UDC 533.9

Particle Detrapping under AC Electric Field Effect as the Resonance Process

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Detrapping / retrapping processes of the particle under the AC parallel electric field is studied by the analytical methods and the numerical integration of guiding center equations. It is shown that these processes can be considered as the resonance between the bounce frequency of the trapped particle and the frequency of AC parallel electric field.

1. Introduction

Trapped particles in the toroidal magnetic trap can be the reason of the enhanced transport coefficients (neoclassical transport) in plasma due to the large (in comparison with the passing particles) deviation of the trapped particles from the initial magnetic surface due to drifts in the inhomogeneous magnetic field. The reduction of the neoclassical transport in the stellarator type devices is the subject of the numerous efforts. Among them the application of the AC electric field launched from outside to convert the trapped particles into the passing particles can be very important [1-3]. Detrapping of the particles under the AC electric field can be also very helpful if one should like to solve the task to control electric field in plasma by loss cone particle injection into the helical field [4]. That is why the process of conversion is under study in this paper.

2. Resonance Phenomena in the Motion of Particle under AC Electric Field

If the particle with the mass M, electric charge q and magnetic moment m moves in the magnetic field $\mathbf{B} = 0,0,B_z$, where $B_z = B_0(1 - e_B \cos kz)$, and under the electric field $\mathbf{E} = 0,0,E_z$, where $E_z = E_0(1 - e_E \cos lz)\sin \Omega t$, the position of the particle is described with the variable x, where $x \equiv kz$, that satisfies the following equation

$$\ddot{x} + \left(\frac{mB_0 e_B k^2}{2\Omega^2} + e_B \dot{x}^2\right)\sin x = \frac{qkE_0}{M\Omega^2}\left(1 - e_E \cos\frac{l}{k}x\right)\sin t \qquad (1)$$

Here derivative is taken on $t \equiv \Omega t$. The equation above can be treated as the nonlinear oscillator equation. The solution of (1) takes the form

$$x = x_0 \cos\{[\frac{w_b}{\Omega} + x_0^2 \frac{1}{4}\frac{w_b}{\Omega}(\frac{w_b^2}{k^2 mB_0}(1 + e_B) - \frac{1}{4})]t + c_0\} + a\frac{\Omega^2(1 - e_E)}{w_b^2 - \Omega^2}\sin t. \qquad (2)$$

Here $w_b = \frac{mB_0 e_B k^2}{2}$ and $a = \frac{keE_0}{M\Omega^2}$; x_0 and c_0 are integration constants.

The expression (2) means that the process of detrapping or more exactly the conversion of the trapped particle into passing under the effect of the externally applied electric field can be treated as the resonance phenomenon in the case when the bounce frequency w_b of the trapped particle is equal to the frequency of the external electric field Ω.

For the new variable x, where $x = e\mathrm{x}$, and the parameter of smallness $e << 1$, equation (1) takes the form

$$\ddot{\mathrm{x}} + \frac{w_b^2}{\Omega^2}\mathrm{x} = e^2[\frac{w_b^2}{6\Omega^2}\mathrm{x}^3 - \frac{2w_b^2}{k^2 mB_0}(1 + e_B)\dot{\mathrm{x}}^2\mathrm{x} + \tilde{a}(1 - e_E)\sin t], \qquad (3)$$

where $a = e^3\tilde{a}$.

Taking into account the difference between w_b and Ω, namely $w_b^2 - \Omega^2 = e\Delta$, it is possible to consider the cases of the exact resonance and the resonance with deviation. The solution is taken in the form

$$\mathrm{x} = a\cos(q + t) + e\mathrm{u}_1(a, q, t) + e^2\mathrm{u}_2(a, q, t) \quad , \qquad (4)$$

Bogolyubov-Mitropol'skiy method is used to obtain the equations for the amplitude a and phase q as the slowly changed variables

$$\frac{da}{dt} = -\frac{e\tilde{a}}{2}(1 - e_E)\cos q,$$

$$\frac{dq}{dt} = \frac{e|\Delta|}{2\Omega^2} + e^2[\frac{a^2}{4}(-\frac{1}{4} + \frac{\Omega^2}{k^2 mB_0}(1 + e_B)) + \frac{\tilde{a}}{2a}(1 - e_E)\sin q]. \qquad (5)$$

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The “phase portrait” of particle detrapping / retrapping process is described with the expression

$$y = -\frac{e|\Delta|}{4\Omega^2}a^2 + e^2(\frac{1}{16}A_B a^4 - \tilde{a}a\frac{1}{2}(1-e_E)\sin q). \qquad (6)$$

where

$$A_B = \frac{1}{4} - \frac{\Omega^2}{k^2 m B_0}(1+e_B). \qquad (7)$$

In the case of the exact resonance ($\Delta = 0$) the amplitude

$$a \cong (a)^{1/3}. \qquad (8)$$

It means the larger deviation of the particle from the state of trapped one to the state of the passing one.

In the case of the resonance with the deviation there exist three singular points

$$a_1 = \frac{e\tilde{a}(1-e_E)\Omega^2}{|\Delta|}, \quad \sin q_1 = -1;$$

$$a_2 = \sqrt{\frac{2|\Delta|}{e\Omega^2 A_B}} + \frac{e\tilde{a}(1-e_E)\Omega^2}{2|\Delta|}, \ \sin q_2 = 1; \qquad (9)$$

$$a_3 = \sqrt{\frac{2|\Delta|}{e\Omega^2 A_B}} - \frac{e\tilde{a}(1-e_E)\Omega^2}{2|\Delta|}, \ \sin q_3 = -1.$$

The positions of these points are shown on Fig.1.

The half-width d of the resonance is evaluated from (6) and

$$d = \sqrt{\frac{2\sqrt{2}a(1-e_E)}{\sqrt{\left|\frac{w_b^2}{\Omega^2}-1\right|}\sqrt{A_B}}}. \qquad (10)$$

As one can see the half-width d of the resonance is proportional to the square root of the AC electric field amplitude and reverse proportional to the square root of the deviation $\Omega^2 - w_b^2$.

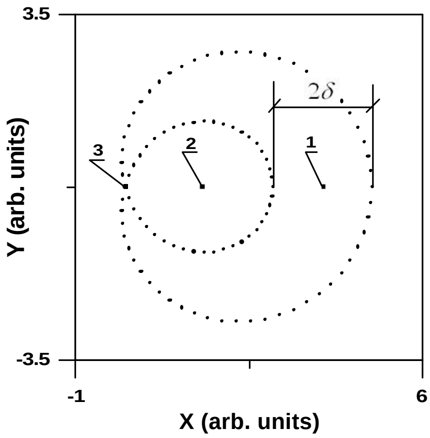

Fig.1. Phase portrait of the particle detrapping under AC electric field

3. Numerical Study of Particle Detrapping

The process of detrapping of particle is demonstrated for the more complicated magnetic geometry that is corresponding to the practical tasks. The heliotron /torsatron configuration is taken with the parameters of Heliotron DR [5]: the number of the helical winding poles $l = 2$, the magnetic field number $m = 15$, the magnetic field at the circular axis $B_0 = 0.05T$, the large radius of torus $R = 90cm$, the small radius of torus $a_h = 13.5cm$.

The main magnetic field ($\mathbf{B} = \nabla\Phi$) is modeled with the use of the magnetic field potential

$$\Phi = B_0\left[Rj - \frac{R}{m}\sum_n e_{n,m}(r/a_h)^n \sin(nJ - mj) + e_{1,0} r \sin J\right] \qquad (11)$$

where B_0 is the magnetic field at the circular axis, R and a_h are the major and minor radii of the helical winding; r, J, j are the coordinates connected with the circular axis of the torus, r is the radial variable, J and j are the angular variables along the minor and major circumference of the torus, J increases in the direction opposite to the main normal of the circular axis of the torus; metric coefficients are the following: $h_r = 1$, $h_J = r$, $h_j = R + r\cos J$; m is the number of the magnetic field periods along the torus, l is the helical winding pole number. The index n assumes the values $n = l$, $l-1$, $l+1$; $e_{n,m}$ are the coefficients of the harmonics of the magnetic field. The coefficients $e_{n,m}$ are chosen in such way that the magnetic surfaces and particle orbits are like those which are obtained in the paper [5].

The AC parallel electric field which effect on the particle is chosen in a such form

$$\tilde{E} = \tilde{E}_0[1-\cos(lJ - mj)]\cos(\Omega t + f_E), \qquad (12)$$

where $\Omega = w_b$, $f_E = p/2$.

The test particle (electron with the energy $W = 750$ eV and $V_{\parallel}/V$ =0.001) is the helically trapped one after the start, then it becomes the blocked particle and comes back to the helically trapped state again (Fig.2).

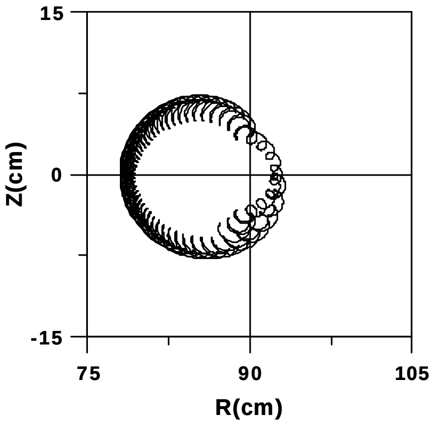

Fig. 2. The projection on vertical plane of the trajectory of electron with the starting point r_0=2 cm, under J_0=p and j_0=0 (without AC electric field)

The corresponding velocity change on time is shown on Fig.3 (top).

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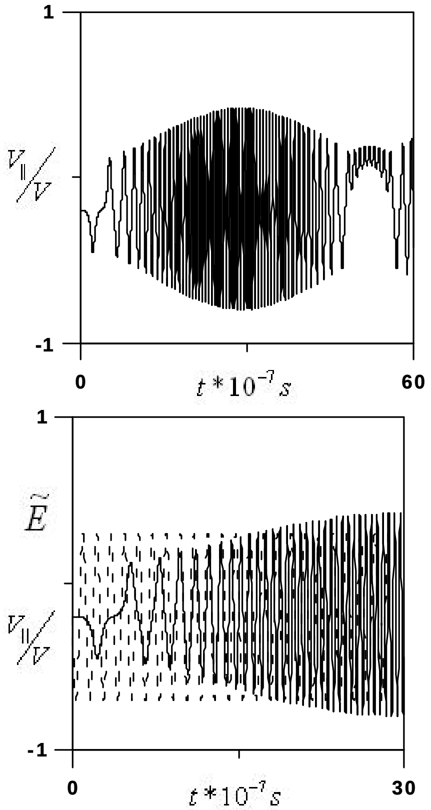

Fig.3 The test particle velocity on time without AC electric field (solid line) and the dependence of AC electric field (dashed line) on time

This test particle is moving without the AC electric field.

In the case of the AC electric field is applied the trajectory of the particle changes remarkably. The particle starts as the passing during some time, then becomes the helically trapped and transforms into the passing particle (Fig.4 and 5).

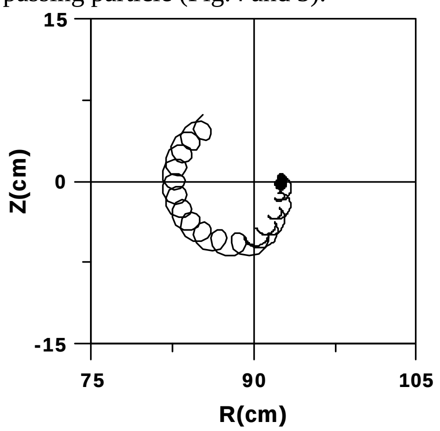

Fig.4. The projection on vertical plane of the trajectory of electron from Fig.2 with the AC electric field.

One can see that point of the trajectory (Fig.4) where the helically trapped particle transforms into the passing one. This conversion takes place when the frequency of the AC electric field is close to the bounce frequency of the helically trapped particle (Fig.3 down). From Fig.5 one can see that the particle starts the motion as the passing one, then becomes the helically trapped and transfers into passing one.

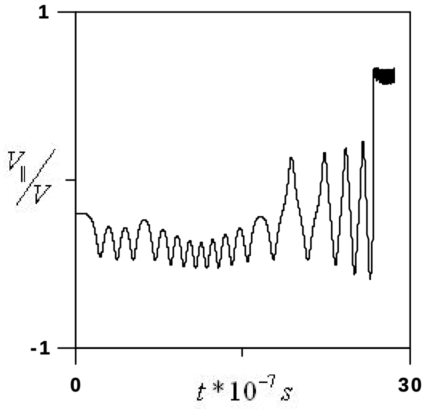

Fig. 5 The dependence of the test electron velocity under the AC electric field

4. Conclusions

4.1. Detrapping of the charged particle under AC electric field can be considered as the resonance process when the bounce frequency of the trapped particle is equal to the frequency of the AC electric field.

4.2. With the use of the analytical methods for the simplified model of the magnetic field it is shown that the equation of the particle motion under the AC electric field is the equation of the non-linear oscillator with the external force. The resonance separatrix in the phase plane separates the states of the deeply trapped particles, barely trapped and “near passing” ones.

4.3. Particle conversion from the helically trapped state into passing ones in the magnetic field with the toroidal and helical contributions is shown by the numerical integration of the guiding center equations in the presence of AC electric field. The reduction of the deviation of the particle from the initial magnetic surface is noticeable.

References

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