PARAMETRIC INSTABILITY INFLUENCE ON ISOTOPE SEPARATION BY ION-CYCLOTRON RESONANCE METHOD

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Solution of the dispersion equation for linear potential ion cyclotron oscillations with characteristic parameters used for selective ion cyclotron resonance separation of gadolinium isotopes has revealed the presence of the short-wavelength unstable oscillation in the ion cyclotron frequency range. Computer simulation of this instability by means of macro particle technique has shown that this instability may be responsible for turbulent heating of both resonant and nonresonant ions obtaining large transversal energy. It can decrease the efficiency of this method.

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1. INTRODUCTION

According to this method [1] in the plasma coming with the thermal velocity from an ion source into the heating region under the antenna the resonant ions which are to be separated obtain their transverse energy from the RF field under the ion cyclotron resonance and are heated up on the average. The nonresonant ions obtain lesser energy due to Doppler effect and one can neglect their heating if the generator frequency (more precisely the magnetic field) is matched so that $\left|\omega_0 - \omega_{ci} - k_{||0}\cdot V_{||}\right| >> k_{||0}\, V_{Ti}$, where $V_{||}$ is the particle velocity along the magnetic field, V_{Ti} is its thermal velocity, $k_{||0} = 2\pi / L$, L is the antenna length. Transverse RF field speeds up the resonant particles and they are heated. For a nonresonant particle the cyclotron frequency differs from one for a resonant particle by the value $\Delta\omega = \omega_0\left(\Delta A/A\right)$, where A is the atomic number of the isotope, and the condition of resonant absorption is not met for them, if

$$L >> L_{\kappa p} = \sqrt{8}\pi\left(V_{Ti}/\omega_0\right)\left(A/\left|\Delta A\right|\right). \qquad (1)$$

In the RF field during the time $\Delta t \sim 1/k_{||}V_{Ti}$ the resonant ions obtain the current velocity of the order of

$$u_{res} \approx \sqrt{\pi/2}\, eE_0/\left(m_i k_{||0} V_{Ti}\right),$$

where E_0 is the complex amplitude of the field rotating in the direction of cyclotron gyration of ions. Current velocity of nonresonant ions will be of the order of $u_{nores} \sim \left(eE_0/B_0\right)\cdot\left(A/\Delta A\right)$. The velocity u_{res} can be, by virtue of inequality (1), considerably greater than the velocity of nonresonant ions.

If the relative velocity of resonant and nonresonant ions will become of the order of initial thermal velocity of ions, then in such plasma, due to the relative oscillations of ions, parametric instabilities will arise associated with potential ion cyclotron oscillations [2].

In the present work the linear growth rates of the parametric ion cyclotron instability, conditioned by the oscillations of different species ions, are found and computer simulation, based on the macro particle model (2D2V model [3]), of this instability is carried out for the gadolinium isotopes.

2. LINEAR THEORY OF PARAMETRIC ION CYCLOTRON INSTABILITIES

Let us consider the particle motion in the pumping field $\mathbf{E} = \mathrm{Re}\left(\mathbf{E}_0 \exp\left[i(k_{||0} z - \omega_0 t\right]\right)$ that is switched on at the moment $t = 0$. Because the particle displacement is small in comparison with the antenna radius, then one can consider the electric field to be uniform. Integrating the equations of motion, neglecting small longitudinal electric field $E_{||}$ and carrying out the averaging over initial Maxwell distribution, we obtain the expression for the average transversal velocity of particles

$$\left\langle \hat{V}_{\alpha\perp}\right\rangle = \sqrt{\frac{\pi}{2}}\,\frac{(e_a/m_a)\hat{E}_0 \exp(-i\omega_{c\alpha} t)}{k_{||0}V_{T\alpha}}\left\{\exp\left[-i(\omega_0-\omega_{c\alpha})\,t\right]\times\right.$$
$$\times\exp\left[-\frac{\left(k_{||0}V_{T\alpha}t\right)^2}{2}\right]\cdot W\left(\frac{\omega_0-\omega_{c\alpha}}{\sqrt{2}k_{||0}V_{T\alpha}} - i\frac{k_{||0}V_{T\alpha}t}{\sqrt{2}}\right) \qquad (2)$$
$$\left. - W\left(\frac{\omega_0-\omega_{c\alpha}}{\sqrt{2}k_{||0}V_{T\alpha}}\right)\right\},$$

where $\hat{V}_{\alpha\perp} = V_{\alpha\, x} + iV_{\alpha\, y}$ is the transversal velocity in the complex form, $\omega_{c\alpha}$ is the ion cyclotron frequency of the ion of α -species, $\hat{E}_0$ is the complex amplitude of the pumping field, ω_0 is the frequency of this field, that is equal to the cyclotron frequency of the ion species under separation. In expression (2)

$$W(z) = \exp\left(-z^2\right)\cdot\left(\frac{k_{||0}}{|\,k_{||0}\,|} + \frac{2i}{\sqrt{\pi}}\int_0^z \exp\left(t^2\right)dt\right).$$

In [4] the device was proposed for isotope gadolinium-157 separation with the parameters: B_0=3 T, n_0=10^{12} cm^{-3}, initial temperature of ions is 10 eV, L=2 m, the frequency of alternating field is $\omega = \omega_{c\,res}$=$2\cdot10^6$ s^{-1}, the plasma radius is 10 cm, the electric field strength is E_φ=3 V/cm. In this case u_{res}~$3\cdot10^6$ cm/s, the nonresonant ions velocity is approximately 2 times smaller; the initial thermal velocity of ions is V_{Ti}~$2.5\cdot10^5$ cm/s.

In Fig. 1 the dependence against time $\tau = \omega_0 t$ of the velocities of the gadolinium isotopes G^{157} and G^{158} and their relative velocity, divided by the initial thermal velocity of ions, is shown for the conditions given above [4]. Oscillations of the relative velocity of the isotopes lead to the appearance of the amplitude beats with the period $\tau' = 2\pi\left(A/\Delta A\right) \approx 10^3$, that arise from “instantaneous” switching on of the pumping field. At the initial stage the velocity growth is almost the same for the isotopes of different species. At the stage, when the relative velocity of different isotopes is close to the maximum value, oscillations of the components of the transversal velocity take place in opposite phase. In this case the value of the relative velocity achieves $20V_{Ti}$. If switching on of the pumping field takes place at the moment $t \to -\infty$, then the oscillations with the frequency ω_{ci} disappear. Only the oscillations with the frequency ω_0 remain. In this case the relative amplitude u at $\tau \geq 400$ is equal to $4V_{Ti}$ approximately.

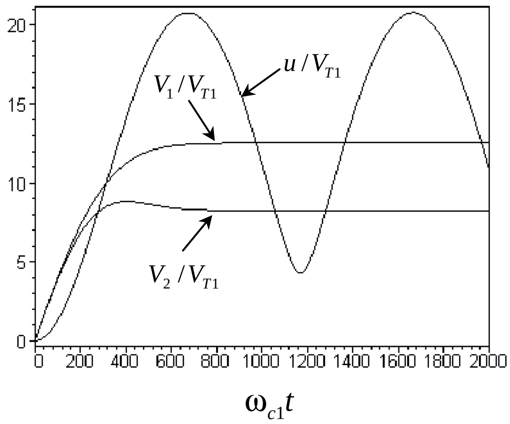

Fig. 1. Time dependence of the velocities of the gadolinium isotopes G^{157} (V_1), G^{158} (V_2) and their relative velocity u divided by the initial thermal velocity of Gd^{157} (V_{T1})

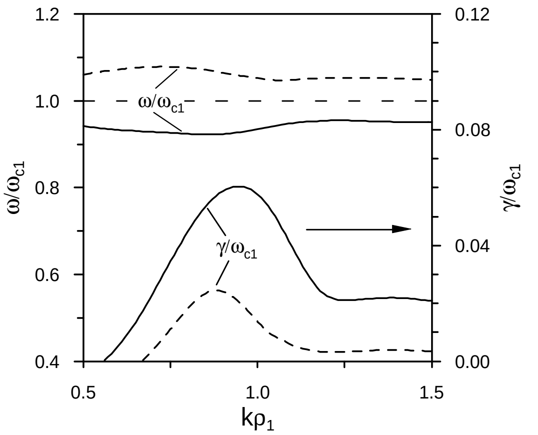

Fig. 2. Dependence of frequencies and growth rates of the unstable oscillations against wave number ($k_{\|}\rho_1$ = 0.01). For oscillations with larger $k_{\|}$ growth rate is 1.5÷3 times less than maximal one

The dispersion equation for unstable oscillations has a form of the infinite determinant that is equal to zero

$$\det|A_{mn}| = 0\,, \qquad (3)$$

$$A_{mn} = \delta_{mn} + \frac{1}{1+\delta\varepsilon_e(\mathbf{k},\omega+m\omega_0)}\sum_{\alpha}\sum_{p=-\infty}^{\infty} J_{p+m}(a_a)J_{p+n}(a_a)\times\,,$$
$$\times\delta\varepsilon_\alpha(\omega+p\omega_0)$$

where a_α is the displacement of ions of α-species in the pumping field, multiplied by the wave vector of the unstable oscillations, $J_p(a_\alpha)$ is the Bessel functions of the first kind, $\delta\varepsilon_\alpha$ is the contribution to the longitudinal plasma permittivity of the particles of α-species in the presence of the oscillations with the average velocity (2).

The solution of the dispersion equation (3) for the parameters [4] in the case of the rest plasma, $\langle V_{\|}\rangle = 0$, is presented in Fig. 2 (it is assumed in computations, that nonresonant ions have a mass of the gadolinium-158 and concentration 0.843, and concentration of gadolinium-157 is equal to its concentration in the natural mixture, 0.157). In Fig. 2 there are two branches of the unstable oscillations, the frequencies of which join the ion cyclotron frequencies (because heavy ions are considered the difference of their cyclotron frequencies is negligible). One can see, that in the region $k\rho_i \approx 1$ (ρ_i - is the Larmor radius of the ions) the difference between unstable oscillations frequencies and the frequency $\omega_0 = \omega_{ci}$ is $\Delta\omega \approx 0.05\omega_{ci}$. The growth rates have a maximum at $k\rho_i \sim 1$. The growth rate of the branch with lesser frequency achieves the value $0.06\omega_{ci}$, and the growth rate of the branch with larger frequency achieves the value $0.025\omega_{ci}$. The longitudinal wave number is small, $k_{\|}\rho_i$ =0.01, but it exceeds the longitudinal wave number of the pumping field $k_{\|0}$=0.03 cm^{-1} three times, that allows to neglect the pumping field nonuniformity .

3. COMPUTER SIMULATION RESULTS

Two-dimensional numerical simulation of nonlinear evolution of the parametric ion cyclotron instability by macro particle technique was carried out for the ions of gadolinium isotopes with the parameters [4]. As distinct from the previous section we will assume, that the oscillations of the ions of different species are defined by formulae (2) with $k_{\|0} \to 0$. In this case the velocity of the resonant isotopes, for which $\omega_{cres} = \omega$, will grow linearly with time, $V_1 = c(E_0/B_0)\cdot t$, and the simulation results obtained can be used up to the time lesser than $\Delta t = (k_{\|0}V_{Ti})^{-1}$, i.e. $\omega_{cres}\cdot\Delta t \approx 300$.

In Fig. 3 the time dependence of the averaged energy density of unstable oscillations of the electric field is shown with the initial value $W_E(0) = n_0T_0\cdot 10^{-3}$.

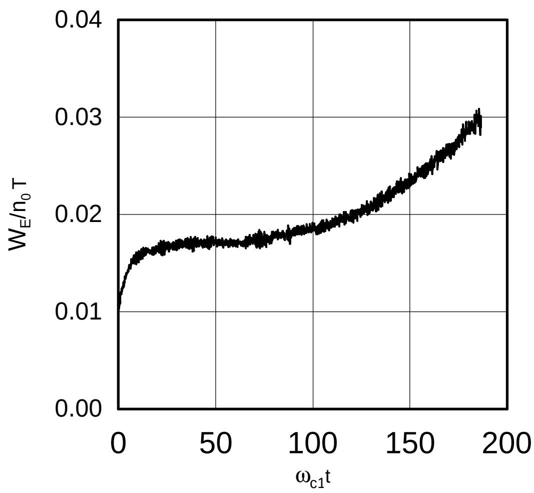

Fig. 3. Time dependence of energy density of the self consistent electric field $W_E = E^2/8\pi$ divided by the initial thermal energy density

The frequencies of these oscillations are of the order of $n\omega_{ci}$. The characteristic wave numbers at $\omega_{ci}t \le 20$ are large, $k\rho_i > 1$. At the large time the oscillations are excited with $k\rho_i \lesssim 1$. Note, the dependence of the value $W_E / n_0 T_0$ against time is nonlinear. It relates to the variation of the current velocities of the particles in time and nonlinear effects in the particles motion in the electric field of unstable oscillations.

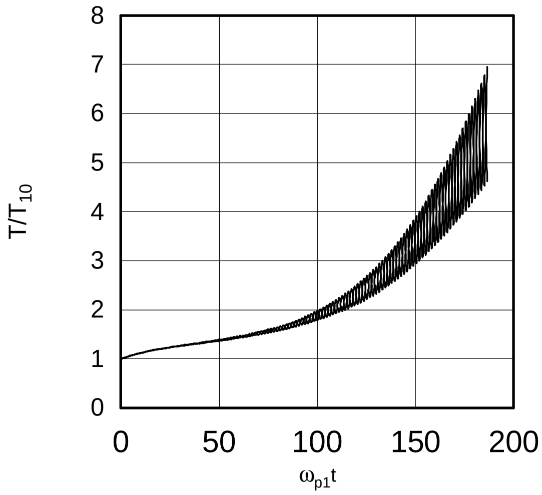

Fig. 4. Time dependence of the averaged chaotic energy of ions of the isotope Gd[157] divided by its initial value

Development of the parametric instability is accompanied by heating the particles. It is illustrated by Fig. 4. In Fig. 4 the time dependence of the relation of transversal chaotic energy of the resonant isotope G^{157} to the initial temperature is shown. At $\omega_{ci}t = 180$ this value exceeds the initial value seven times. The heating is conditioned by nonlinearity of the equations of the particles motion and by the dynamic chaos that can arise. In Fig. 4 at $\omega_{ci}t > 50$ one can see the oscillations with the frequency $2\omega_{ci}$ and with the amplitude growing in time. It is related to high energy ions generation. Transversal chaotic energy of the nonresonant isotope G^{158} grows in time too and it has the value of the order of that is obtained for the G^{157}isotope.

4. CONCLUSIONS

On the ground of computer simulation by means of macro particle technique it is shown, that for the typical parameter values of plasma, magnetic field and alternating electric filed under condition of the ion cyclotron resonance for the gadolinium isotopes mixture, the small-scale potential ion cyclotron oscillations with transversal wave number of the order of the inverse Larmor radius of ions arise in plasma as a result of the parametric instability after the relative velocity of the different isotopes achieves the value of the order of their thermal velocity. This instability results in the turbulent heating of both resonant and nonresonant ions and it can reduce the efficiency of the isotope separation method given.

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ВЛИЯНИЕ ПАРАМЕТРИЧЕСКОЙ НЕУСТОЙЧИВОСТИ ПЛАЗМЫ НА ЭФФЕКТИВНОСТЬ РАЗДЕЛЕНИЯ ИЗОТОПОВ МЕТОДОМ ИОННОГО ЦИКЛОТРОННОГО РЕЗОНАНСА

В.В. Ольшанский, К.Н. Степанов

Решение дисперсионного уравнения для линейных потенциальных ионных циклотронных колебаний с параметрами, характерными при использовании селективного ионного циклотронного резонанса для разделения изотопов гадолиния, показало наличие коротковолновых неустойчивых колебаний в области ионных циклотронных частот. Моделирование этой неустойчивости методом макрочастиц показало, что эти неустойчивости могут быть ответственными за турбулентный нагрев как резонансных, так и нерезонансных ионов, приобретающих большую поперечную энергию, что может снизить эффективность этого метода.

ВПЛИВ ПАРАМЕТРИЧНОЇ НЕСТІЙКОСТІ ПЛАЗМИ НА ЕФЕКТИВНІСТЬ РОЗДІЛЕННЯ ІЗОТОПІВ МЕТОДОМ ІОННОГО ЦИКЛОТРОННОГО РЕЗОНАНСУ

В.В. Ольшанський, К.М. Степанов

Розв'язання дисперсійного рівняння для лінійних потенційних іонних циклотронних коливань з параметрами, характерними при використанні селективного іонного циклотронного резонансу для розділення ізотопів гадолінію, показало наявність короткохвильових нестійких коливань в області іонних циклотронних частот. Моделювання цієї нестійкості методом макрочастинок показало, що вони можуть бути відповідальними за турбулентне нагрівання як резонансних, так і нерезонансних іонів, які отримують велику поперечну енергію, що може знизити ефективність цього методу.