COMPARATIVE ANALYSIS OF EXCITATION OF LSM AND LSE WAVES BY A BUNCH TRAIN IN DIELECTRIC LOADED RECTANGULAR RESONATOR

T.C. Marshall[1], I.N. Onishchenko[2], G.V Sotnikov[2]

[1]Columbia University, New York City, USA;
[2]NSC "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

The results of a three-dimensional analysis of wake field excitation in a slab-symmetric dielectric-loaded resonator by rigid electron bunches are presented. The complete set of solutions, including the solenoidal and potential parts of the electromagnetic field, consists of LSM and LSE modes. Each of the LSM and LSE modes contains odd and even waves. A numerical analysis of wake field excitation by symmetric electron bunches is carried out. The three-dimensional spatial structure of the longitudinal electric field is investigated. The influence of the drift vacuum channel on the wake field amplitude and on the coherent summation of wakefields for a regular sequence of bunches is studied.
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1. INTRODUCTION

Dielectric-lined structures show promise for generation of strong accelerating fields by relativistic electron bunches. Recently attention of specialists in the area of accelerating techniques has been directed to dielectric-lined waveguides with rectangular configuration. Simplicity of manufacturing, possibility of realizing a multimode regime of excitation with equally-spaced frequencies resulting in a significant increase of accelerating field amplitude, easy fine tuning of working frequency, additional intrinsic focusing, and other advantages make dielectric-lined structures in a rectangular configuration attractive for excitation of accelerating fields by a laser pulse or electron bunches.

In a dielectric loaded waveguide of finite length that is excited by a train of electron bunches, the important factors restricting the summation of Cherenkov wakefields of bunches are the transition radiation and the "quenching wave". With the purpose of eliminating these factors, we proposed to use the dielectric resonator [1].

The principles of a resonator concept for the planar dielectric wakefield accelerator have been reported before [2]. They were based on a two-dimensional analysis, neglecting the influence of the bunch vacuum channel upon the eigen-frequencies of the dielectric resonator.

In this work we investigate the excitation of the rectangular resonator, loaded with two symmetric dielectric slabs (DLRR). The separation of waves into LSM and LSE modes [3] is very effective at studying such problems. A numerical analysis of wakefield excitation by symmetric electron bunches of LSM and LSE mode is carried out.

2. FIELD EXPRESSIONS IN THE DIELECTRIC RESONATOR

Let's consider a rectangular metal resonator loaded with oppositely placed dielectric slabs of permittivity ε. The transverse size of a resonator in y-direction is b and in x-direction is a. The transverse size of the vacuum channel in the y-direction is b_1 (the thickness of slabs is $d=(b-b_1)/2$). The length of the resonator is L. Parallel to the dielectric slabs along the z-axis a regular sequence of electron bunches is injected at the plane z=0, which travel through the resonator.

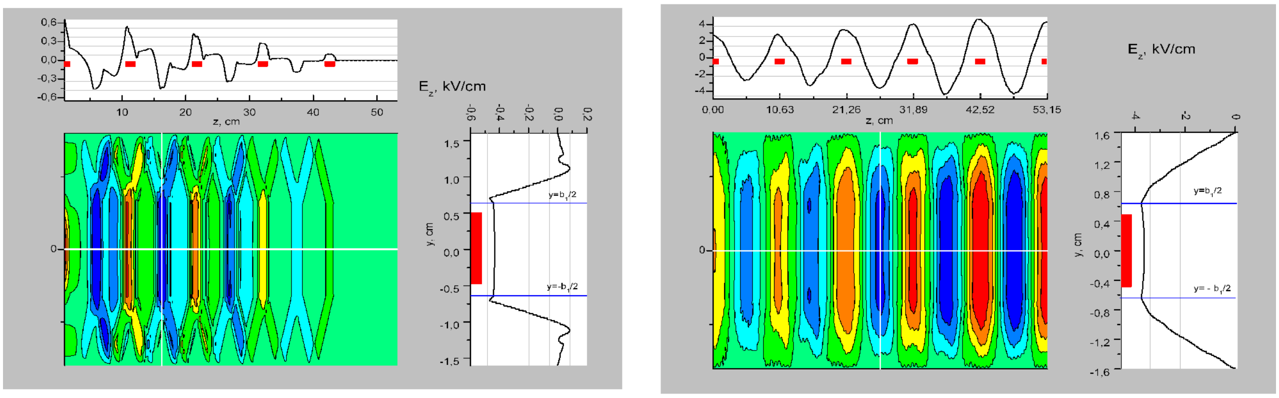
We shall proceed from the following equations for electromangnetic field components transverse to the slabs:

$$\Delta E_y + \frac{\partial}{\partial y}\left[\frac{1}{\varepsilon_y}\frac{\partial \varepsilon_y}{\partial y}E_y\right] - \frac{\varepsilon_y}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 4\pi\frac{\partial}{\partial y}\left(\frac{\rho}{\varepsilon_y}\right),$$
$$\Delta H_y - \frac{\varepsilon_y}{c^2}\frac{\partial^2 H_y}{\partial t^2} = \frac{4\pi}{c}\frac{\partial j_z}{\partial x}, \quad (1)$$

where: $j_z=\rho v_0$, ρ is charge density and $\varepsilon_y=\varepsilon$ for $b_1\le|y|\le b$ or $\varepsilon_y=1$ for $|y|<b_1$.

Having executed a time Fourier transformation in time of the equations (1), (2) and having expanded into a series in harmonic functions with respect to x-,y-coordinates, we shall obtain the equations in ordinary derivatives with respect to the y-coordinate. Solutions of each of the equations jointly contain two sets of independent eigenfunctions. The component H_y contains odd eigenfunctions (symmetric with respect to plane $y=0$) and even eigenfunctions (antisymmetric). The component E_y contains odd eigenfunctions (antisymmetric) and even eigenfunctions (symmetric). All these eigenfunctions are orthogonal with respect to a certain weight factor among themselves; eigenvalues for them are determined from the corresponding four dispersion equations. Having expanded E_y and H_y in a series on eigenfunctions and having executed the inverse Fourier transformation, we can obtain final expressions. Other components of the field can be obtained from the Maxwell equations through E_y and H_y. To obtain expressions for the fields of a bunch having finite transverse size it is necessary to integrate these expressions over the transverse locations of the composing point bunches. We consider symmetric (with respect to planes $x=0$ and $y=0$) electron bunches, therefore upon integration over the locations x_{0i}, y_{0i} of bunches there will remain only the symmetric solutions. For different components of field the symmetric solutions correspond both to even and odd LSE and LSM modes. The longitudinal component E_z of the electric field contains only odd LSE and LSM modes:

$$
\begin{aligned}
E_z = {} & E_0 \frac{v_0}{L}\sum_{l,m,n=1}^{\infty} \frac{(p_v^n c)^2}{\omega_{mnl}^2-\omega_l^2}\,\frac{\omega_l^2 \cos(k_z^l z)}{(\omega_\parallel^{ml})^2}\Bigg\{\Big[\frac{1}{\omega_l}\sin\omega_l(t-t_{0i}) - \frac{1}{\omega_{mnl}}\sin\omega_{mnl}(t-t_{0i})\Big]\theta(t-t_{0i}) - \Big[\frac{1}{\omega_l}\sin\omega_l(t-t_{0i}) \\
& -(-1)^l \frac{1}{\omega_{mnl}}\sin\omega_{mnl}(t-t_{0i}-L/v_0)\Big]\theta(t-t_{0i}-L/v_0)\Bigg\} G_x^m(x_b)G_{y,n}^{LSM}(y_b)\cos(k_x^m x)\frac{e_{\parallel o}^n(y)}{\|D_{mnl}^o\|^2} \\
& - E_0 \frac{v_0}{L}\sum_{\substack{m,n=1\\ l=0}}^{\infty}\frac{\delta_l \cos(k_z^l z)}{\varpi_{mnl}^2-\omega_l^2}\,\frac{\varpi_{mnl}^2}{(\omega_\parallel^{ml})^2}\Bigg\{\Big[\omega_l\Big(1-\frac{\omega_\parallel^{ml2}}{\varpi_{mnl}^2}\Big)\sin\omega_l(t-t_{0i}) - \frac{(k_x^m v_0)^2}{\varpi_{mnl}}\sin\varpi_{mnl}(t-t_{0i})\Big]\theta(t-t_{0i}) - \Big[\omega_l\Big(1-\frac{\omega_\parallel^{ml2}}{\varpi_{mnl}^2}\Big) \\
& \times\sin\omega_l(t-t_{0i}) - (-1)^l \frac{(k_x^m v_0)^2}{\varpi_{mnl}}\sin\varpi_{mnl}(t-t_{0i}-L/v_0)\Big]\theta(t-t_{0i}-L/v_0)\Bigg\} G_x^m(x_b)G_{y,n}^{LSE}(y_b)\cos(k_x^m x)\frac{h_{yo}^n(y)}{\|H_{mnl}^o\|^2}
\end{aligned}
\qquad (2)
$$



a b

Fig.1. Wakefield in the DLRR, shown in the z-y plane (x=0): a) after injection of 5-th bunch, b) after injection of 61-th bunch. Crossline line shows sections of image/contour plot. The corresponding 1D structures of wakefield are presented at the top and at the right of images. Rectangles show bunch locations

where: $E_0 = 32\pi Q_b / ab$; Q_b is bunch charge;
$(p_v^n)^2 = \omega_{mnl}^2/c^2 - (k_\parallel^{ml})^2$, $(p_d^n)^2 = \omega_{mnl}^2\varepsilon/c^2 - (k_\parallel^{ml})^2$,
$(q_v^n)^2 = \varpi_{mnl}^2/c^2 - (k_\parallel^{ml})^2$, $(q_d^n)^2 = \varpi_{mnl}^2\varepsilon/c^2 - (k_\parallel^{ml})^2$;
$G_x^m(x_b) = \sin^2(\pi m/2)\sin(\pi m x_b/2a)/(\pi m x_b/2a)$,
$G_{y,n}^{LSM}(y_b) = \sin(p_v^n y_b/2)/(p_v^n y_b/2)$, $k_\parallel^{ml2} = k_x^{m2} + k_z^{l2}$,
$G_{y,n}^{LSE}(y_b) = \sin(q_v^n y_b/2)/(q_v^n y_b/2)$; δ_l is equal 1 if $l=0$ and is equal 2 if $l \neq 0$; $\omega_l = k_z^l v_0$, $\omega_\parallel^{ml} = k_\parallel^{ml} v_0$.
Functions $e_{\parallel o}^n(y) = \varphi(p^n y)$ and $h_{yo}^n(y) = \varphi(q^n y)$ describe the transverse structure of wakefield

$$
\varphi(g^n y) = \begin{cases} \cos(g_v^n b_1/2)\sin g_d^n(b/2-|y|)/\sin g_d^n d, \\ \text{if } b_1/2 \le |y| \le b/2 \\ \cos(g_v^n y), \text{ else} \end{cases},
$$

and norms $\|D_{mnl}^o\|^2$, $\|H_{mnl}^o\|^2$ of odd LSM and LSE modes are defined in accordance with:

$$
\|D_{mnl}^o\|^2 = \frac{b_1}{b}\Big(1-\frac{\sin(p_v^n b_1)}{p_v^n b_1}\Big) + d\,\frac{(p_v^n)^2\varepsilon^2}{(p_d^n)^2}\,\frac{\cos^2(p_v^n b_1)}{\sin^2(p_d^n d)} \times\Big(1+\frac{\sin(2p_d^n d)}{2p_d^n d}\Big); \qquad \|H_{mnl}^o\|^2 = \frac{b_1}{b}\Big(1+\frac{\sin(q_v^n b_1)}{q_v^n b_1}\Big) + 2\frac{\varepsilon}{d}\,\frac{\cos^2(q_v^n b_1)}{\sin^2(q_d^n d)}\Big(1-\frac{\sin(2q_d^n d)}{2q_d^n d}\Big)
$$

Eigenfrequencies ω_{mnl} of odd LSM modes are determined from the dispersion equation

$$
p_d^n tg(p_v^n b_1/2) - \varepsilon\, p_v^n ctg(p_d^n d) = 0, \qquad (3)
$$

frequencies ϖ_{mnl} of odd LSE modes from the equation

$$
q_d^n ctg(q_d^n d) - q_v^n tg(q_v^n b_1/2) = 0 \qquad (4)
$$

It should be noted, that when

$$
\omega_{mnl} = \omega_l, \quad \varpi_{mnl} = \omega_l \qquad (5)
$$

the relevant items in the expression (2) have removable singularities. The conditions (5) are nothing else than the conditions of Cherenkov radiation in a slowing medium.

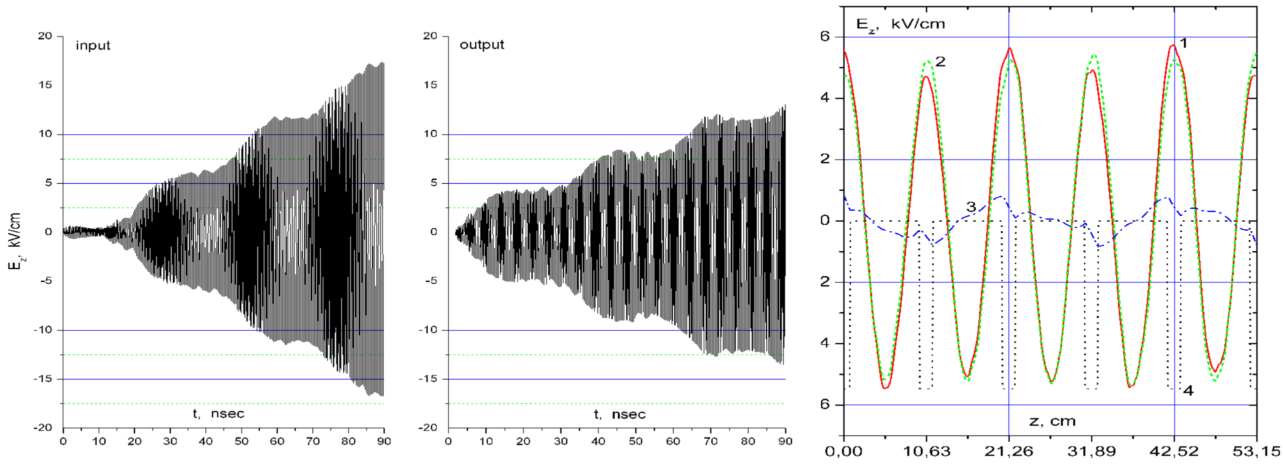
3. NUMERICAL RESULTS

For a numerical analysis of wakefield excitation, a metal resonator with the sizes $a = 1.6\,cm$, $b = 7.4\,cm$ was chosen. Parameters of a sequence of bunches are: repetition frequency $f_0 = 2805\,MHz$, energy of electrons $4.5\,MeV$, macropulse current is $1\,A$, bunch length is $L_b = 1.6\,cm$, transverse sizes $x_b \times y_b = 1\times 1\,cm^2$. For a coherent summation of bunch wakefields, at using a bunch multiplicity $N = 10$, it is necessary [3] to choose the length of the resonator $L = 53.16\,cm$. The thickness of the slab ($\varepsilon = 8.2$) slabs was chosen from the condition (5) For the excitation of the $LSM_{1,1,10}$-mode with frequency f_0 from the equation (3) it follows $b_1 = 6.41\,mm$.

In Fig.1 the structure of the excited wave is presented. During the time before the bunches reach the exit end of the resonator, a wakefield is formed which is similar to the wakefield in a semi-infinite waveguide. The field grows from the head of a bunch, and Cherenkov cones with reflections from the metal walls of the resonator are easily seen. The presence of the Cherenkov cone is the typical sign of multimode radiation in a slow-wave medium. Near the resonator entrance the field of the transition radiation is appreciable. In the direction traverse to the slabs the field is practically homogeneous.

As the next new bunches are injected, the field in the resonator builds up and its amplitude becomes more homogeneous in the longitudinal direction, oscillating in time with the frequency of the $LSM_{1,1,10}$-mode. The dynamics of the longitudinal electric field at the entrance and exit of the resonator is given in Fig.2,a. It is seen that the field in the resonator grows in time on both ends of the resonator and the field of the transition radiation is no longer significant.

In Fig.2,b the axial distribution of wakefields at time t=32.11 ns, after injection of 91 bunches, is shown. From these plots it follows that the dominant contribution to the total field is given by LSM modes.



a b

Fig. 2. a) Dynamics of the E_z at exit (left) and at entrance (right) of DLRR axis (x=y=0); b) Electric field E_z at time t=32.11ns along the axis of resonator ($x = y = 0$) after injection of 91 bunches. Solid line (1) - total field, dashed line (2) - of all LSM modes, dash-dot line (3) - all LSE modes, dotted line (4) - the shape and position of bunches

We also carried out calculations of DLRR excitation in the case where the size of the slabs is chosen so that the frequency of the $LSE_{1,1,10}$-mode is equal to the frequency of bunch repetition ($b_1 = 5.24\,mm$). Qualitatively, the structure of the longitudinal electric field is similar to the one presented above. However, the rate of increase is a little smaller than in the case of the LSM-modes. Again the field of the transition radiation does not appreciably affect the rate of increase of the wakefield.

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REFERENCES

1. T.C. Marshall, J-.M. Fang, J.L. Hirshfield, S.J. Park. Multi-mode, Multi-bunch Dielectric Wake Field Resonator Accelerator// *AIP Conference Proceedings 569*, American Institute of Physics, New York. 2001, p. 316.
2. T.C. Marshall, I.N. Onishchenko, N.I. Onishchenko, G.V. Sotnikov. Mode-Locking In A Dielectric Wake Field Resonator Accelerator//*Proc. of VI Int. Workshop Strong Microwaves in Plasma, Inst. of Appl. Phys. RAS, Nizhny Novgorod, 2006*. V.1, p.277.
3. L. Pincherle. Electromagnetic Waves in Metal Tubes Filled Longitudinally with Two Dielectrics// *Phys.Rev*. 1944, v. 66, N5-6, p. 118-130.

СРАВНИТЕЛЬНЫЙ АНАЛИЗ ВОЗБУЖДЕНИЯ LSM И LSE ВОЛН ЦЕПОЧКОЙ СГУСТКОВ В ДИЭЛЕКТРИЧЕСКОМ ПРЯМОУГОЛЬНОМ РЕЗОНАТОРЕ

Т.С. Маршалл, И.Н. Онищенко, Г.В. Сотников

Представлены результаты сравнительного анализа возбуждения кильватерных полей последовательностью сгустков в прямоугольном резонаторе с симметрично расположенными в нем диэлектрическими пластинами.

ПОРІВНЯЛЬНИЙ АНАЛІЗ ЗБУДЖЕННЯ LSM ТА LSE ХВИЛЬ ЛАНЦЮЖКОМ ЗГУСТКІВ У ДІЕЛЕКТРИЧНОМУ ПРЯМОКУТНОМУ РЕЗОНАТОРІ

Т.С. Маршалл, І.М. Онищенко, Г.В. Сотніков

Представлено результати порівняльного аналізу збудження кільватерних полів послідовністю згустків у прямокутному резонаторі із симетрично розташованими в ньому діелектричними пластинами.