THE KINEMATIC CORRECTION TO A REACTION OF QUASIELASTIC ELECTRON SCATTERING BY INTRANUCLEAR NUCLEONS

S.V. Dementiy

National Sciece Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

The problem of compatibility and interpretation of the data on quasi-elastic electron scattering on nuclei in a reaction $A(e,e'p)$, obtained in essentially various kinematic conditions is considered. It is shown that in such cases it is necessary to introduce the kinematic correction into the intranuclear nucleon momentum defined by the laws of energy and momentum conservation. The correction value can be defined as a first approximation from the inclusive $A(e,e')$ reaction. The reasoning are illustrated by an example of a reaction 4He($e,e'p$)T.

PACS 25.30. Fj

The reactions of quasi-elastic scattering on the nuclei from the very beginning of their investigation were considered as a rather promising tool for studying the structure of atomic nuclei. In particular, the quasi-elastic electron scattering, owing to relative weakness and well-known (both experimentally, and theoretically) character of electromagnetic interaction of a scattered particle with intranuclear nucleons, has allowed at once to make particular deductions concerning the momentum distribution of the latters inside the nuclei.

However, with upgrading the experimental technique and improving of precision of experiment, the difficulties in interpretation of the data obtained have arisen. So, the processing of data from papers [1,2], (most precise results fort he reaction ^{4}He(e,e′p)T), have met considerable difficulties not only when analyzing them together but also separately within the framework of each of these works. For the question be clear, we refer to a graph in Fig. 1. The figure presents the values of the spectral function $S(E_m, p_m)$ of the ^{4}He nucleus, measured under various kinematic conditions, which in a plain-wave approximation also represents momentum distribution of intranuclear nucleons. Two series of points are measured at the momentum transfer $q = 251$ *MeV/c* (kinematics I [1]) and $q = 435$ *MeV/c* (kinematics II [1]) with the varying parameter $p_m = 15 - 205$ *MeV/c* and $p_m = 115 - 345$ *MeV/c,* respectively (see Table 1). Three series of points are measured with three constant values of the parameter $p_m = 30$, 90 [2] and 100 *MeV/c* [1], but with varying values $q = 299 - 650$ *MeV/c*. One point is measured at $q = 680$ *MeV/c* and $p_m = 190$ *MeV/c* [2]. The curve shows the theoretical momentum distribution of a proton in a proton–triton system, bound in the ^{4}He nucleus [3]. Besides, in order to be possible to consider the data at small and at large values of p_m, the graph is shown in two vertical scales: linear (Fig. 1, *a*) and logarithmic (Fig. 1, *b*)

Here it is necessary to notice the following. To have a possibility to compare the data of paper [2], obtained with separation on longitudinal and transverse parts, and the data of paper [1], obtained without such separation, the nonseparated values $S(E_m, p_m)$ were calculated using the cross sections measured under forward scattering angles of electrons from [2].

As is seen from Fig. 1, points obtained at $q = 251$ *MeV/c* (open circles) or 435 *MeV/c* (solid circles) [1] are perfectly consistent within the frame of one branch. At the same time, the mean ratio of values of these branches in the overlapping area is about 1.7. Similarly, the points obtained with constant values p_m, but with varying values of q, that in principle should give the same values $S(E_m, p_m)$, give essentially different values. Scattering of the points in these cases noticeably exceeds the experimental errors.

Such situation has allowed the authors of papers [1,2] to confirm the presence of the dependence of values $S(E_m, p_m)$ on kinematic conditions, under which these values were obtained.

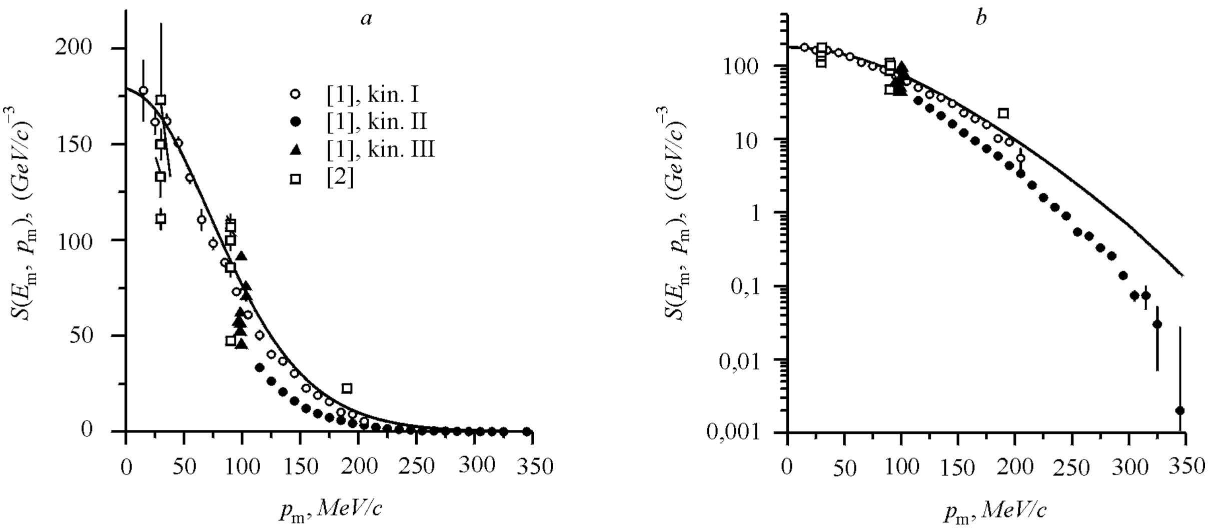

Fig. 1. *Spectral function $S(E_m, p_m)$ of ^{4}He nucleus versus p_m. Experimental points obtained in kinematics I, II, III [1] (see table 1) at $p_m = 15 - 205$, $115 - 145$ and 100 MeV/c, respectively and at $p_m = 30$, 90 and 190 MeV/c [2]. The curve shows the theoretical calculation for the model of ^{4}He nucleus with a potential Urbana V14NN [3].*

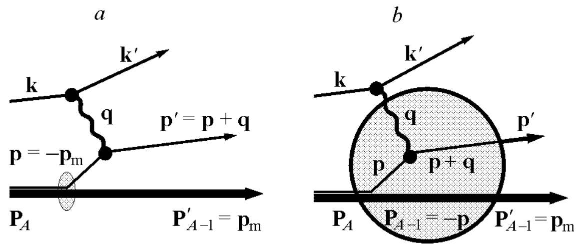

Fig. 2

To understand an essence of the arisen problem, we shall consider some details of the quasi-elastic scattering process. As a major part of time, the nucleons in the nucleus do not interact (are quasi-free), for the main approximation one usually takes the elementary diagram (Fig. 2, *a*; see for example [4]). Then the cross section of the process is expressed as:

$$\frac{d^5\sigma}{dk' d\Omega_e d\Omega_p} = K\sigma_{ep} \sum_i S(E_{mi}, p_m) . \qquad (1)$$

Here K is the known kinematic factor, σ_{ep} is the cross section of elastic electron-proton scattering with the proton being off the mass shell [5]. Then the proton is considered to be free (on mass shell). E_{mi} is the energy of separation of a nucleon (mass M) from the nucleus (mass M_A) with formation of the residual nucleus of a definite mass $M_{A\text{-}1,i}$:

$$E_{mi} = M + M_{A-1,i} - M_A, \qquad (2)$$

and p is the value of a nucleon momentum inside the nucleus. In the case of proton knocking-out from the nucleus in the ^{4}He($e,e'p$)T reaction there is a only bound state of a residual nuclear system, i.e. the tritium nucleus. Thus E_{mi} = 19,8 MeV.

The laws of energy and momentum conservation determine a kinematics of the process:

$$\omega + M_A = \sqrt{(\mathbf{p}')^2 + M} + \sqrt{\mathbf{p}_m^2 + M_{A-1}^2} , \qquad (3)$$

$$\mathbf{p}' = \mathbf{q} - \mathbf{p}_m . \qquad (4)$$

Here q and ω are the momentum and energy transferred by an electron, p' is the momentum of a knocked out nucleon and p_{mi} is the momentum of the residual nucleus.

To make a kinematics definite and to have a possibility to determine the momentum of an intranuclear nucleon, the following natural supposition is usually made. As stated above, the knocked out nucleon practically does not interact with residual nucleus. Therefore, it is possible to consider that the recoil nucleus does not change its momentum. But since the initial nucleus in a laboratory system is at rest, it follows at once that (Fig. 3, *a*):

$$\mathbf{p} = - \mathbf{p}_m . \qquad (5)$$

The supposition we have made is quite enough for the missing momentum p_m.in the reaction ($e,e'p$) at given momenta of initial and final electrons and recoil nucleon be restored.

In such a case it can be supposed, that all discrepancies, which was observed above (see Fig. 1), have in the basis a poor correctness of the statement (5).

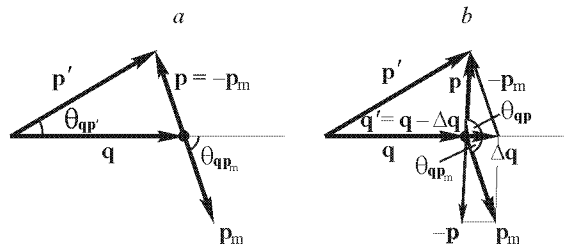

Fig. 3. *Vector diagrams for 3-momenta.*

Really, at all naturalness of reasons, which have resulted in the statement (5), it should be recognized, that they and together with them equation (5) do not take into account the fact, that the particles, components of the nucleus, are outside of a mass shell. In particular, if we consider the reaction ^{4}He($e,e'p$)T, with a proton knocking-out then both particles (proton and triton) should be simultaneously put onto the mass shell. In the above given reasoning it is implicitly supposed, that the electron energy transfer ω puts onto the mass shell only the proton, that can not in any way affect a state of the triton, as we have neglected the interaction between them. Since one should to substitute the mass of a free triton in equation (3), then it is implicitly supposed that the triton already is on the mass shell.

Thus, we should conclude, that during transition of final particles onto the mass shell they should necessarily interact. Thus the redistribution of the transferred momentum is quite possible similarly to redistribution of the transferred energy according to equation (3).

The diagram (Fig. 2, *b*) represents the above said as absorption of a virtual γ-quantum inside the nuclear volume (conventionally shown by the shaded area). It is obvious that at this stage the process has the same character, as in the diagram on the left (Fig. 2, *a*). In other words, electrodynamics of process, in essence, is the same. After the proton exits from the nuclear volume the proton and residual nucleus become free and parameters of their motion should be submit to equations (3) and (4).

One can imagine, that we have created a new difficulty for ourselves, since we do not know, how the transferred momentum is shared between reaction products without having received anything instead. To leave from this difficult position, we appeal to an inclusive reaction (e,e'), which has the same nature. Its cross section is an integral of cross section (1) on all the knocked-out protons:

$$\frac{d^3\sigma}{dk' d^2\Omega_e} = N[z\sigma_{ep} + (A - z)\sigma_{en}]\ F(y), \qquad (6)$$

where z is the number of protons and $(A - z)$ is the number of neutrons in the nucleus; $\sigma_{ep(n)}$ is the cross section of elastic scattering of an electron on a proton (neutron), N is a known kinematic factor, $F(y)$ is a scaling function, symmetric relatively to the value y=0.

$$F(y) = \int_{|y|}^{\infty} \rho(p)\, p\, dp \qquad (7)$$

where $|y| = p_{min}$ is the minimum value of nucleon momentum inside the nucleus, at which the interaction between the electron and the intranuclear nucleon is possible.

The value y is defined from the equation, which expresses the law of energy conservation at a lower limit of the integral in (7):

$$\omega + M_A = \sqrt{(q+y)^2 + M^2} + \sqrt{y^2 + M_{A-1}^2}\ . \quad (8)$$

It can take both positive, and negative values relevant to cases of vectors $\mathbf{p}_{min}$ and $\mathbf{q}$ being parallel ($y > 0$) or antiparallel ($y < 0$).

Since $\sigma_{ep(n)}$ and N at a fixed energies of initial and final electrons and scattering angle are slowly varying functions of ω, from (6) and (7) follows that the cross section should reach a maximum at $y = 0$. At the same time, it is known that in practice in many cases a maximum of a scattered electron spectrum is placed at negative values of y. In particular, such situation was observed when investigating the inclusive reaction of quasielastic electron scattering by the nuclei ^{4}He and ^{12}C [6,7]. The shift of the maximum of scattered electrons spectrum (at least in a case of ^{4}He) to the $y < 0$ region, observed in these works, could not be caused by the presence of any competing channel, since a two-particle disintegration of ^{4}He by a proton and triton (or by a neutron and ^{3}He) goes with a minimum separation energy (~20 MeV), and no states with a smaller excitation energy exist. Therefore this shift was interpreted in [6,7] as a modification of momentum effectively transferred to a proton by the Δq value ($\Delta q = y_{max}$):

$$\mathbf{q}' = \mathbf{q} - \Delta\mathbf{q}\ . \quad (9)$$

Simultaneous change of a recoil nucleus momentum on magnitude $\Delta\mathbf{q}$ is supposed. As is seen from Fig. 3, *b*, such modification of particle momenta, does not break a law of momentum conservation, and allows restoring their initial values. By means of that there is a possibility to correct values p_{min}.

In [6,7] with the experimental accuracy reached in these papers, is shown, that the values of Δq, obtained at the same values q, coincide, i.e. Δq is a function of q. Thus, there is a possibility to determine by behavior of a reaction (*e,e'*) cross section, the kinematic correction in interpretation of data for the reaction (*e,e'p*).

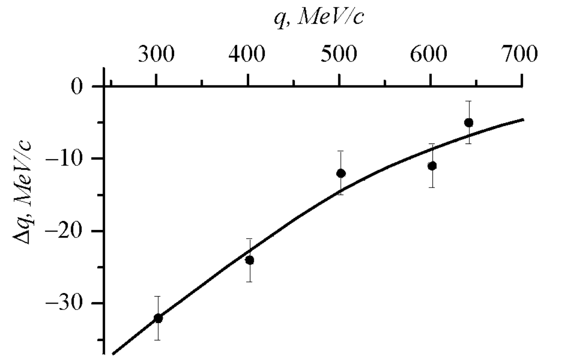

Fig. 4. *Experimental estimation of dependence Δ q(q) based on results of [8].*

To obtain an estimation of Δq values, the results of investigation of y-scaling on the ^{4}He nucleus [8] were used. In this paper experimental values of longitudinal $F_L(y)$ and transverse $F_T(y)$ scaling functions are presented. For each of five of presented in [8] experimental functions $F_L(y)$ the values y_{max} were defined and taken as Δq. The obtained estimation of experimental dependence $\Delta q(q)$ is shown in a Fig. 4. The smooth curve, drown through the points, was used for interpolation Δq.

In Table 1 the calculated kinematic conditons under which in [1] the reaction ^{4}He(e, e'p)T was investigated are shown. In the first column of the table the designation of a kinematics, as assumed in [1], is given. In next five columns the values of kinematic parameters, related with an electron scattering are shown: magnitudes of initial k and final k' momenta of electron, scattering angle $\theta_{\mathbf{k}'}$, momentum transfer q and angle $\theta_{\mathbf{q}}$ between vectors $\mathbf{k}$ and $\mathbf{q}$. The following three columns contain magnitudes of a recoil proton momentum p', angle $\theta_{\mathbf{p}'}$

Table 1. *All momentums and angles are given in MeV/c and degrees, respectively*

Kine-matiks	k	k'	$\theta_{\mathbf{k}'}$	q	$\theta_{\mathbf{q}}$	p'	$\theta_{\mathbf{p}'}$	p_m	Δq	p	Δp
I–1	425,6	310	70,42	435	–42,24	430	–47,05	37	–20	38	2
I–2	425,6	310	70,42	435	–42,24	427	–53,75	87	–20	85	–1
I–3	425,6	310	70,42	435	–42,24	423	–60,58	137	–20	134	–3
I–4	438,9	426	66,90	432	–43,96	417	–67,64	175	–20	171	–5
II–1	425,6	426	35,99	251	–49,81	382	–57,70	138	–37	173	35
II–2	425,6	426	35,99	251	–49,81	376	–73,17	185	–37	203	23
II–3	425,6	426	35,99	251	–49,81	368	–84,55	216	–37	228	12
II–4	481,7	382	31,07	250	–51,91	359	–97,52	253	–37	256	3
II–5	425,6	326	35,99	251	–49,81	347	–106,48	296	–37	291	–5
II–6	425,6	326	35,99	251	–49,81	434	–106,48	336	–37	324	–12
III–A	425,6	351	63,88	416	–49,30	313	–48,89	103	–21	84	–21
III–B	425,6	306	93,31	538	–34,57	434	–34,30	104	–11	92	–11
III–C	425,6	335	38,09	263	–51,92	361	–51,09	99	–36	134	36
III–D	425,6	282	62,19	386	–40,74	484	–40,15	98	–24	120	24
III–E	481,7	409	52,99	403	–54,21	308	–60,17	102	–23	83	–21
III–F	481,7	384	53,99	402	–50,51	380	–64,45	97,5	–23	92,5	–5
III–G	481,7	359	54,86	402	–46,86	442	–58,72	96	–23	104	10
III–H	481,7	334	55,58	402	–43,25	498	–45,38	97	–23	117	23

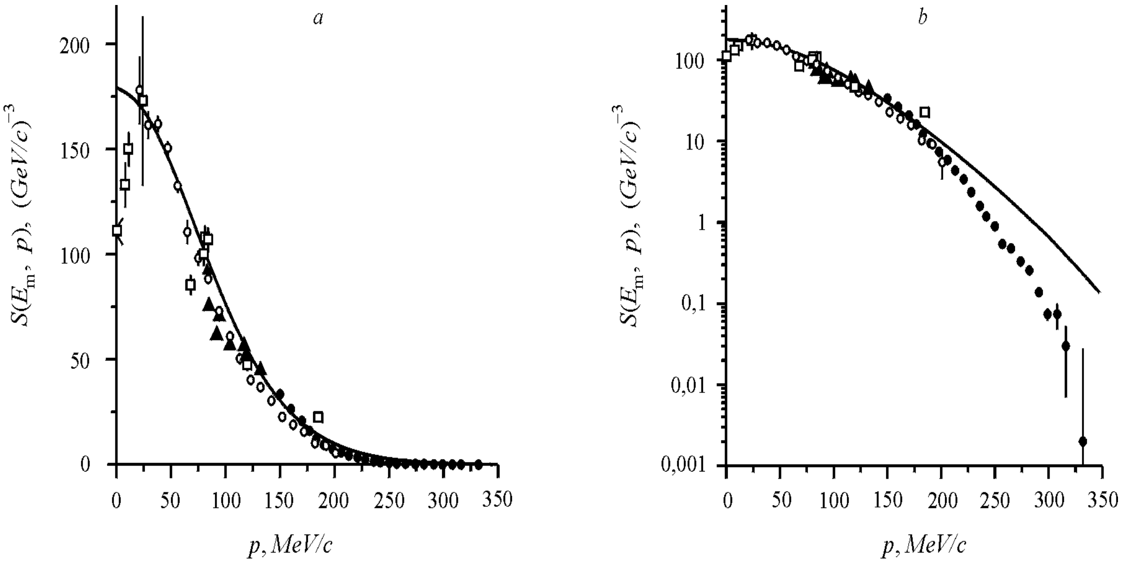

Fig. 5. *The same data, as in Fig. 1, but versus a modified nucleon momentum p.*

between **k** and **p**′ and mean value of p_m in the given kinematics [1]. In last three columns the values of Δq used in calculation, mean value of an initial proton momentum **p** and mean resulting correction to six values of p_m recorded at each installation [1] are given. The calculations were based only on geometric considerations, clear from Fig. 3.

Table 2

#	*q*, *MeV/c*	ω, *MeV*	*p*′, *MeV/c*	p_m, *MeV/c*	Δ*q*, *MeV/c*	*p*, *MeV/c*
1	299	57,78	269	30	–30	0
2	380	83,13	350	30	–24	6
3	421	98,19	391	30	–21	9
4	650	206,32	620	30	–6	24
5	299	98,70	389	90	–32	122
6	380	65,06	290	90	–25	65
7	544	125,60	454,6	90	–12	78
8	572	137,82	482	90	–10	80
9	650	176,67	560	90	–6	84
10	680	146,48	490	190	–5	185

In Table 2 the results of calculations of the intranuclear proton momenta for kinematic conditions of [2] are given. The columns 2 and 3 contain the adjusting values of transfer momentum and energy. In 4 and 5 columns the values of the final proton momentum which was registered in the direction of momentum transfer, and the magnitude $p_m = |\mathbf{q} - \mathbf{p}'|$ are given. Last two columns contain the values Δq, corresponding to each q value, and corrected values of the initial proton momentum, which accordingly to the collinearity **q** and **p**′ are equal to $\mathbf{p} = |\mathbf{q}_1 - \mathbf{p}'|$ (see Fig. 3).

The influence of the correction to the proton momentum on the result is shown in Fig. 5. It is seen, that the situation in comparison with that shown in Fig. 1 is essentially improved, and all 41 points obtained in the wide area of kinematic conditions describe a smooth dependence on *p*. Thus the hypothesis taken in a basis of calculation of the correction proves to be true.

At last modification of the *p* gives us an unexpected deep at $p < 30$ MeV/c. It is not the aim of this publication to explain existence of the deep, but it is impossible to leave the fact without any comments.

The first two points (kinematics 1 and 2, see Table 2), as well as the point at $p = 68$ MeV/c (kinematics 6) were measured at low energy transfer $\omega < 100$ MeV/c, where considerable suppression of the quasielastic scattering process is expected [6,7]. Estimation of the suppression made from the ^{4}He(*e,e*′) reaction [6] shows a possible effect of about 20%. Obviously, such value is not able to explain the situation.

REFERENCES

1.J.F.J. van den Brand et al. The two-body electrodisintrgration of ^{4}He studied through the (*e,e'p*) reaction // *Nuclear Physics*, 1991, v. A534, p. 637-679.
2.J.F. Ducret et al. The longitudinal and transverse structure functions of the ^{4}He(*e,e'p*)T reaction // *Nuclear Physics*, 1993, v. A556, p. 73-395.
3.R. Shiavilla, V.R. Pandharipande, R.B. Wiringa. Momentum distributions in A = 3 and 4 nuclei // *Nuclear Physics*, 1986, v. A449, p. 219-242.
4.S. Frullani, J. Mougey. Single-particle properties of nuclei through (*e,e'p*) reactions. // *Advances in Nuclear Physics*, 1984, v. 14, 289 p.
5.T. De Forest. Off-shell electron-nucleon cross section. The momentum approximation // *Nuclear Physics*, 1983, v. A392, p. 232-248.
6.S.V. Dementiy. Experimental investigation of inclusive quasielastic scattering of electrons on the nucleus 4He // *Yadernaya Fizica*, 1988, v. 48, p. 609 – 617 (English translation: *Soviet Journal of Nuclear Physics*, 1988, v. 48, p. 389-398).
7.S.V. Dementiy. Experimental y-scaling on nuclei. // Proceedings of the 7-th seminar "*Electromagnetic Interactions of Nuclei at Low and Medium Energies*". (Moscow, December 12–14 1988 г). Moscow, INR AC USSR, 1990, p. 5-13.
8.A. Zghiche et al. Longitudinal and transverse reactions in quasi-elastic electron scattering from ^{208}Pb and ^{4}He // *Nucl. Phys.*, 1994, v. A572, p. 513-559.