwell equations two independent deferential equations of the second order. One of them is for the B_z component. of wave and by analogy to the theory of waveguides we shall call these waves as TE coaxial modes. And another is for E_z component of wave. We shall call these waves as TB coaxial modes. At the boundary plasma - the metal TB coaxial modes does not exist. Moreover even in systems with vacuum gap it was not considered as a rule. The real plasma devices have final length along confining magnetic field. Therefore it is worthwhile to take into account $k_{||} \neq 0$ in coaxial modes studies.

The paper presented concerns with analytical studies of the excitation and propagation of this waves. We consider a homogeneous plasma cylinder of radius $r = a$ with axial magnetic field $B_0 || Oz$. It is separated from metal wall of $r = b$ $(a < b)$ by vacuum gap. The waves are exited by an azimuth surface current $\vec{j} = \vec{e}_j\, j_0 \exp i(mj + k_{||}z - wt)$ of radius $r = d$ $(a < d < b)$. We take into account presence of a surface charge $r = -i\, div\vec{j}\, / w$. The RF field takes the form $\vec{B},\vec{E} \sim \vec{B},\vec{E}(r)\exp i(mj + k_{||}z - wt)$. From Maxwell equations we have

$$(e_1 - N_\|^2)\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)-\frac{m^2}{r^2}\right]B_z + \frac{w^2}{c^2}\left[\left(e_1 - N_\|^2\right)^2 - e_2^2\right]B_z - iN_\| e_2\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)-\frac{m^2}{r^2}\right]E_z = 0$$

$$iN_\| e_2\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)-\frac{m^2}{r^2}\right]B_z + \left[e_1\left(e_1 - N_\|^2\right) - e_2^2\right]\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)-\frac{m^2}{r^2}\right]E_z + \frac{w^2}{c^2}e_3\left[(e_1 - N_\|^2)^2 - e_2^2\right]E_z = 0 \qquad (1)$$

Here e_a are components of dielectric permeability tensor of cold two-component (ions and electrons) plasma. The collisional absorption of waves is taken into account. The effective collision frequency $n << w$ is included in e_a. We carry out our research for $w_{ci} < w < w_{ce}$ (w_{ca} is cyclotron frequency). The solutions of (1) can be written as $B_z(r) = BI_m(k_\perp r)$, $E_z(r) = EI_m(k_\perp r)$ (*B*, *E* are constants, $I_m(x)$ is modified Bessel function). Then for $N_\perp = k_\perp c / w$ we have

$$N^2_{\perp TE,TB} = \frac{1}{2e_1}\left[e_2^2 - (e_1+e_3)\left(e_1 - N_\|^2\right) \pm \left[\left[e_2^2 - (e_1+e_3)\left(e_1 - N_\|^2\right)\right]^2 - 4e_1e_3\left[\left(e_1 - N_\|^2\right)^2 - e_2^2\right]\right]^{1/2}\right] \qquad (2)$$

The sign “+” meets TE coaxial modes and sign “-“ - TB coaxial modes. The electro-magnetic field in vacuum consist of two independent modes. One of them is TE_m mode with $E_z = 0$ and another is TB_m with $B_z = 0$. In the space between plasma and surface current they look like

$$B_z(r) = AV(k_v r, k_v b) - \frac{p}{2} j_0 V(k_v r, k_v d) k_v d \,,$$

$$E_z(r) = CW(k_v r, k_v b) + \frac{ip}{2} j_0 W(k_v r, k_v d) m N_{\|} . \qquad (3)$$

Here $W(k_v r, k_v b) = Y_m(k_v b) J_m(k_v r) - Y_m(k_v r) J_m(k_v b)$, $V(k_v r, k_v b) = Y'_m(k_v b) J_m(k_v r) - Y_m(k_v r) J'_m(k_v b)$, $Y_m(x)$, $J_m(x)$ - Bessel functions. Equating tangential components of electric and magnetic fields at the plasma - vacuum boundary we have a system of four linear equations to obtain *A,C,B* and *E*. The right side of this system is proportional to j_0. Putting to zero the real part of determinant of this system we get the eigenfrequencies that is resonance curves of coaxial modes. We investigate resonance properties of coaxial modes using $\Omega - N_A^2$ plane (here $\Omega = w / w_{ci}$ and $N_A = c / v_A$, v_A is Alfven velocity) (see Fig.1). To number of parameters which characterize device and remain fixed at resonance curve we shall refer a, b, d, B_0 and $k_{\|}$. They are included in simulations as $b_0 = b w_{ci} / c$, a / b, d / b and $N_{\|0} = k_{\|} c / w_{ci}$. Then changing the plasma density we change N_A^2.

There are two ways of coaxial modes absorption. One of them is absorption of power, oscillating in the vacuum gap by the metallic wall due to finite conductivity of the wall. As the calculations show this absorption is negligibly small both for TE_m and for TB_m modes. Another one is collisional absorption of the power oscillating in the plasma. We shall characterize this absorption by $g = P_{ab} / P_{os}$. Here P_{ab} is the power absorbed in the plasma and P_{os} is the power oscillating in the plasma. The value of g changes with frequency within the resonance range (see Fig.2). This is due to different absorption value of TE mode and TB mode. But this difference strongly depends on plasma density as it is shown in Fig.3.

Now we turn to resonance properties of the coaxial modes of magnetized plasma cylinder. In principle these properties are well described by the relations

$$\frac{V'(k_v b, k_v a)}{V(k_v b, k_v a)} + \frac{1}{N_{\perp F}} \frac{I'_m(k_{\perp F} a)}{I_m(k_{\perp F} a)} + \frac{m}{k_{\perp F} a} \frac{1}{N_{\perp F}} \frac{e_2}{e_1 - N_{\|}^2} = 0$$

(4)

$$\frac{W'(k_v a, k_v b)}{W(k_v a, k_v b)} - N_{\perp S} \frac{I'_m(k_{\perp S} a)}{I_m(k_{\perp S} a)} = 0 .$$

These equations turn out from (1) ignoring the interaction of TE and TB coaxial modes in the plasma.
As we can see in Fig.1, the difference between the eigenfrequencies of TB coaxial mode and TE coaxial mode approaches to zero with density increase. This easy to understand analyzing Eqs.(4). With N_A^2 growth $N_\perp^2$ is increased both for TE coaxial mode and TB coaxial mode. Therefore zero of the first equation in (4) are determined by zero of a numerator of the first addend and the zero of the second equation are determined by zero of a denominator of the first addend of this equation. The roots of these functions are very close. That is why it is interesting to investigate excitation of coaxial modes in a range of such "collective" resonance. In this range we can consider the plasma cylinder as the two resonators system. Neglecting the coupling of resonators we get the eigenfrequencies w_{TE1} and w_{TB1} from (4). Taking into account the coupling of coaxial modes we have another pare of resonance frequencies w_{TE2} and w_{TB2}. The differences between these pares are rather small (Fig.4) but very important from the point of view of theory of oscillations.

Let us consider the system of two coupling contours and put x_{TE} and x_{TB} as the generalized coordinates of contours. Then the potential and kinetic energy of the system are

$$U(x_{TE}, x_{TB}) = a_{11} x_{TE}^2 + 2a_{12} x_{TE} x_{TB} + a_{22} x_{TB}^2 ,$$

$$T = b_{11} \dot{x}_{TE}^2 + 2b_{12} \dot{x}_{TE} \dot{x}_{TB} + b_{22} \dot{x}_{TB}^2 .$$

The partial resonance frequencies are defined as $v_{TE} = \sqrt{a_{11} / b_{11}}$ and $v_{TB} = \sqrt{a_{22} / b_{22}}$. The resonance frequencies of system can be obtained from

$$(b_{11} b_{22} - b_{12}^2) w^4 - (a_{11} b_{22} + a_{22} b_{11} - 2a_{12} b_{12}) w^2 + a_{11} a_{22} - a_{12}^2 = 0$$

Consequently, having partial resonance frequencies and resonance frequencies of coupling system, we can calculate the coupling coefficients $g_1^2 = b_{12}^2 / (b_{11} b_{22})$ (inductive) and $g_2^2 = a_{12}^2 / (a_{11} a_{22})$ (capacitive). They characterize the swap time between the resonators $t_s \approx p / (v \bar{g})$, $\bar{g}^2 = g_1^2 + g_2^2 - g_1 g_2$ (Fig.5). As we can see from Fig.5 the time of power transfer from one mode to another is the order of few periods.

Now we define A_{TE} - amplitude of TE mode with frequency w_{TE}, B_{TE} - amplitude of TB mode with frequency w_{TE}, A_{TB} - amplitude of TE mode with frequency w_{TB}, B_{TB} - amplitude of TB mode with frequency w_{TB}, and coefficients of amplitudes distribution $k_1 = A_{TE} / B_{TE}$ and $k_2 = A_{TB} / B_{TB}$. These coefficients are connected to the contours tie-up coefficient s like

$$k_1 = \sqrt{\frac{b_{11}}{b_{22}} \frac{1 - \sqrt{1 + s^2}}{2s}}, \quad k_2 = \sqrt{\frac{b_{11}}{b_{22}} \frac{1 + \sqrt{1 + s^2}}{2s}} ,$$

where $s = \bar{g} 2 v_{TE} v_{TB} / \left| v_{TB}^2 - v_{TE}^2 \right|$. In the case of weak coupling or great difference between partial frequencies of resonators, $s \to 0$, $k_1 \to 0$, $k_2 \to \infty$ and excitation of resonator (TE mode) does not influence another resonator (TB mode). But this is not our case (Fig.6). Even if the external currents is managed so as to excite only TE mode, then in the swap time t_s RF power transfers to the TB mode (and then back).

CONCLUSION

- strong coupling of fast (***TE_m***) and slow (***TB_m***) coaxial modes determined
- this coupling does not depend on wave damping value
- dependencies of coupling on plasma density and longitudinal wave number established
- absorption of coaxial modes in the wall is negligible
- collisional absorption of these modes in plasma is rather weak.

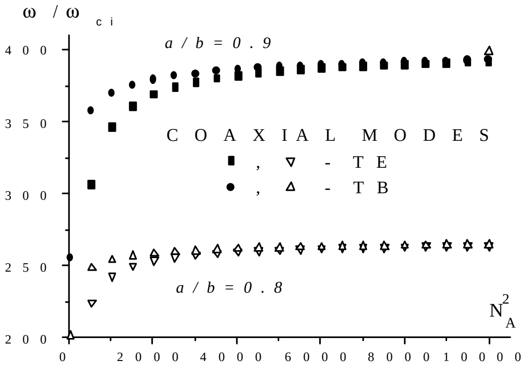

Fig.1. Resonance curves of TE and TB modes, calculated with use of system (1).

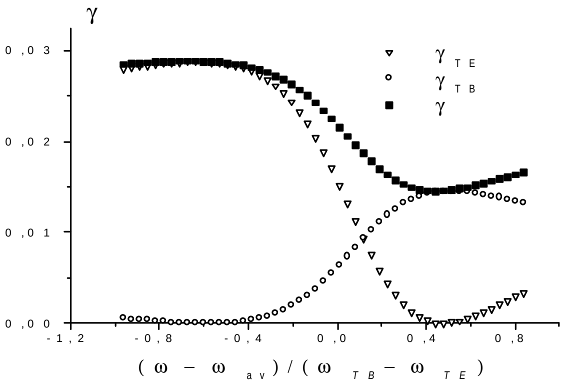

Fig.2. Dependencies of $g = P_{ab} / P_{os}$ *on frequency for TE, TB modes and total. Here* w_{TE}, w_{TB} *- resonance frequencies of TE , TB modes,* $w_{av} = (w_{TE} + w_{TB}) / 2$ *.*

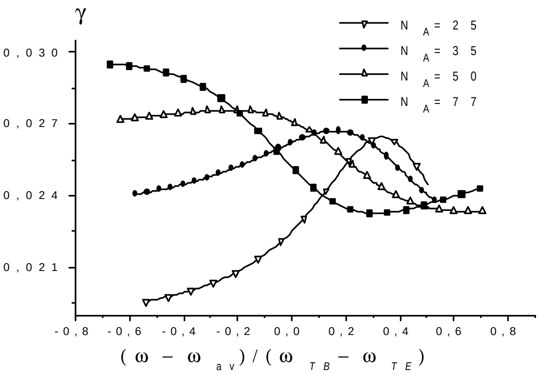

Fig.3. Dependencies of $g = P_{ab} / P_{os}$ *on frequency for different density values. Here* w_{TE}, w_{TB} *- resonance frequencies of TE , TB modes,* $w_{av} = (w_{TE} + w_{TB}) / 2$ *.*

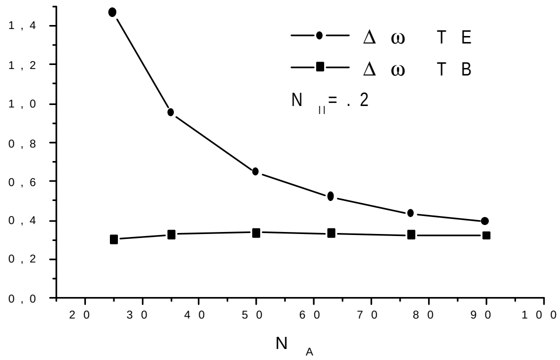

Fig.4. Dependencies of difference Δw *between resonance frequencies, calculated for coupling and isolated TE and TB modes, on plasma density.*

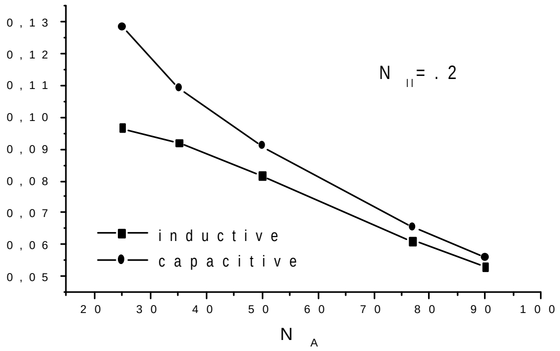

Fig.5. Dependence of the coupling of TE and TB coaxial modes on plasma density.

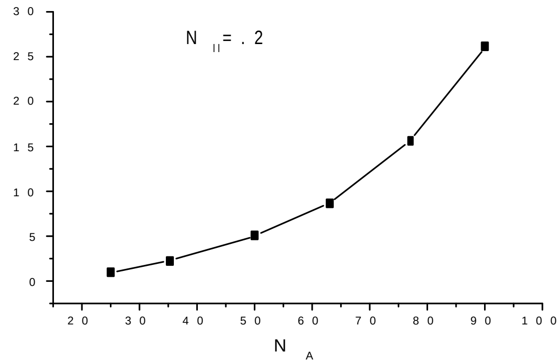

Fig.6. Dependence of the tie-up of TE and TB coaxial modes on plasma density

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