APPLICATION OF THOMAS-FERMI MODEL TO EVALUATION OF THERMODYNAMIC PROPERTIES OF MAGNETIZED PLASMA

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The aim of this work is the evaluation of thermodynamic and transport properties of fusion plasmas in an externally applied strong magnetic field ($10\ldots10^3$ T). For these purpose the Thomas-Fermi model for substances with a given temperature and density was used. The effect of such strong magnetic field on the transport properties of plasmas and the view of the inner shells of atoms and ions was analyzed. Isotherms of the pressure and the specific internal energy of the tungsten plasma are obtained. Quantum and exchange corrections to the pressure of this plasma have been taken into account.

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INTRODUCTION

Thermodynamic and transport properties of fusion plasmas, consisting of the atomic mixture of substances, in an externally applied strong magnetic field ($10\ldots10^3$ T) are important. Such plasma may be used as cylindrical [1] and spherical [2, 3] target for magnetized target fusion or magneto-inertial fusion [4-6]. Note that the degeneracy of the electron gas in such plasmas is possible [7].

Transport properties of the plasma are the coefficients of thermal and electrical conductivity. Thermodynamic properties are described by thermal and caloric equations of state: $P = P(T, \rho)$, $E = E(T, \rho)$, $S = S(T, \rho)$, where P is the pressure of the plasma; E, S are its internal energy and entropy per unit mass. T and ρ are the temperature and the density of the plasma.

The approximate method is used for the mathematical description of plasmas thermophysical properties. This method is based on the generalization of statistical Thomas-Fermi method to the case of zero temperature [7] and the external magnetic field [8]. This method is simple (for example, compared with the Hartree-Fock-Slater method), while it provides acceptable accuracy for practice, especially when the quantum oscillatory and exchange corrections are taken into account [9].

Currently, various software packages and databases for determining the thermophysical properties of substances have been developed: ASTEROID (IPMech RAS), SESAME (LANL, LLNL Livermore, and Sandia), TEFIS (KIAM RAS), IVTANTERMO (JIHT RAS), TERMOS (KIAM), program complexes «TUR» (RFNC-VNIITF) and EIP EOS (Ioffe Institute) and etc. The Thomas-Fermi (T-F) model with the quantum and exchange corrections is used in many of these databases as a theoretical model. However, some of these databases are for private use only or they are incomplete. Also in these databases (except the program complex EIP EOS [10]) T-F model is not extended to the case of the external magnetic field impact on thermonuclear plasma.

1. THE THOMAS-FERMI MODEL: DESCRIPTION AND APPLICATIONS

The physical model of matter, which is the basis of the T-F model, assumes that the difference between "free" and "bound" electrons is absent and a substance is considered to be composed not of ions and electrons, but of nuclei and electrons. The interaction energy of the particles in matter is determined by the electrons. Calculations of fusion plasmas thermodynamic properties are based on the model of local thermodynamic equilibrium (LTE). Systems for a large number of noninteracting nuclei comply with Boltzmann statistics.

The calculation of electronic parts of the energy and the pressure is based on the LTE model, according to which a substance is separated into the system of atomic cells; each contains Z electrons (e is the electron charge) and a nucleus of charge $Z\cdot e$. For simplicity, the shape of the atomic cell is taken to be spherical.

Electrons in the atomic cell are considered as a gas in the slowly varying radially self-consistent electrostatic field $V(r)$, generated by the nuclear charge and the charge of electrons. Thus, the nonideal electron gas is taken into account. The Fermi-Dirac statistics is applied to the electron-ion gas.

Assume, that the T-F potential distribution $\varphi(x) = x\cdot(V(r)+\mu)/\theta$ is known. Here θ is the $k_B\cdot T$ (k_B is the Boltzmann constant); μ is the chemical potential of the plasma. Then the pressure of electrons P_e at the atomic cell boundary can be calculated as the average momentum carried by them per unit e per unit area through the atomic cell with the radius r_0.

It is necessary to take into account the pressure created by the nuclei to find the total pressure of the system of particles in the atomic cell. At high temperatures the system (gas, consisting of cores) is typically treated as an ideal gas. Therefore the total pressure P is defined as $\left(P_e + \frac{\theta}{\nu}\right)$, where v is the volume of the atomic cell.

For details (e.g. for calculating the specific internal energy and the entropy of the plasma), refer to the [9, 11].

Quantum and exchange corrections are required to consider because the T-F model is the approximate method (for details, refer to the [12]). To find them we use the method, described here [9].

2. ESTIMATIONS OF THE EXTERNAL MAGNETIC FIELD INFLUENCE ON THERMOPHYSICAL AND TRANSPORT PROPERTIES OF PLASMAS

The degree of the magnetic field influence on plasma transport coefficients (the electrical conductivity, the thermal conductivity) depends on the ratio of the collision frequency of electrons ν_e to the cyclotron frequency $\omega_e = \dfrac{e}{m_e} B = 1,759 \cdot 10^{11} \cdot B$ of electrons [13], here, B is the magnetic induction in T, m_e is the mass of the electron.

The magnetic field will have a noticeable effect on the transport properties of plasmas if $\nu_e/\omega_e << 1$. We obtain the condition of the strong magnetic field influence on the properties of plasmas:

$$B >> 2 \cdot 10^{-16} \frac{\Lambda}{10} \frac{Zn}{T_e^{3/2}},$$

where Z is the ion charge; Λ is the Coulomb logarithm; T_e is the electron temperature in K; n is the plasma density in m^{-3}.

For the tungsten plasma (Z = 74) with parameters of our interest $T_e \sim 10^7$ K, $n \sim 10^{25}$ m^{-3} which corresponds to the plasma density $\rho \sim 10$ kg/m^3 – crown density of the target in the inertial confinement fusion) we have the following estimate ($\Lambda \approx 15$) $B >> 7$ T.

In the magneto-inertial fusion it is experimentally obtained magnetic field $B_{fus} \sim 10^3$ T and it can be achieved even higher values [14-19]. Therefore it is required to consider the influence of strong magnetic fields on the transport properties of plasmas.

The magnetic field affects the orientation of spins of electrons or atoms in the gas which has a temperature T defined by the condition

$$\mu B >> k_B T \text{ or } B >> 1,49 \cdot T,$$

where μ is the Bohr magneton.

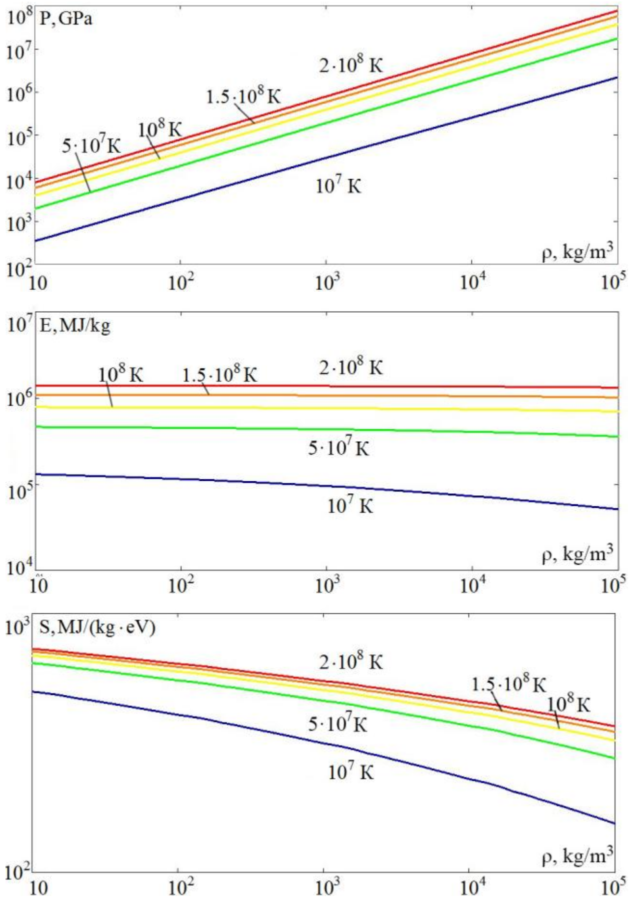
In our case, the characteristic temperature is $T_{char} \sim 10^7$ K. Then $B >> 10^7$ T, that is significantly more than values reached in the thermonuclear fusion.

The magnetic field $B \sim 10^5$ T, in which the energy of the magnetic moment μB is larger than the characteristic energy of the atom or molecule (It has the order of the number $Ry = m_e e^4/2\hbar^2$) significantly affects the structure of atoms and molecules and strongly modifies their binding energy and ionization energy. Thus, we assume that the magnetic field does not affect the orientation of spins of electrons or atoms in a gas. Considered fields are also much smaller than the field $B \sim 10^9$ T, so we can neglect the relativistic effects.

3. RESULTS

The boundary value problem, which determines the $V(r)$ potential distribution, is solved by the sweeping method with iterations [9, 11].

The pressure $P(T, \rho)$, the specific internal energy $E(T, \rho)$ and the entropy $S(T, \rho)$ of the tungsten plasma are shown on Fig. 1.



Isotherms of the pressure P, the specific internal energy E and the entropy S depending on the density ρ for the tungsten plasma at different temperatures

CONCLUSIONS

Evaluations we made showed that the magnetic field up to 10^3 T only affects the transport properties of the plasma, but does not change the view of the inner shells of atoms and ions.

Isotherms of the pressure and the specific internal energy of the tungsten plasma at the temperature range $T = 10^7 \ldots (2 \cdot 10^8)$ K and the density range $\rho = 10 \ldots 10^5$ kg/m^3 are obtained. The maximum values of relative quantum and exchange corrections to the pressure of these plasmas are ~ 1% at 10^7 K and 10^5 kg/m^3.

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ПРИМЕНЕНИЕ МОДЕЛИ ТОМАСА-ФЕРМИ ДЛЯ ОПРЕДЕЛЕНИЯ ТЕРМОДИНАМИЧЕСКИХ СВОЙСТВ ЗАМАГНИЧЕННОЙ ПЛАЗМЫ

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Определены термодинамические и транспортные свойства термоядерной плазмы, находящейся в сильном внешнем магнитном поле (10…1000 Тл). Для этой цели использовалась модель Томаса-Ферми для веществ с заданным температурой и плотностью. Проанализировано действие сильного магнитного поля на транспортные свойства плазмы и вид внутренних оболочек атомов и ионов. Построены изотермы давления, удельной внутренней энергии и энтропии плазмы вольфрама. Учтены квантовые и обменные поправки к давлению плазмы.

ЗАСТОСУВАННЯ МОДЕЛІ ТОМАСА-ФЕРМІ ДЛЯ ВИЗНАЧЕННЯ ТЕРМОДИНАМІЧНИХ ВЛАСТИВОСТЕЙ ЗАМАГНІЧЕНОЇ ПЛАЗМИ

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Визначено термодинамічні і транспортні властивості термоядерної плазми, що знаходиться в сильному зовнішньому магнітному полі (10...1000 Тл). Для цієї мети використовувалася модель Томаса-Фермі для речовин із заданною температурою і щільністю. Проаналізовано дію сильного магнітного поля на транспортні властивості плазми і вид внутрішніх оболонок атомів і іонів. Побудовані ізотерми тиску, питомої внутрішньої енергії і ентропії плазми вольфраму. Враховані квантові і обмінні поправки до тиску плазми.