RESPONSE FUNCTION EXTRAPOLATION OF ^{2}H NUCLEUS IN THE REGION OF HIGH TRANSFER ENERGY

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The experimental response functions of the ^{2}H nucleus are extrapolated along the ω-region by the power function for q=1.05 fm^{-1}.

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1. In the modern e,e'-experiment measured are the transversal ($R_T(q,\omega)$) and longitudinal ($R_L(q,\omega)$) response functions of the atomic nuclei (e.g., see [1]). The moments of these functions are calculated by sum rules, being one of the most model-independent theoretical methods.

By definition the moment of a response function is

$$S^{(k)}(q) = \int_0^\infty R(q,\omega)\omega^k d\omega \ , \qquad (1)$$

where k is the moment number, q and ω are the 3-momentum and the energy transfer to the nucleus, respectively. But values of the R_T-function (R^e_T) and the R_L-function (R^e_L) practically are measured as high as $\omega \approx 1/2q$. Therefore, for determination of the moments value, it is necessary to extrapolate the experimental data along ω by some function $R^t(q,\omega)$.

The problem of this extrapolation is very important. This can be seen from the fact that the behaviour of the R^t-function determines the maximum number, k_{max} corresponding to the finite moment value. In this connection it should be noted that G. Orlandini and M. Traini (paper [1], p.275) classified the k_{max} problem as one of main crucial points until clarifying of which "the sum rules risk remaining of only academic interest".

2. In this paper the R^t-function of ^{2}H nucleus is studied. We consider as such function the expression in the form

$$R^t_q(\omega) = C_q \omega^{-\alpha}, \qquad (2)$$

suggested for the R_L-function in the theoretical papers [1], [3], [4] and for the R_T-function in [3]. Here α is the parameter and C_q for q = const is the parameter too. For determination their values we used the R^e_L- and R^e_T-data of the ^{2}H nucleus for q = 1.05 fm^{-1} (Fig. 1) produced at the linear accelerator LUE-300 KhFTI [2]. Since it is not clear from what value of the energy transfer the expression (2) starts to correspond to the behaviour of the experimental data, then at first for the fitting we used a more general expression proposed by V.D. Efros in [5]:

$$R^t(q,\omega) = \omega^{-\alpha} \sum_{n=1}^{N} C_{q,n}\, \omega^{-(n-1)} \ . \qquad (3)$$

The fittings of this expression to different groups of the experimental data demonstrated that the magnitude N = 1 is sufficient both for the R^e_L- and the R^e_T-data in the region of ω ≥ 34 MeV. That is the experimental data of the ^{2}H nucleus reduce the multiparameter expression (3) to the form (2) beginning from the ω value corresponding to the quasi-elastic peak half-altitude. Note that such extrapolation function simplification is not rule. So, in the case of studying of the ^{4}He nucleus R^t_T-function the similar analysis required N = 3 [6].

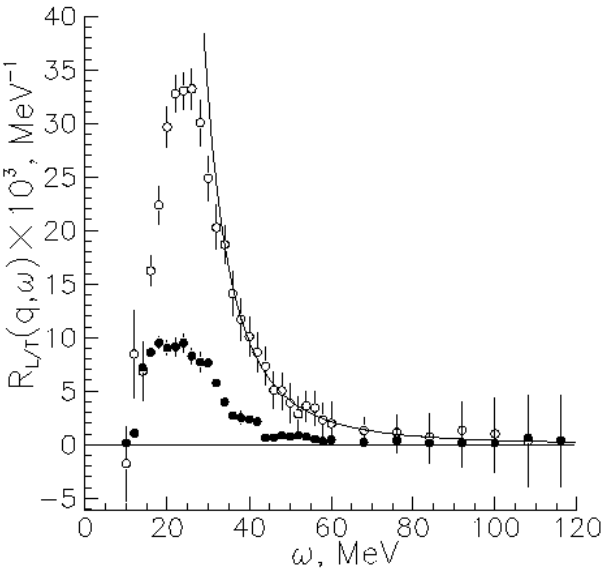

***Fig. 1**. Experimental value of the ^{2}H nucleus response functions: (°) is $R_L(q,\omega)$, (•) is $R_T(q,\omega)$. The curve demonstrates the fitting result of the expression (2) with the R^e_L-data for ω ≥ 34 MeV .*

We examine a dependence of the parameter α_L on the choice of lower (ω_{min}) and upper (ω_{max}) limits of the ω range, where the expression (2) is fitted with the R^e_L-data.

The variations of ω_{max} have shown that α_L and $\Delta\alpha_L$ values are unchangeable for different ω_{max} > 70 MeV. It is explained by a low relative accuracy of the experimental data in this region of the energy transfer.

Fig. 2 shows α_L, $\Delta\alpha_L$ and $\overline{\chi^2_i}$ values as functions of ω_{min} for ω_{max} = 100 MeV. One can see that $\overline{\chi^2_i}$ quickly increases with decreasing ω_{min}, beginning from 34 MeV, where the expression (2) is not able to approximate the experimental data (see Fig. 1). The function $\alpha_L(\omega_{min}) = a + b\omega_{min}$ was fitted with $\alpha_L \pm \Delta\alpha_L$ value defined on different ω_{min} from the interval ω_{min} = 34 – 56 MeV. The fitting result is b = –0,0006 ± 0,0060 MeV^{-1} (Fig.2). The inequality $|\Delta b| > |b|$ shows that the dependence of α_L on ω_{min} is not observable (at least the linear dependence) in the mentioned interval. From here we can conclude that the expression (2) with the parameter α_L = const

describes the response function adequately, and evidences the independence (in limits of the statistical errors $\Delta_s\alpha_L$) of α_L value obtained on the choice of ω_{min}.

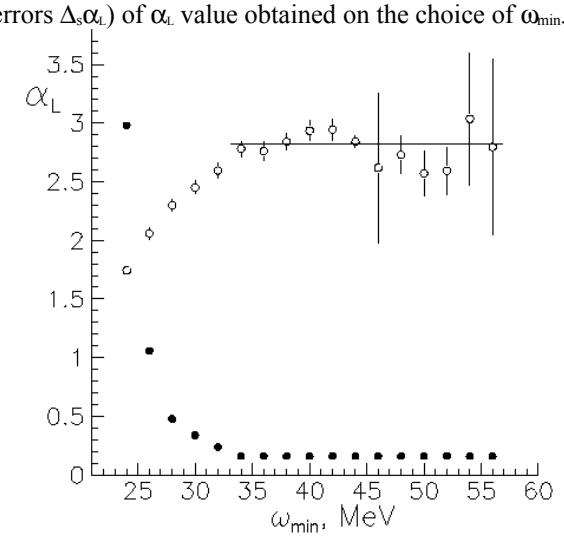

Fig. 2. *The parameter* α_L (°) *and* $\overline{\chi_i^2}$ *(its values* (•) *are given in arbitrary units) are shown as functions of* ω_{min}, *being is the lower limit of the fitting region of the expression (2) with the* R^e_L*-data. The solid line demonstrates the fitting result of the linear function with* α_L *value.*

Analysis of influence on α_L value of the errors of the cross-section normalization, the background-correction and the radiation-correction shows that the statistical error $\Delta_s\alpha_L$ makes a main contribution to the $\Delta\alpha_L$ value and specifies it almost completely.

The solution of the problem of the approximating the R^e_T-data by the expression (2) is slightly differed (methodologically as well as by results) from the above-mentioned case of the longitudinal response function and therefore it is not presented. Also, we have considered the R_T-function in the form proposed in paper [4]:

$$R^t_{T,q}(\omega) = C_q\, \omega^{-\beta} e^{\nu\omega}. \qquad (4)$$

However, the fitting of expression (4) with R^e_T-data for the different ω_{min} gives a wide spread in values of the parameters: $\beta = 1.5 \div 2.6$; $\nu = -0.02 \div -0.10$ MeV^{-1}. The more detail examination of the expression (4) apparently requires the using of the additional experimental data.

With regard to the expression (2), one can conclude that it describes adequately the used R^e_L- and R^e_T-data with the values of parameters:

$$\alpha_L = 2.82 \pm 0.07; \quad \alpha_T = 2.93 \pm 0.15.$$

3. Let us extract from the equation (1) the part where $R(q,\omega) = R^t(q,\omega)$ and write down it for the case of q = const as

$$S^{t\,(k)}_q(\omega_x) = \lim_{\omega_f \to \infty} \int_{\omega_x}^{\omega_f} R^t_q(\omega)\, \omega^k d\omega, \qquad (5)$$

where ω_x is the lower limit of the interval in which the R^t-function is determined. The substitution of Eq.(2) into integral (5) transforms one to

$$S^{t\,(k)}_q(\omega_x) = \frac{C_q}{\alpha - 1 - k}\left[\omega_x^{-\alpha+1+k} - \lim_{\omega_f \to \infty} \omega_f^{-\alpha+1+k}\right]. \qquad (6)$$

We can see that $S^{t\,(k)}_q(\omega_x)$, and therefore the total integral $S^{(k)}_q$, are converged if k is less then $ent(\alpha)$ i.e. integer part of α value:

$$k_{max} = \begin{cases} ent(\alpha) - 1 & \text{if } \alpha \neq ent(\alpha) \\ \alpha - 2 & \text{if } \alpha = ent(\alpha) \end{cases} \qquad (7)$$

According to Eq. (7) the derived α values correspond to $k_{max} = 1$.

4. The extrapolation functions in the light of sum rules give support to experimental defining of the $S^{(k)}_q$ moments. These moments usually are calculated either in the laboratory coordinate system (l.c.s.) or nuclear coordinate system (n.c.s.) connected with the nuclear centre-of-mass after electron interaction. The nonrelativistic coordinate systems (c.s.), in particular l.c.s., are defined by a ω_{el} observable:

$$\omega = \omega' + \omega_{el}, \qquad (8)$$

where ω' is the energy transfer in the n.c.s. For the l.c.s. $\omega_{el} = q^2/(2AM)$, where A is the atomic weight of nucleus, M is the nucleon mass. The moment values and in particular their errors are not always simply to convert from one c.s. to another, thus we suggest better to obtain the experimental $S^{(k)}_q$ values both in the n.c.s. and in the l.c.s. to compare them with the calculated ones. To obtain $S^{(k)}_q$ it is necessary to have the R^t-function in the corresponding c.s.. The R^t-function was considered in the n.c.s. as in ref. [1], [3,4] and in the present paper. Let us find this function in other c.s.

The properties of the R^t-function follow from the invariance of the response function zero moment $S^{(0)}_q$ (here and below it means the invariance respectively to the choice of some c.s. of those which are correlated by Eq. (8)). Write down this moment as

$$S^{(0)}_q = S^{e\,(0)}_q(\omega_x) + S^{t\,(0)}_q(\omega_x), \qquad (9)$$

where the integral $S^{t\,(0)}_q(\omega_x)$ is defined by Eq. (5) and the integral $S^{e\,(0)}_q(\omega_x)$ covers the range of the experimental values of the response function and therefore it can be represented as a histogram area. Since the area of every histogram pole is the product of $S^e_q(\omega_i)$ and $\omega_i - \omega_{i-1}$ and these quantities are not depend on the choice of c.s. then $S^{e\,(0)}_q(\omega_x)$ is not connected with any c.s.. From the invariance of $S^{(0)}_q$ and $S^{e\,(0)}_q(\omega_x)$ follows the $S^{t\,(0)}_q(\omega_x)$ invariance.

The value of the R^t-function at the point ω_x which is the lower limit of the integral $S^{t\,(0)}_q(\omega_x)$ can be represented as

$$R^t_q(\omega_x) = \left.\frac{d\, S^{t,(0)}_q(\omega)}{d\,\omega}\right|_{\omega = \omega_x}, \qquad (10)$$

If it is right that $S^{t\,(0)}_q(\omega_x)$ and $d\omega$ are invariant then the R^t-function is invariant too. The quantity ω_x can assume arbitrary values therefore $R^t_q(\omega_x)$ is invariant for the all domain of the function.

We denote that $R^{t\,el}_q(\omega)$ the invariant R^t-function for some fixed q. This function can be derived from expression (2) if in a power base of this expression to make independent relatively to c.s. The ω_{el} value defines the belonging of the ω to some c.s. Thus $\omega - \omega_{el}$ does not depend on c.s. and the desired function can be written as

$$R^{t\,el}_q(\omega) = C_q(\omega - \omega_{el})^{-\alpha}. \qquad (11)$$

Now we consider this problem by another point of view. The R-function describes a nuclear response on the energy transfer due to the electron scattering on it and in the common case this function depends on c.s. The energy gained by nucleus is distributed on the kinetic energy of the nuclear of centre-of-mass motion, ω_{el} and on the energy of the changing inner nucleus state (the excitation energy) which coincidences with the total energy transfer in the n.c.s., ω'. The excitation energy of nuclear states is the structure characteristic of a nucleus and this energy does not depend on c.s. So, if there is $R(\omega')$ and ω' is the invariant parameter then this function itself is the invariant one. The response function in some c.s., $R(\omega) = R(\omega'+\omega_{el})$ is invariant function $R(\omega')$ shifted along the transfer energy on the ω_{el} magnitude. The power base in the expression (11) is $\omega-\omega_{el}$ that is equal to ω'. From this fact follows: $R(\omega') \equiv R^{t}_{q}{}^{el}(\omega)$. We note, that $R(\omega'+\text{const})$ is the invariant too, but there is no special meaning at such presenting when const $\neq 0$.

The total moment $S_q^{(k)}$ is not invariant at $k \neq 0$. Therefore there is a question: how the k_{max} value is connected with choice of c.s.? To answer this question we substitute the R^t-function, in the form (11), into the integral (5). As the result we derive:

$$S_q^{t,\,(k),\,el}(\omega_x) = \sum_{n=0}^{k} \binom{n}{k} \frac{C_q \omega_{el}^n}{\alpha + n - k - 1} \chi \left[(\omega_x - \omega_{el})^{-\alpha - n + k + 1} - \lim_{\omega_f \to \infty} (\omega_f - \omega_{el})^{-\alpha - n + k + 1} \right] . \quad (12)$$

The term where $n = 0$ is the most slowly decreasing with the increase of ω_f, thus it defines a condition when the total sum has a finite value. This summand itself is identically equal to expression (2). So, Eq.(7) is valid for the form (11). Therefore from the invariance of α parameter follows the invariance of k_{max}.

5. We compare the obtained data with the theoretic calculations. In [3] $\alpha_L = \alpha_\mu = 2.5$ was defined basing on the relatively simple model. The parameter α_μ is related with the contribution into the R_T-function of electron scattering on nucleon magnetic moments. This contribution at the $q = 1$ fm^{-1} according to the calculation of [4] is about 90%, thus $\alpha_\mu \approx \alpha_T$ and $\alpha_L \approx \alpha_T$. The experimental data give $\alpha_T - \alpha_L = 0.11 \pm 0.17$. The more exact calculation with the using the nucleon Reid soft-core potential gives $\alpha_L = 3 - 4$ [1]. Taking into account the spread of calculation values this result does not contradict to the experimental one: $\alpha_L = 2{,}82$.

The moment $S^{(k)}$ is calculated for $k = 2$, 3 in a number of some theoretic papers (see [1], [7], [8], [9]). We have derived that the moment $S^{(k)}$ is diverged at the $k > 1$, basing on the response function extrapolation by form (2) and experimental values of α. In view of the complexity and importance of the k_{max} problem we suppose that it is expedient to restrict ourselves in this paper by the fact of so low experimental k_{max} value [1]/ and to point out that it is a good idea to make research, using the greater experimental data body.

[1]/ The fact that $k_{max} = 1$ follows from Eq.(7) for the magnitudes of $\alpha = 2.5$, 3.0 of [3], [1] too.

In terms of the experimental determination of $S^{(k)}$ values we note that since, in accordance with [1], the extrapolation function does not depend on the transfer momentum and faintly depends on the atomic number of nucleus, the defined parameters α_L, α_T and also expression (11) for the R^t-function in the l.c.s. can be used for studying not only ^{2}H nuclei but nuclei of other elements too. The revision of the R^t-function should have the most effect while determining the moments with $k = 1$ when the extrapolation part is of about 15-40% of the measured value.

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