FEATURES OF THREE WAVE INTERACTION IN THE MAGNETOACTIVE PLASMA

V.A. Buts, I.K. Koval'chuk

National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukrane

E-mail: vbuts@kipt.kharkov.ua

The decay processes of waves propagating in the nonlinear gyrotropic media were investigated. It was shown that the dynamics of the process does essentially change if conditions for Faraday effect are realized. The most important result is the conclusion that the presence of gyrotropy can lead to suppression of the decay instability. This means that in such conditions the plasma wave will propagate without attenuation. The analytical conclusions are confirmed by numerical results.

PACS: 52.35.Mw

INTRODUCTION

By the present time, nonlinear wave interaction in different media has been investigated quite minutely. As examples, optics, hydrodynamics, plasma physics, etc. can be exemplified. Especially thoroughly were studied so called three wave processes, when three natural waves of a physical system are taking part in nonlinear interaction. The well-known example of such interaction is the decay instability. On the other hand, there is the Faraday effect in a gyrotropic plasma. Separately both the Faraday effect and processes of three wave interaction has been investigated quite fully and in detail. However, the processes of nonlinear wave interaction in magnetoactive plasma when Faraday effect is taken into account were not investigated yet. In this report the results of investigation of such processes are presented.

First of all the equations describing such processes were obtained. In the common case these are complicated equations and their analysis may be carried out by means of numerical methods only. But there are some conditions when consideration of these processes is essentially simplified. In particular, such simplify-cation is possible at weak gyrotropy. In this case the dispersion properties of the system are close to properties of vacuum dispersion. Such conditions may exist in rare plasma ($\omega_p^2 \ll \omega^2$) and frequencies (ω) are not close to electron cyclotron or to upper hybrid frequencies.

The Faraday effect may be considered as energy exchange between two linear oscillators which are *E*- and *H*-waves accordingly. It may be expected that, if the period of this exchange process is sufficiently short, then all other processes having longer characteristic time will be suppressed. The mechanism of stabilization of such states is described in [1, 2].

Below we present: the increments of *E*- and *H*-wave decays and parameter of linear energy exchange, the ranges of parameters where linear transfer and decay processes are possible, the assessments of threshold amplitudes when decay process is possible, and the results of numerical investigation of the equations describing nonlinear processes in magnetoactive plasma. It was shown that linear interaction between *E*- and *H*-wave can result in suppression of decay instability.

The obtained results can be useful to overcome the limitation on amplitudes of waves excited at beam-plasma interaction [3, 4].

1. PROBLEM DEFINITION AND BASIC EQUATIONS

Let us consider an infinite magnetoactive plasma. We suppose that external uniform magnetic field is directed along z- axis. The permittivity of such plasma is described by means of tensor of the form (see, for example [5]):

$$\begin{pmatrix} \varepsilon_\perp & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \ \varepsilon_\perp = 1 - \frac{\omega_p^2}{\omega^2 - \omega_e^2}, \tag{1}$$

$$\varepsilon_\parallel = 1 - \frac{\omega_p^2}{\omega^2}, \qquad \varepsilon_2 = -\frac{\omega_p^2 \omega_e}{\omega\left(\omega^2 - \omega_e^2\right)},$$

where ω_p is plasma frequency, ω_e – electron cyclotron frequency, ω – frequency of wave. If the waves propagate at some angle to magnetic field the coordinate axes can be directed in such a way that wave vector will be in the *yz* plane. Only the case when electromagnetic waves do not depend on *x* coordinate will be analyzed, with additional supposition that wave vectors of interacting waves are in the same plane.

To investigate dynamics of nonlinear wave interaction, the Maxwell equations describing electro-magnetic field components and hydrodynamic equations for plasma electrons will be used. It is supposed that spatial-temporal dependence of all electromagnetic field components has the form:

$$E, H \sim E_i, H_i(z) \exp\left(i\omega_i t - ik_{yi} y\right), \tag{2}$$

where ω_i is frequency of *i*th wave, k_{xi}, is *y* component of its wave vector. Expressing from Maxwell equations all *x* and *y* components of electromagnetic field through E_z and H_z, the following set of ordinary differential equations can be obtained for these components:

$$-\frac{\varepsilon_{\perp i}^2 - \varepsilon_{2i}^2}{\varepsilon_{\perp i}} \frac{\omega_i^2}{c^2} H_{zi} - \frac{d^2 H_z}{dz^2} + k_{yi}^2 H_{zi} =$$

$$= \frac{\varepsilon_{2i} \varepsilon_{\parallel i}}{\varepsilon_{\perp i}} \frac{\omega_i}{c} \frac{dE_{zi}}{dz} + \frac{4\pi}{c} \left(rot\, \vec{j}\right)_z,$$

$$-\varepsilon_{\parallel i}\frac{\omega_i^2}{c^2}E_{zi}-\frac{\varepsilon_{\parallel i}}{\varepsilon_{\perp}}\frac{d^2E_{zi}}{dz^2}+k_{yi}^2E_{zi}= \\ =-\frac{\varepsilon_{2i}}{\varepsilon_{\perp i}}\frac{\omega_i}{c}\frac{dH_{zi}}{dz}-i\frac{4\pi\omega}{c^2}j_z. \tag{3}$$

The second terms in the right parts of these equations describe nonlinear interaction. The index *i* means that corresponding value belongs to characteristics of *i*th wave. It is following from this set that in gyrotropic plasma the *E*- and *H*-waves are not independent. Below we will be interested in weak girotropy, when $\varepsilon_2 << 1$. This is the case of low density plasma when frequencies of interacting wave are not near the electron cyclotron frequency or upper hydride one. In such case dispersion properties of media are close to vacuum dispersion. Let's consider the influence of linear coupling on the nonlinear interaction of eigenmodes. To move further, the dependence of z-components of the electric and magnetic fields on z coordinate are presented as follows:

$$E_{zi},H_{zi}\rightarrow E_{zi}(z),H_{zi}(z)\exp\left(-ik_{zi}z\right), \tag{4}$$

where $E_i(z)$, $H_i(z)$ are dimensionless and slowly varying amplitudes of *z* components of electric and magnetic fields for *i*th wave. We will limit ourselves by three wave nonlinear interaction. To obtain shorted equations describing dynamics of slowly varying amplitudes in the three wave nonlinear interaction process, the method presented, for example, in [6…8] can be used. The conditions of synchronism may be taken according to:

$$\begin{aligned}&\omega_1=\omega_2+\omega_3,\\&k_{y1}=k_{y2}+k_{y3},\\&k_{z1}=k_{z2}+k_{z3}+\delta k_z,\end{aligned} \tag{5}$$

where δk_z – detuning for *z* component of wave vector ($\delta k_z \ll k_{zi}$).

After some mathematic transformation the set of ordinary differential equations can be obtained, which describe the dynamics of nonlinear interaction of natural modes in magnetoactive plasma taking into consideration the influence of energy exchange between *E*- and *H*-waves:

$$\begin{aligned}
\frac{dH_{z1}}{dz}&=\frac{\omega_p^2\omega_e(\omega_1^2-\omega_p^2)}{2\omega_1^2c\left(\omega_1^2-\omega_e^2-\omega_p^2\right)}E_{z1}+\\
&+\frac{k_{y1}\omega_p^2\omega_1\exp(-\delta k_z z)}{2k_{z1}(\omega_2^2-\omega_e^2)(\omega_3^2-\omega_e^2)}\left[a_{h23}H_{z2}H_{z3}+\right.\\
&\left.b_{h23}E_{z2}H_{z3}+c_{h23}H_{z2}E_{z3}+d_{h23}E_{z2}E_{z3}\right],\\
\frac{dE_{z1}}{dz}&=\frac{-\omega_p^2\omega_1^2\omega_e}{2c\left(\omega_1^2-\omega_e^2\right)\left(\omega_1^2-\omega_p^2\right)}H_{z1}-\\
&-i\frac{\omega_p^2\omega_1^4\left(\omega_1^2-\omega_e^2-\omega_p^2\right)\exp(-\delta k_z z)}{2k_{z1}c\left(\omega_1^2-\omega_e^2\right)\left(\omega_1^2-\omega_p^2\right)}\left[b_{e23}E_{z2}H_{z3}+\right.\\
&\left.+c_{e23}H_{z2}E_{z3}+d_{e23}E_{z2}E_{z3}\right],\\
\frac{dH_{z2}}{dz}&=\frac{\omega_p^2\omega_e(\omega_2^2-\omega_p^2)}{2\omega_2^2c\left(\omega_2^2-\omega_e^2-\omega_p^2\right)}E_{z2}+\\
&+\frac{k_{y2}\omega_p^2\omega_1\exp(\delta k_z z)}{2k_{z2}(\omega_1^2-\omega_e^2)(\omega_3^2-\omega_e^2)}\left[a_{h13}H_{z1}H_{z3}^*+\right.\\
&\left.+b_{h13}E_{z1}H_{z3}^*+c_{h13}H_{z1}E_{z3}^*+d_{h13}E_{z1}E_{z3}^*\right],\\
\frac{dE_{z2}}{dz}&=\frac{-\omega_p^2\omega_2^2\omega_e}{2c\left(\omega_2^2-\omega_e^2\right)\left(\omega_2^2-\omega_p^2\right)}H_{z2}+\\
&\frac{\omega_p^2\omega_1\omega_2^3\left(\omega_1^2-\omega_e^2-\omega_p^2\right)\exp(\delta k_z z)}{2k_{z2}c\left(\omega_2^2-\omega_e^2\right)\left(\omega_2^2-\omega_p^2\right)}\left[b_{e13}E_{z1}H_{z3}^*+\right.\\
&\left.+c_{e13}H_{z1}E_{z3}^*+d_{e13}E_{z1}E_{z3}^*\right],\\
\frac{dH_{z3}}{dz}&=\frac{\omega_p^2\omega_e(\omega_3^2-\omega_p^2)}{2\omega_3^2c\left(\omega_3^2-\omega_e^2-\omega_p^2\right)}E_{z3}+\\
&+\frac{k_{y3}\omega_p^2\omega_1\exp(\delta k_z z)}{2k_{z3}(\omega_1^2-\omega_e^2)(\omega_2^2-\omega_e^2)}\left[a_{h12}H_{z1}H_{z2}^*+\right.\\
&\left.+b_{h12}E_{z1}H_{z2}^*+c_{h12}H_{z1}E_{z2}^*+d_{h12}E_{z1}E_{z2}^*\right],\\
\frac{dE_{z3}}{dz}&=\frac{-\omega_p^2\omega_3^2\omega_e}{2c\left(\omega_3^2-\omega_e^2\right)\left(\omega_3^2-\omega_p^2\right)}H_{z3}+\\
&+i\frac{\omega_p^2\omega_1\omega_3^3\left(\omega_3^2-\omega_e^2-\omega_p^2\right)\exp(\delta k_z z)}{2k_{z1}c\left(\omega_3^2-\omega_e^2\right)\left(\omega_3^2-\omega_p^2\right)}\left[b_{e12}E_{z1}H_{z2}^*+\right.\\
&\left.+c_{e12}H_{z1}E_{z2}^*+d_{e12}E_{z1}E_{z2}^*\right].
\end{aligned} \tag{6}$$

The values *a, b, c, d* with different indexes are quite complicated. They characterize the influence of nonlinear terms. These coefficients, entering into the equation for dE_{zi}/dz, contain denominators of the type $\left(\omega^2-\omega_e^2\right)$. It is following from these equations that process of three wave interaction in magnetoactive plasma is substantially more complicated than it is described, for example, in [6-8]. The linear interaction between the *E*- and *H*- components plays significant role in this case. The linear terms describing such interaction can much exceed the terms describing the decay instability. However, it may be expected that at some electromagnetic field strength the contions can be realized when the decay instability dominates over the linear transfer.

2. RESULTS OF ANALYTICAL AND NUMERICAL INVESTIGATIONS

By neglecting the nonlinear terms, the set (6) is transformed into three set of equations, each describing the linear energy transfer between *E*- and *H*-components of one eigenmode. Every such set of first order equations can be reduced to equation of the view:

$$\frac{d^2H_{zi}}{dz^2}+\lambda_{Li}^2H_{zi}=0, \tag{7}$$

where

$$\lambda_{Li}^2=\frac{\omega_p^4\omega_e^2}{4c^2(\omega_i^2-\omega_e^2)\left(\omega_i^2-\omega_e^2-\omega_p^2\right)}. \tag{8}$$

It follows from expression (8) that energy exchange between *E*- and *H*-waves of one mode is possible in the frequency region $\omega<\omega_e$ and $\omega>\sqrt{\omega_e^2+\omega_p^2}$, the latter is the upper hybrid frequency. Linear transfer becomes faster when approaching the electron cyclotron frequency or the upper hybrid one. However, it should be taken into account that in the vicinity of these frequencies the approximation of slow varying amplitudes is not correct. From expression (8) the characteristic length for energy exchange between *E*- and *H*-waves can be estimated: $L=1/\lambda_i$..

As it was pointed out in the previous section, the subject of our interest is the case when the terms of questions describing the effect of magnetic field on the permittivity tensor are small. With that the dispersive properties of investigated system are close to those of vacuum. This is possible if the conditions, mentioned in the previous paragraph, are true. In the selected model when wave vectors of interacting waves are in one plane, the synchronism conditions (5) are satisfied if all interacting waves are propagating in one direction (and the dispersion properties are close to the vacuum dispersion properties).

To define the increment of decay instability it is supposed that the decaying mode amplitude is a constant value. In this case the set of shorted equations is linear. In this case, every of natural modes has *E*- and *H*-components which take place in the process of linear energy exchange. The decay instability can be realized differently depending on what components of natural mode is dominant. The estimation of increment was performed for two cases: (i) at initial moment the decaying wave contains only *H*-component, and (ii) only *E*- component. The expressions for these increments are rather complicated. Both, *E*- and *H*-component amplitudes, enter as multipliers in these expressions, and this allows to point out those amplitude values when the nonlinear decay dominates over the linear transfer.

If initially the *H*-wave dominates in the decaying mode, then there are two increments of its decay into *E*- and *H*-waves in second and third modes. On the contrary, if *E*-wave is dominant initially, there is one increment of decay into modes 2 and 3. The expressions for these increments are complicated, especially if *E*-wave decays. The expressions for these increments allow to conclude that there are ranges of parameters where decay is not possible although synchronism conditions are hold. For every of indicated decays it is possible to define a threshold value for the amplitude of the decaying wave when the decay instability is possible.

Thus the expressions for increments of decay H-wave instability (for made above supposition) are:

$$\lambda_{he}^2=\frac{\omega_p^4\omega_1^2\omega_e^2\left|H_{z1}\right|^2}{4k_{z2}k_{z3}c^4\left(\omega_1^2-\omega_e^2\right)^2},$$

$$\lambda_{hh}^2=\frac{k_{y2}^2k_{y3}^2\omega_p^4\omega_1^6\omega_e^2\left|H_{z1}\right|^2}{4k_{y1}^4k_{z2}k_{z3}c^4\left(\omega_1^2-\omega_e^2\right)^2\left(\omega_2^2-\omega_e^2\right)\left(\omega_3^2-\omega_e^2\right)},\qquad(9)$$

where λ_{he}, λ_{hh} – increments of decay instability H-wave into *E*- and *H*-wave of the modes 2 and 3, respectively. As can be seen from equations (9) there is frequency range where transfer of *H*-wave into *H*-wave is not possible. The increment of the decay instability of *E*-wave is more complicated, so it was not shown here. It should be noted that square of increment of this instability can be not only a real value, but complex value also. Thus, the decay of *E*-wave has a more more complicated character, depending of frequencies of natural modes that do take part in interaction.

Using expressions (8) and (9) together with condition $\lambda_{L1}>\lambda_{he}$ it is possible to obtain threshold value for dimensionless strength of H-wave of decaying mode when the decay instability is possible. The criterion for the decay instability $H\to E$ to exist is:

$$\left|H_{z1}\right|>\frac{c\sqrt{k_{z2}k_{z3}}}{\omega_1}.\qquad(10)$$

As follows from these results the dynamics of the decay instability in the magnetoactive plasma may be more complicated comparing with the well-known cases (e.g., see, [6-8]).

In addition to what was said above, we want to know how the decay instability is influenced by the process of linear transfer between *E*- and *H*-modes.

One might expect that if the period of this exchange is below the inverse increment of the decay instability, then the latter will be suppressed.

This expectation was confirmed by results of numerical investigation of the equation (6). The plots demonstrating influence of linear transfer on decay instability are presented in Figs. 1-3.

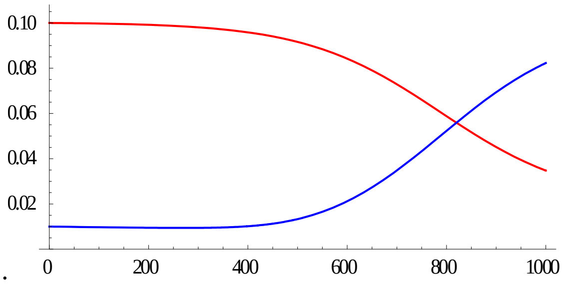

Fig. 1. Nonlinear dynamics of decaying mode where H-wave is dominant. The process, corresponding to linear transfer is neglected. Red curve corresponds to H-wave; blue curve – E-wave. The vertical axes corresponds to amplitudes of the waves. The horizontal axes corresponds z coordinate

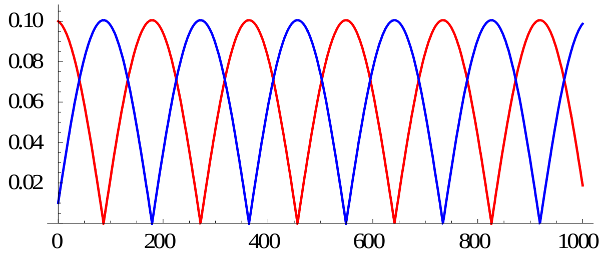

Fig. 2. Linear transfer between E- and H-wave of mode 1. Red curve corresponds to H-wave; blue curve – E-wave. The vertical axes corresponds to amplitudes of the waves. The horizontal axes corresponds z coordinate

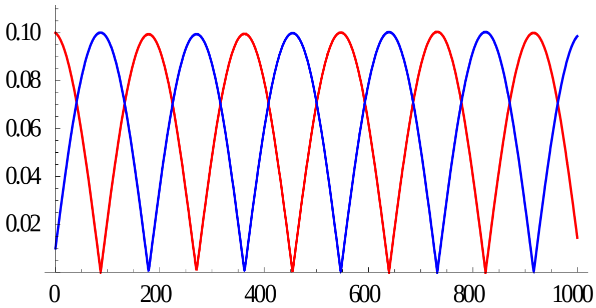

Fig. 3. The dynamics of the mode 1 when both linear and nonlinear processes are taken into account. Red curve corresponds to H-wave; blue curve – E-wave. Nonlinear decay is suppressed. The vertical axes corresponds to amplitudes of the waves. The horizontal axes corresponds z coordinate

CONCLUSIONS

Nonlinear equations describing interaction of the waves in magnetoactive plasma which propagate under some angle to the magnetic field were obtained. The set of equations describing slow varying amplitudes of interacting modes was derived. As distinct from the well-known three-wave approximation ([6-8]), in our case the resulting system contains six equations. This is because the every interacting mode contains two connected components: *E*- and *H*-wave.

The expression for the length where linear transfer takes place is presented. The increments of the decay instability when the *H*-wave amplitude exceeds the *E*-wave amplitude are presented. The estimation of decaying mode amplitude was found when the decay instability dominates on the process of linear energy exchange.

The most important result of this study, on our point, is the conclusion that the presence of gyrotropy can lead to suppression of the decay instability, what means that in such conditions the plasma wave will propagate without attenuation.

REFERENCES

1. V.A. Buts. Stabilization of classic and quantum systems // *Problems of Atomic Acience and Technology*. 2012, № 6(82), p. 146-148.
2. V.A. Buts. Stabilization of unstable states. *20th International Conference on Microwaves, Radar and Wireless Communications, June 16-18, 2014*, Gdansk, Poland, p. 681-685.
3. A.N. Antonov, B.A. Buts, I.K. Koval`chuk, O.F. Kovpik, E.A. Kornilov, B.G. Svichenskii, D.V. Tarasov. Regular and stochastic decay waves in the plasma resonator // *Plasma Physics*. 2012, № 38, v. 8, p. 693-708.
4. V.A. Buts, I.K. Koval`chuk, E.A. Kornilov, and D.V. Tarasov. Stabilization of Beam Instability by a Local Instability Developing due to a Wave-Wave Interaction // *Plasma Physics Reports.* 2006, v. 32, № 7, p. 563-571.
5. *Electrodynamics of plasma* / Editor A.I. Akhiyezer. Moscow: ”Nauka”. Gl. Red. Phys.-mat. Lit, 1974, p. 720 (in Russian).
6. B.B. Kadomtsev. *Collective phenomena in plasma*. Moscow: ”Nauka”. Gl. Red. Phys.-mat. Lit. 1988, p. 394 (in Russian).
7. H. Wilhelmsson, J. Weiland. *Coherent non-linear interaction of waves in plasmas*. M: “Energoizdat”. 1981, 223 p. (in Russian).
8. V.N. Tsitovich. *Nonlinear collective effects in plasma*. Moscow: ”Nauka”, p. 268 (in Russian).

Article received 03.12.2014

ОСОБЕННОСТИ ТРЕХВОЛНОВОГО ВЗАИМОДЕЙСТВИЯ В МАГНИТОАКТИВНОЙ ПЛАЗМЕ

В.А. Буц, И.К. Ковальчук

Исследованы процессы распадов волн, которые распространяются в гиротропных нелинейных средах. Показано, что в условиях реализации эффекта Фарадея динамика процессов существенно изменяется. Наиболее важным результатом является утверждение, что наличие гиротропии может привести к подавлению распадной неустойчивости. Это означает, что в таких условиях волны в плазме распространяются без затухания. Выводы аналитического рассмотрения подтверждаются результатами численного анализа.

ОСОБЛИВОСТІ ТРИХВИЛЕВОЇ ВЗАЄМОДІЇ В МАГНІТОАКТИВНІЙ ПЛАЗМІ

В.О. Буц, І.К. Ковальчук

Досліджені процеси розпаду хвиль, що поширюються в гіротропних нелінійних середовищах. Показано, що в умовах реалізації ефекту Фарадея динаміка процесів істотно змінюється. Найбільш вагомим висновком є висновок, що наявність гіротропії може привести до зриву розпадної нестійкості. Це означає, що в таких умовах хвилі в плазмі розповсюджуються без згасання. Висновки аналітичного розгляду підтверджуються результатами чисельного аналізу.