SUPPRESSION OF EXCITED VORTICAL TURBULENCE IN INHOMOGENEOUS PLASMA IN CROSSED RADIAL ELECTRICAL AND LONGITUDINAL MAGNETIC FIELDS

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Properties and excitation of the vortical turbulence, excited in a cylindrical radially inhomogeneous plasma in crossed radial electric and longitudinal magnetic fields, are considered. The dispersion relation, which allows to determine the range of parameters for which the vortical turbulence is suppressed, is derived from the general non-linear equations for the vorticity.

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INTRODUCTION

It is well known from numerous numerical simulations (see, for example, [1]) and from experiments (see, for example [2]) that electron density nonuniformity in kind of discrete vortices are long-living structures. In experiments [2] a rapid re-organization of discrete electron density nonuniformity has been observed in the spatial distribution of vorticity in pure electron plasma when a discrete vortex has been immersed in an extended distribution of the background vorticity. In plasma lens [3-5] for ion beam focusing a small-scale turbulence has been excited in crossed radial electrical and longitudinal magnetic fields by unremovable gradient of external magnetic field. This turbulence is a distributed vorticity. In this paper the properties and excitation of similar vortical turbulence, excited in cylindrical radially inhomogeneous plasma in crossed radial electrical E_{0r} and longitudinal magnetic H_0 fields [6], is investigated theoretically. From general nonlinear equation, presented in article [5], for vorticity the dispersion relation, which determines the range of experimental installation parameters, for which the vortical turbulence is damped.

1. EXCITATION OF VORTICES

Hydrodynamic equations for electrons and Poisson equation are used

$$\partial_t \vec{V} + (\vec{V}\vec{\nabla})\vec{V} = (e/m_e)\vec{\nabla}\varphi + [\vec{\omega}_{He}, \vec{V}] - (V_{th}^2/n_e)\vec{\nabla}n_e, \quad (1)$$

$$\partial_t n_e + \vec{\nabla}(n_e\vec{V}) = 0, \ \vec{\nabla}\varphi \equiv \vec{\nabla}\phi - \vec{E}_{0r}, \ \Delta\varphi = 4\pi e(n_e - n_i). \quad (2)$$

Here $\vec{\omega}_{He} = e\vec{H}_0/m_e c$ is the electron cyclotron frequency; e, m_e are the electron charge and mass; **V**, n_e and V_{th} are the velocity, density and thermal velocity of electrons in the plasma, n_i is the ion density; E_{or} is the external radial electric field, ϕ is the electric potential of vortical perturbation. From equations (1), (2) one can derive, neglecting $(V_{th}^2/n_e)\vec{\nabla}n_e$ in (1), equations

$$d_t[(\alpha - \omega_{He})/n_e] = [(\alpha - \omega_{He})/n_e]\partial_z V_z, \quad (3)$$

$$d_t V_z = (e/m_e)\partial_z\varphi, \ d_t = \partial_t + (\vec{V}_\perp\vec{\nabla}_\perp), \quad \alpha \equiv \vec{e}_z \mathrm{rot}\vec{V}.$$

Also from equation (1) one can derive expression for transversal ($\vec{V}_\perp \perp \vec{H}_0$) electron velocity.

$$\vec{V}_\perp = = (e/m\omega_{He})[\vec{e}_z, \vec{\nabla}\varphi] -$$
$$-\omega_{He}^{-1}\partial_t[\vec{e}_z, \vec{V}_\perp] - \omega_{He}^{-1}[\vec{e}_z, (\vec{V}\vec{\nabla})\vec{V}_\perp]$$
$$\approx (e/m\omega_{He})[\vec{e}_z, \vec{\nabla}\varphi] + (e/m\omega_{He}^2)\partial_t\vec{\nabla}_\perp\varphi, \quad (4)$$

$$\alpha = 2eE_{r0}/rm\,\omega_{He} + (e/m\omega_{He})\Delta_\perp\phi. \quad (5)$$

From (2), (5) it approximately follows $\alpha \approx (\omega_{pe}^2/\omega_{He})\delta n_e/n_{0e}$ that the vortical motion begins, as soon as there appears a plasma density perturbation δn_e.

From (3) one can derive

$$d_t(\omega_{He}/n_e) = (\omega_{He}/n_e)\partial_z V_z. \quad (6)$$

Taking into account the effect of unmagnetized ions, moving in longitudinal direction, from (2) one can obtain, searching the following dependence of perturbations δn_e, $\delta n_i \propto \exp[i(k_z z + \ell_\theta\theta - \omega t)]$.

$$\beta\Delta\varphi/4\pi e = \delta n_e, \ \beta = 1 - \omega_{pi}^2/(\omega - k_z V_{i0})^2,$$
$$n_e = n_{0e} + \delta n_e. \quad (7)$$

Here ω_{pi} is the ion plasma frequency; V_{i0} is the longitudinal velocity of the ion flow.

At first let us consider instability development. From (3) we derive

$$d_t(\omega_{He}/n_e) = -(e\omega_{He}/m_e n_{0e})ik_z^2\varphi/(\omega - \ell_\theta\omega_{\theta0}),$$
$$\omega_{\theta0} = V_{\theta0}/r. \quad (8)$$

$V_{\theta o}$ is the electron drift velocity in crossed radial electrical and longitudinal magnetic field. From (4), (7), (8) we obtain equation for ϕ

$$(\omega_{pe}^2/\omega_{He}^2)\nabla_\theta\phi\nabla_r\omega_{He} + \beta(\partial_t\Delta\phi + \omega_{\theta0}\partial_\theta\Delta\phi) =$$
$$= ik_z^2\omega_{pe}^2\phi/(\omega - \ell_\theta\omega_{\theta0}). \quad (9)$$

Assuming that the value $A = r^{-1}\partial_r(\omega_{pe}^2/\omega_{He})$ is approximately independent on radius and looking for a transverse structure with the help of the Bessel functions from (9) one can find the linear dispersion relation, which describes the development of the instability ω_{pe} is the electron plasma frequency.

$$1 - \omega_{pi}^2/(\omega - k_z V_{i0})^2 - \ell_\theta A/k^2(\omega - \ell_\theta\omega_{\theta0}) -$$
$$-k_z^2\omega_{pe}^2/k^2(\omega - \ell_\theta\omega_{\theta0})^2 = 0. \quad (10)$$

For short magnetic coil

$$\omega_{He}(r) \approx \omega_{He0}\left(1+Br^2/R^2\right)$$

at B<<1 we have

$$A = r^{-1}\partial_r\left(\omega_{pe}^2/\omega_{He}\right) \approx -\left(\omega_{pe0}^2/\omega_{He0}\right)2B/R^2 .$$

For plasma density, decreasing on radius

$$n_e(r) = n_{0e}\left(1-r^2/R^2\right),$$

we have

$$A = r^{-1}\partial_r\left(\omega_{pe}^2/\omega_{He}\right) \approx -\left(\omega_{pe0}^2/\omega_{He0}\right)2/R^2 .$$

Let us take into account that the ions pass through the system of length L during $\tau_i = L/V_{i0}$ and electrons are renovated during τ_e. (10) can be presented as

$$1-\omega_{pi}^2/\left(\omega-k_zV_{i0}-i/\tau_i\right)^2-\ell_\theta A/k^2\left(\omega-\ell_\theta\omega_{\theta0}-i/\tau_e\right)-$$
$$-k_z^2\omega_{pe}^2/k^2\left(\omega-\ell_\theta\omega_{\theta0}-i/\tau_e\right)^2=0 . \quad (11)$$

Let us mean quick perturbations those, which phase velocity approximately equals $V_{ph} \approx V_{\theta0}$. For them from (11) we derive in approximation $k_z=0$, $\omega=\omega^{(o)}+\delta\omega$, $|\delta\omega|<<\omega^{(o)}$ and neglecting τ_e, τ_i

$$\omega^{(0)} = \omega_{pi} = \ell_\theta\omega_{\theta0},\ \omega_{\theta0} = \left(\omega_{pe}^2/2\omega_{He}\right)\left(\Delta n/n_{0e}\right),$$

$$\delta\omega = i\gamma_q,\ \gamma_q = k^{-1}\sqrt{\left(\omega_{pi}/2\right)\ell_\theta|A|},\ \Delta n = n_{0e}-n_{0i} . \quad (12)$$

From (12) it follows

$$\ell_\theta = \sqrt{m_i/m_e}\left(\omega_{He}/\omega_{pe}\right)n_{0e}/\Delta n , \quad (13)$$

that for typical parameters of experiments the perturbations with $\ell_\theta > 1$ are excited at a large magnetic field and at small electron density.

For slow perturbations it is fulfilled $V_{ph} << V_{\theta0}$. We derive for them from (11) in approximation $k_z=0$ and neglecting τ_e, τ_i the following expressions

$$\gamma_s = \left(\sqrt{3}/2^{4/3}\right)\left[\omega_{pi}^2\ell_\theta\left(\omega_{pe}^2/2\omega_{He}\right)\left(\Delta n/n_{0e}\right)\right]^{1/3},$$

$$k^2 = -A/\omega_{\theta0},\ \mathrm{Re}\,\omega_s = \gamma_s/\sqrt{3} . \quad (14)$$

Here γ_s is the growth rate of slow perturbation excitation.

2. SPATIAL STRUCTURE OF VORTICES

Let's describe structure of a fast vortex, placed on radius r_q, in a rest frame, rotated with angular rate $\omega_{ph} \equiv V_{ph}/r_q$. Let's consider a chain (on θ) of interleaving vortices – bunches and vortices – cavities of electrons. Neglecting non-stationary and non-linear on ϕ terms, we receive the following equation

$$\vec{V}_\perp = -\left(e/m\omega_{He}\right)\left[\vec{e}_z,\vec{E}_{r0}\right]+\left(e/m\omega_{He}\right)\left[\vec{e}_z,\vec{\nabla}\phi\right], \quad (15)$$

describing quasistationary dynamics of electrons in fields of crossed fields and vortical perturbation. From (15) we receive expressions for radial and azimuth velocities of electrons

$$V_r = -\left(e/m\omega_{He}\right)\nabla_\theta\phi,$$

$$V_\theta = V_{\theta o}+(e/m_e\omega_{He})\nabla_r\phi ,$$

$$V_{\theta o} = -(e/m_e\omega_{He})E_{ro} = (\omega^2_{pe}/2\omega_{He})(\Delta n/n_{oe})r . \quad (16)$$

V_θ can been presented as the sum of the phase velocity of the perturbation, V_{ph}, and velocity of azimuth oscillations of electrons, δV_θ, in the field of the perturbation, $V_\theta=V_{ph}+\delta V_\theta$. As $V_\theta=rd\theta/dt$, we present $d\theta/dt$ as $d\theta/dt=d\theta_1/dt+\omega_{ph}$, here $\omega_{ph}=(\Delta n/n_{oe})(\omega^2_{pe}/2\omega_{He})|_{r=rq}$. Then from (16) we obtain

$$d\theta_1/dt=(e/m_e)[E_{ro}(r)/r\omega_{He}(r)-E_{ro}(r_q)/r_q\omega_{He}(r_q)]+$$
$$+(e/rm_e\omega_{He})\partial_r\phi ,\ dr/dt=-(e/m_e\omega_{He}r)\partial_\theta\phi . \quad (17)$$

At small deviations r from r_q, decomposing $\Delta n(r)/\omega_{He}(r)$ on $\delta r\equiv r-r_q$ and integrating (17), we obtain

$$(\delta r)^2+4\phi/r_q\omega_{He}\partial_r[E_{ro}(r)/r\omega_{He}(r)]|_{r=rq}=\mathrm{const.} \quad (18)$$

The vortex boundary separates the trapped electrons, formed the vortex and moving on closed trajectories, and untrapped electrons moving outside the boundary of the vortex and oscillating in its field. For vortex boundary we receive the following expression from the condition $\delta r|_{\phi=-\phi o}=\delta r_{cl}$,

$$\delta r=\pm[-4(\phi+\phi_o)/r_q\omega_{He}\partial_r[E_{ro}(r)/r\omega_{He}(r)]|_{r=rq}+(\delta r_{cl})^2]^{1/2}. \quad (19)$$

Here δr_{cl} is the radial width of the vortex-bunch of electrons. From (19) the radial size of the vortex-cavity of electrons follows

$$\delta r_h=[-8\phi_o/r_q\omega_{He}\partial_r[E_{ro}(r)/r\omega_{He}(r)]|_{r=rq}+(\delta r_{cl})^2]^{1/2}. \quad (20)$$

From the equation of electron motion and Poisson equation one can derive approximately expression for the vorticity $\alpha\equiv\mathbf{e}_z\mathrm{rot}\mathbf{V}$, which is characteristic of the vortical motion of electrons

$$\alpha\approx-2eE_{ro}/rm\omega_{He}+(\omega^2_{pe}/\omega_{He})\delta n_e/n_{eo}.$$

From here it follows that up to certain amplitude of vortices the structure of electron trajectories in the field of the chain on θ of fast vortices in the rest frame, rotated with $\omega_{ph}\equiv V_{ph}/r_q$, looks one kind and for large amplitude another kind.

For large amplitudes of fast vortices in the region of electron bunches the reverse flows are formed. The vortex-cavity is rotated in the rest frame, rotated with frequency $\omega_{ph}\equiv V_{ph}/r_q$, in the same direction as nonperturbed plasma. The vortex-bunch is rotated in the opposite direction of rotation of nonperturbed plasma at $\delta n_e>\Delta n\equiv n_{oe}-n_{oi}$. One can see that the size of the vortex is inversely proportional to $[-8/r_q\omega_{He}\partial_r[E_{ro}(r)/r\omega_{He}(r)]|_{r=rq}]^{1/2}$ and is proportional to $\phi^{1/2}_o$. That is the size of the vortex essentially depends on a gradient of the magnetic and electrical fields. At small $\partial_r[E_{ro}(r)/r\omega_{He}(r)]|_{r=rq}]^{1/2}$ already at small perturbations of electron density the size of the vortex, δr_h, can reach $\delta r_h\approx R/2$, R is the radius of the plasma.

Eq. (17) can be integrated without decomposing $\Delta n(r)/\omega_{He}(r)$ on $\delta r\equiv r-r_q$. For this purpose in approximation $\Delta n\neq\Delta n(r)$ we approximate $\omega_{He}(r)=\omega_{Ho}(1+\mu r^2/R^2)$. Then, integrating (17), we obtain

$$2\phi+\pi e\Delta nr^2[1-\omega_{Ho}/2\omega_{He}(r_q)-\omega_{He}(r)/2\omega_{He}(r_q)]=\mathrm{const.} \quad (21)$$

From the condition $r|_{\phi=-\phi o}=r_q+\delta r_{cl}$ and (21) we derive the expression, determining the boundary of the vortex - cavity of electrons,

$$[r^2-(r_v+\delta r_{cl})^2][1-\omega_{Ho}/\omega_{He}(r_v)]-$$
$$[r^4-(r_v+\delta r_{cl})^4]\omega_{Ho}\mu/2R^2\omega_{He}(r_v)+2(\phi+\phi_o)/\pi e\Delta n=\mathrm{const.} \quad (22)$$

From (22) and $r|_{\phi=\phi o}=r_q+\delta r_h$ we derive the expression, determining the radial width of the vortex - cavity of electrons,

$$\phi_o 4R^2\omega_{He}(r_v)/\pi e\Delta n]\omega_{Ho}\mu=$$
$$=(\delta r_h-\delta r_{cl})(2r_v+\delta r_h+\delta r_{cl})[r_v(\delta r_h+\delta r_{cl})+(\delta r_h^2+\delta r_{cl}^2)/2]. \quad (23)$$

Let's consider the vortex with the small phase velocity V_{ph} in comparison with drift velocity of electrons, $V_{ph} << V_{\theta o}$. The spatial structure of electron trajectories in its field for small amplitudes of the vortex looks like as corrugated structure. It is determined by that in all system α has the identical sign, $\alpha>0$. In other words, radial electrical field created by the vortex is less, than external electrical field, $E_{rq}<E_{ro}$. Then in all system the azimuth velocities of electrons have the identical sign and there are no reverse flows of electrons. There is no separatrix in slow vortex of small amplitude. For the description of spatial structure of electron trajectories we use (16). Using in them $V_\theta=rd\theta/dt$ and excluding θ, we obtain for vortex boundary $r(\theta)$

$$r=[r_s^2+(\phi_o-\phi)2/\pi e\Delta n]^{1/2} . \qquad (24)$$

In the case of small amplitudes (24) becomes

$$\delta r \equiv r-r_s=(\phi_o-\phi)/\pi e\Delta n r_s . \qquad (25)$$

From (24) we derive the radial size of the slow vortex

$$\delta r_s \equiv r|_{\phi=-\phi o}-r_s=[r_s^2+4\phi_o/\pi e\Delta n]^{1/2}-r_s . \qquad (26)$$

In the case of small amplitudes (26) becomes

$$\delta r_s \approx 2\phi_o/\pi e\Delta n r_s . \qquad (27)$$

Because on $r=r_v$, $\delta n(r=r_v)=0$, then the electron moves on it with $V_{\theta o}$ without radial perturbations. At $r>r_v$ radial displacement is positive, and when $r<r_v$ – negative radial displacement of the electrons.

At large amplitudes, $\delta n_e>\Delta n$ (or $E_{rv}>E_{ro}$), in the region, where the electron cavities place, the characteristic of the vortical motion α obtains the inverse sign, $\alpha<0$. In other words, on the axis, connecting the vortex-cavity and the vortex-bunch, the inequality $E_{rv}>E_{ro}$ is executed and there is an azimuth reverse flow of electrons. Then in some regions the electrons are rotated in the direction inverse to their rotation in crossed fields. The slow vortex is a dipole perturbation of electron density, disjointed on radius. At $\delta n_e>\Delta n$ the structure of the slow vortex is similar to the structure of Rossby vortex.

3. SUPRESSION OF EXCITATION OF VORTICAL PERTURBATIONS IN CASE OF MAGNETIZED IONS

Similarly (10) one can derive dispersion relation

$$1-\omega_{pi}^2/\left(\omega_{\theta o}\ell_\theta-\omega\right)^2-\ell_\theta A/k^2\left(\omega-\ell_\theta\omega_{\theta 0}\right)=0 ,$$

which demonstrates the suppression of the instability in the case of magnetized ions $eE_r/m_i\omega_{Hi}^2 \le R$.

CONCLUSIONS

Properties and excitation of the vortical turbulence, excited in a cylindrical radially inhomogeneous plasma in crossed radial electric and longitudinal magnetic fields, have been described. The dispersion relation, which allows to determine the range of parameters for which the vortical turbulence is suppressed, is derived from the general non-linear equations for the vorticity.

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ПОДАВЛЕНИЕ ВОЗБУЖДАЕМОЙ ВИХРЕВОЙ ТУРБУЛЕНТНОСТИ В НЕОДНОРОДНОЙ ПЛАЗМЕ В СКРЕЩЕННЫХ РАДИАЛЬНОМ ЭЛЕКТРИЧЕСКОМ И ПРОДОЛЬНОМ МАГНИТНОМ ПОЛЯХ

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Рассматриваются свойства и возбуждение вихревой турбулентности, возбуждаемой в цилиндрической радиально-неоднородной плазме в скрещенных радиальном электрическом и продольном магнитном полях. Из общего нелинейного уравнения для завихренности получено дисперсионное уравнение, позволяющее определить диапазон параметров, для которых вихревая турбулентность подавляется.

ПРИДУШЕННЯ ЗБУДЖЕННЯ ВИХРОВОЇ ТУРБУЛЕНТНОСТІ В НЕОДНОРІДНІЙ ПЛАЗМІ В СХРЕЩЕНИХ РАДІАЛЬНОМУ ЕЛЕКТРИЧНОМУ І ПОЗДОВЖНЬОМУ МАГНІТНОМУ ПОЛЯХ

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Розглядаються властивості і збудження вихрової турбулентності, яка збуджується в циліндричній радіально-неоднорідній плазмі в схрещених радіальному електричному і поздовжньому магнітному полях. Із загального нелінійного рівняння для завихренності отримано дисперсійне рівняння, що дозволяє визначити діапазон параметрів, для яких вихрова турбулентність пригнічується.