NON-LINEAR PHENOMENA CONNECTED WITH PROPOGATION OF THE DRIFT WAVES INTO PLASMA

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Non-linear heat and particle transport and restriction of wave amplitudes are discussed. Non-linear transformation of drift wave is considered under influence of outer non-uniform static electric field.

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1. PLASMA DIFFUSIVITY FORMED BY DRIFT WAVE

The past decades of research in high temperature magnetized plasma has shown that transport of particles and energy across the magnetic field B_0 is largely controlled by low-frequency drift waves [1]. In spit of existence a lot of different formulas for plasma diffusivity usually the reference formula is the Bohm diffusivity

$$D_B = \frac{T_e}{eB_0}.$$

For estimations particles transport in tokamak plasma is used often so-called the gyro-Bohm diffusivity

$$D_{gB} = \left(\frac{\rho_s}{L_n}\right)\left(\frac{cT_e}{eB_0}\right).$$

Here factor ρ_s / L_n <<1 (ρ_s is the ion cyclotron radius, $L_n^{-1} = (1/n_0)(dn_0/dx)$ is the density gradient scale.

In this work the simple model of the plasma diffusivity is proposed. The model is based on non-linear analysis of particles dynamics under the influence of the drift waves. Collisionless motionless magnetized slab plasma with $T_e = T_i$ is considered. It is supposed that the uniform magnetic field is parallel to *z*-axis. Outward electric field is absent. Low ion temperature approach is used: finite size of Larmor radius is neglected. Plasma beta is assumed to be β<<1. The only mode is considered. The wave amplitude is supposed to be enough large and constant. Non-linearity of consideration presented below follows from taking into account of particles moving along magnetic force lines. It is the main feature of this work. Drift waves propagate in two directions simultaneously along magnetic force lines and across them. Consideration longitudinal particles moving in combination with across oscillations of drift waves allows to explain phenomenon of the collisionless diffusivity in the magnetized plasma and derive corresponding formula for plasma diffusivity across magnetic field.

Originally we proceed from usual consideration of drift waves picture. It is harmonic distribution of electrostatic field and perturbed plasma density. We present wave along density gradient (*x*-axis) as sum of narrow layers so that in every layer one can suppose perturbed plasma density is constant. But in general case it is incorrectly.

Then we take into account an added non-linear process in the plane *xOz*. Forming of drift waves results in non-uniformity of the perturbed plasma density along of the magnetic field force line (*Oz*) because there is plasma of different layers with different perturbed plasma densities. Consequently it is possible to take into account process of mutual penetration particles between neighboring layers along force lines. Therefore collisionless transfer of particles from domains of high density to domains of low density arises because of influence of the drift waves.

So we consider the drift wave picture in a certain moment in plane *xOz*. There are zero line and harmonic lines. Harmonic lines share three narrow layers by thickness h. It is supposed average plasma perturbed density in this moment are δn_0 (middle layer), $\delta n_1 = \delta n_0 + h[d(\delta n_0)/dx]$ (upper layer) and $\delta n_{-1} = \delta n_0 - h[d(\delta n_0)/dx]$ (lower layer). Therefore perturbed plasma density along magnetic field lines is non-uniform. The particles transport is considered along straightforward layers with thickness h parallel by magnetic field.

The particles transport between harmonic layers number (0), (1) and (-1) is estimated. One is produced because of the ion collisionless motion (sound velocity is$\upsilon_{ti} = \sqrt{2T_i/m_i}$). Thickness h is chosen from non-equality $h << \lambda_z$ (wave length along magnetic field direction). Choose some part inside harmonic layer (in considered case inside layer (0), length $l \approx 3h$). This choice plays important role because one determines time of ions penetration of ions into chosen part harmonic layer. Taking into account relation $\omega / k_z >> \upsilon_{Ti}$ it is clear that penetration into chosen part from neighboring layer is not much because of condition that $l \le 3h$. It means that perturbation of plasma density in chosen part connected with this penetration is no much, too. Consequently below we restrict ourselves by simple linear relations.

Features of change between chosen part and neighboring layer depends on location of chosen part from layer. It is important to consider three cases. First case is: chosen part belongs to inclined domain of sinusoidal layer. In this case plasma density doesn't change because increasing of density through particles exchange (for example between layers number (0) and (1)) is compensated by decreasing of density through contrary particles exchange (in our case between layers number (0) and (-1)). Second case is: chosen part is in maximum domain. In this case neighboring layer of chosen part is the only the layer with density more than density inside chosen layer (for example layers number (0) and (1)). As a result plasma density inside chosen layer increases. Third case is: chosen part is in minimum domain. It is the particles exchange between chosen part and layer number (-1). As a result of such a particles change plasma density inside chosen part decreases.

Therefore real transport across magnetic field can be realized because of particles exchange between layers in maximum and minimum domains. Estimation of the particles flow presents following result

$$\Gamma = -D_{\perp}\left(\frac{d(\delta n)}{dx}\right),$$

were $D_{\perp} = \frac{h^2}{\xi}\omega_* = \frac{h^2}{\xi}\frac{2\pi}{\lambda_y L_n}\frac{cT}{eB_0}$, $\xi = \omega / k_z \upsilon_{Ti}$.

More convenient expression is

$$\Gamma = -\tilde{D}_{\perp}\left(\frac{dn_0}{dx}\right)$$

with the diffusivity

$$\tilde{D}_{\perp} = \frac{h^2}{\xi}\frac{2\pi}{\lambda_y n_0}\left(\frac{d(\delta n)}{dx}\right)\frac{cT}{eB_0} = \frac{h^2}{\xi}\frac{2\pi}{\lambda_y n_0}\left(\frac{d(\delta n)}{dx}\right)D_B .$$

It is necessary to note some features of the last two formulas. Firstly Γ is diffusive flow. Secondly $\tilde{D}_{\perp}$ contains T / eB_0 like to Bohm diffusivity. Thirdly factor before Bohm diffusivity contains both the wave parameters ($\lambda_y, \omega / k_z, d(\delta n)/dx$) and the plasma parameters (υ_{Ti}, n_0). Finally obviously quantitative value of factor previous D_B in formula for $\tilde{D}_{\perp}$ is much less than 1. But disadvantage of this formula for $\tilde{D}_{\perp}$ is availability of indeterminate value h.

Therefore above presented analysis of transport processes in plasma is based on consideration of ion dynamics both under influence wave drift moving and free moving along magnetic field direction.

Below on base of presented model features of transport of impurities are discussed. It is well-known so-called “ballistic” diffusivity [2]. “Ballistic” means velocity propagation of impurities inside plasma much more than velocity estimated using usual diffusive factors obtained from solution of Boltzman equation. If impurities density $n_{i0} << n_0$ (n_0 is plasma density), then their velocity propagation is determinate by wave processes similar to diffusivity considered above. Consequently diffusive velocity turns out is very large.

Other phenomenon recovered under investigation of propagation of impurities is so-called non-local diffusivity [3]. It means impurities flows don’t write as usual diffusive flows. This phenomenon can be explained by non-linearity of spatial distribution of impurities inside plasma. Then in general case expression of diffusive flow is $\Gamma = -\left\langle \tilde{D}_{i\perp}\frac{dn_{i0}}{dx}\right\rangle$.

2. INFLUENCE OF OUTER ELECTRIC FIELD

The influence of the strong non-uniform electric field perpendicular to magnetic field on drift wave propagating inside plasma is usually analyzed as an it’s influence on growth rate γ of a sinusoidal wave [4]. In agreement with [4] γ decreases because of arising of non-uniform flows of the background plasma under influence of perpendicular magnetic and electrical fields. But such an approach not takes into account possibility of non-linear influence of electric and magnetic fields on wave shape that is to say deviation one from sinusoidal shape.

Plasma waves arise as a result appearing of ensemble of perturbed ions and electrons. Obviously outer stationary electromagnetic field affects both on perturbed particles and on the background plasma. Taking into account of this influence is content of the presented work.

Usual approximation of outer electrical field is $E(x) = \alpha x$, where x is the co-ordinate coinciding with direction of deviation of wave particles and $-A \le x \le A$, A is the wave amplitude that supposed by constant. Influence of electrical field $E(x)$ and magnetic field *B* on wave particles leads to appearing of particle drift velocity $V = E(x)/B = \alpha x / B$.

Direction of this velocity coincides with the wave propagation direction for $0 \le x \le A$ and it has opposite direction for $-A \le x \le 0$. Consequently the wave shape is change in time. Like situation occurs in hydrodynamics: it is problem of increasing wave inclination and following for one toppling over [5].

There are two distinctions between problem [5] and considered here problem.

1) In [5] *V* is linear increasing function of *y* in bound $0 \le y \le \lambda / 2$ (*y* is co-ordinate along direction of wave propagation). In this work *V* is non-linear function and maximal velocity corresponds to *y* for *x=A*.

2) In [5] only upper part of wave is considered. In this work both upper and lower parts of wave are considered. As above was indicated in lower part of drift velocity has opposite direction to upper part direction.

Let’s consider drift wave in the plasma confined in uniform magnetic field **B** directed along *z*-axis and non-uniform static electric field **E**(*x*) directed along *x*-axis. Wave propagates along *y*-axis. In this case particle drift displacement along *x*-axis $X = X(y,t)$ is proportional to the perturbed density $n^{\sim}$ and wave potential $\varphi^{\sim}$. One can suppose

$$\varphi^{\sim} = CX , \qquad (1)$$

where *C* some generally complex constant, which takes into account phase shift between $n^{\sim}$ and $\varphi^{\sim}$ in drift wave.

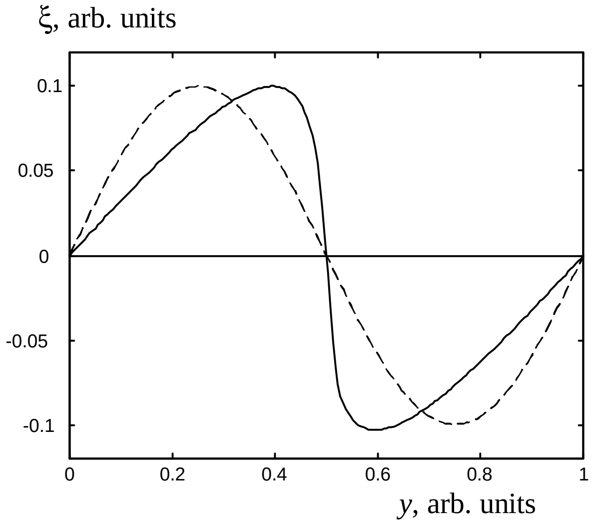

Fig. 1. Wave profile for small growth rate

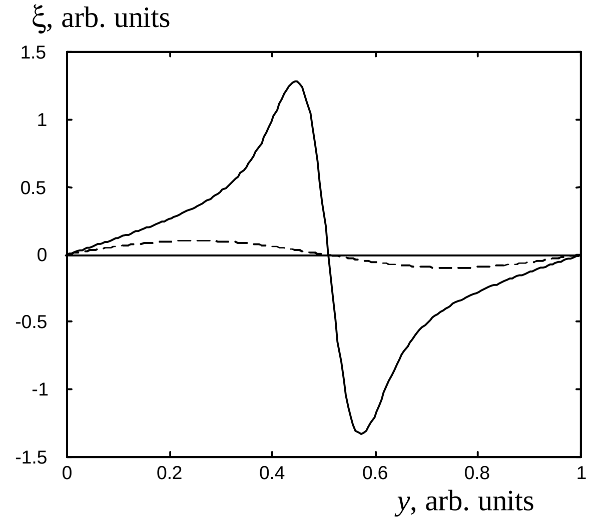

Fig. 2. Wave profile for large growth rate

In the system of coordinates moving together with the wave, equation of guiding center drift are:

$$\frac{dX}{dt} = -\frac{1}{B}\frac{\partial\tilde{\varphi}}{\partial y}, \tag{2}$$

$$\frac{dy}{dt} = -\frac{E(X)}{B}. \tag{3}$$

Using (1), (2), (3) and operator $\frac{dX}{dt} = \frac{\partial X}{\partial t} + \frac{\partial X}{\partial y}\frac{dy}{dt}$, we obtained equation

$$\frac{\partial X}{\partial t} + [C - V_E(X)]\frac{\partial X}{\partial y} = 0, \tag{4}$$

where $V_E(X = E(X)/B$.

Note, Eq. (4) practically coincides with non-linear drift wave equation [6]. Besides such an equation describes an perturbation in a beam of non interacting particles. Non-linear wave profiles are illustrated in Figs. 1 and 2, where dashed curves are initial linear wave profiles. One can see formation of the steep wave inclination.

In this paper Sec. 1 is written by V.I. Khvesyuk, Sec. 2 is written by V.I. Khvesyuk and A.Yu. Chirkov.

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НЕЛИНЕЙНЫЕ ЯВЛЕНИЯ, СВЯЗАННЫЕ С РАСПРОСТРАНЕНИЕМ ДРЕЙФОВЫХ ВОЛН В ПЛАЗМЕ

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Обсуждается нелинейный перенос тепла и частиц, а также ограничение амплитуды волны. Рассматривается нелинейная трансформация дрейфовой волны под действием неоднородного внешнего статического электрического поля.

НЕЛІНІЙНІ ЯВИЩА, ЗВ'ЯЗАНІ З ПОШИРЕННЯМ ДРЕЙФОВИХ ХВИЛЬ У ПЛАЗМІ

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Обговорюється нелінійний перенос тепла і часток, а також обмеження амплітуди хвилі. Розглядається нелінійна трансформація дрейфової хвилі під дією неоднорідного зовнішнього статичного електричного поля.