EQUILIBRIUM AND STABILITY OF CYLINDER PLASMA CONSISTING OF UNMAGNETIZED IONS BEEN BORN AT REST AND MAGNETIZED ELECTRONS IN CROSSED FIELDS

Yu.N. Yeliseyev

Institute of Plasma Physics, NSC "Kharkov Institute of Physics and Technology", Akademicheskaya Str. 1, 61108 Kharkov, Ukraine, e-mail:eliseev2004@rambler.ru

The equilibrium and the non-local stability problem of cylinder plasma, consisting of magnetized electrons and unmagnetized ions, having been born at rest under the ionization of the residual gas, are considered. The equilibrium distribution function of such ions is used. The dispersion equation of eigen helical plasma waves is obtained analytically and solved numerically for various azimuth wave numbers. For the lowest radial modes the instability of «negative mass»-type is possible having the frequency of the order of ion radial oscillation one and the growth rate up to 0.5 of it.
PACS: 52.20.Dq; 52.25.Dg; 52.27.Jt; 52.35.-g; 52.35.Fp; 52.35.Qz; 41.20.Cv

1. INTRODUCTION

Plasmas in crossed fields, consisting of magnetized electrons and unmagnetized ions, are formed in various devices: in plasma lenses, in plasma ion sources based on Penning cell, in the channel of electron and ion beam (secondary plasma). The peculiarity of ions of these plasmas is that they are being born at rest under ionization of residual gas. The peculiarity of operating regimes of these devices is strong radial electric field for ions over a wide range of field changing. Under action of such a field the born ions perform radial oscillating motion along strongly extended on radius trajectories. The frequency of these particle oscillations is called the "modified" cyclotron frequency of Ω ($\Omega >> \omega_{ci}$, ω_{ci} - is the ion cyclotron frequency). Their amplitude is comparable to the radius of the plasma cylinder a. The transversal wave length is of the same order of magnitude. Under these circumstances the non-local treatment of plasma stability is required. It has been carried out in [1-3] under strong restrictions using the variables "cylindrical coordinates – momentum" (r, φ, v_r, v_φ, z, v_z). In present work the non-local problem on the plasma stability is considered using the independent variables R,θ (cylindrical coordinates of Larmor centre of a particle), ρ,ϑ (its coordinates on Larmor circle), z, v_z [4] without these restrictions.

2. PLASMA EQUILIBRIUM

Plasma consists of electrons and of one sort of ions of small density and is bounded with a metal cylinder casing of radius a. It is supposed, that the electric potential in plasma $\Psi(r)$ obeys a square-law function of radius r ($\Psi(r)=\Psi(a)(r^2/a^2)$, $\Psi(a)$ - electric potential on border of the plasma). The radial electric field can be caused by non-neutrality of the plasma ($n_e \neq n_i$), by a spatial charge of a beam of the charged particles penetrating through plasma, by introducing in plasma of special electrodes biased corresponding potentials.

The electrons are magnetized being distributed uniform in radius with density $n_e = const$, $r<a$. In crossed fields they rotate rigidly around the axis of the plasma cylinder with the frequency of $\omega_e = -cE_r/Br = 2c\Psi(a)/(a^2B) = const > 0$ having the Maxwellian distribution in rotating frame of reference. In present article the electrons are assumed to be "hot", $(\omega - m\omega)/(k_z v_{Te}) << 1$ (v_{Te} is the thermal velocity of electrons, m is the azimuth wave number, k_z is longitudinal wave vector and ω is wave frequency).

The ions have been born at rest radially homogeneous with the total density $N = const$ ($0<r<a$) under ionization of the residual gas and move then without collisions under action of crossed fields. The ions can be magnetized or unmagnetized depending on the strength ratio between electric and magnetic fields. The equilibrium distribution function of such ions has the form [1]

$$F(\varepsilon_\perp, M, v_z) = \frac{N}{T}\frac{m_i}{\omega_{ci}} Y\left(e\Psi(a) - \varepsilon_\perp\right)\cdot\delta\left(\varepsilon_\perp - \omega_e M\right)\cdot\delta\left(v_z\right) \quad (1)$$

In (1) $T = 2\pi/\Omega$ is the period of radial oscillation of an ion, $\Omega = \left(\omega_{ci}^2 - 4eE_r/m_i r\right)^{1/2} = \left(\omega_{ci}^2 + 8e\Psi(a)/m_i a^2\right)^{1/2}$, $\varepsilon_\perp$, M - the transversal energy and the generalized angular momentum of an ion in a magnetic field, m_i - mass of an ion, Y - Heaviside step function, δ - Dirac delta function. The factor $\delta(\varepsilon_\perp - \omega_e M)$ reflects the fact of a birth of ions at rest and makes the distribution function similar to "rigid rotator" one. The factor $Y(e\Psi(a)-\varepsilon_\perp)$ reflects the fact of absence of ions with energy $\varepsilon_\perp > e\Psi(a)$ and makes the distribution function similar to Fermi–Dirac one.

3. PROBLEM of PLASMA STABILITY

Running helix waves, having the potential $\tilde{\Phi} = \tilde{\Phi}_m(r)\exp\left[i(m\varphi + k_z z - \omega t)\right]$ are considered. Unknown radial function $\tilde{\Phi}_m(r)$ is represented in Bessel function expansion: $\tilde{\Phi}_m(r) = \sum_{l=1}^{\infty} C_m^l J_m\left(\kappa_{m,l} r/a\right)/N_m^l$ (C_m^l - the expansion coefficients, $\kappa_{m,l}$ - l -s root of Bessel function J_m , $N_m^l = (1/\sqrt{2})\left|J_{|m|+1}(\kappa_{m,l})\right|$).

For considering the plasma stability Vlasov equation for ions and Poisson one should be solved jointly. Obtained in [5] the solution of linearized Vlasov equation in the independent variables R,θ,ρ,ϑ, introduced according to the relation $r\exp(i\varphi) = R\exp(i\theta) + \rho\exp(i\vartheta)$ ($\tilde{\vartheta} = \omega_l^+$,

$\tilde{\vartheta} = \omega_i^-$), is used. The equilibrium ion distribution function (1) should be expressed over new variables R, ρ. By integrating the solution of linearized Vlasov equation over the velocity space, the ion density disturbance $\tilde{n}_i$ is obtained. Substituting $\tilde{n}_i$ in Poisson equation and multiplying both parts on Bessel function $J_m\left(\kappa_{m,k} r/a\right)$ one should integrate the obtained equation over coordinate space. It is necessary to proceed from variables r, v to variables $R, \theta, \rho, \vartheta, z, v_z$ and take into account, that Jacobian is equal to $\Omega^2 R\rho$ [5]. In result it is obtained the homogeneous system of the linear equations for expansion coefficients C_m^l:

$$(L_{kl} - b^2 A_{kl}) C_m^l = 0 \,. \qquad (2)$$

The diagonal matrix $L_{kl} = (1+\kappa_{m,k}^2 b^2)\delta_{kl}$ describes the contribution to Poisson equation of Laplacian of $\tilde{\Phi}$, which takes into account charge separation, and density disturbance of "hot" electrons ($b^2 = ((a^2/r_{De}^2) + k_z^2 a^2)^{-1}$, r_{De} - the electron Debye radius, δ_{kl} - Kronecer symbol). A symmetric matrix A_{kl} describes the contribution of ions ($\omega_{pi}^2 = 4\pi e^2 N/m_i$, $z_l^\pm = \kappa_{m,l}|\omega_\pm|/\Omega$, $\omega' = \omega - m\omega_+$)

$$A_{kl} = \frac{\omega_{pi}^2}{N_m^l N_m^k} \sum_{p=-\infty}^{\infty} \left\{ \frac{1}{\omega_+^2} \frac{p}{\left(\dfrac{\omega'}{\Omega} - p\right)} J_{m+p}(z_l^-) J_{m+p}(z_k^-) J_p(z_l^+) J_p(z_k^+) + \frac{1}{\Omega^2} \frac{k_z^2 a^2}{\left(\dfrac{\omega'}{\Omega} - p\right)^2} \int_0^1 x dx J_{m+p}(z_l^- x) J_{m+p}(z_k^- x) J_p(z_l^+ x) J_p(z_k^+ x) + \right.$$

$$\left. + \left[\frac{m}{\omega_-^2} + p\left(\frac{1}{\omega_-^2} - \frac{1}{\omega_+^2} \right) \right] \cdot \frac{1}{\left(\dfrac{\omega'}{\Omega} - p\right)} \int_0^1 dx \frac{d}{dx} \left[J_{m+p}\left(z_l^- x\right) J_{m+p}\left(z_k^- x\right) \right] J_p\left(z_l^+ x\right) J_p\left(z_k^+ x\right) \right\}. \qquad (3)$$

Expression (3) is valid within the total range of the "modified" cyclotron frequencies of ions ($\omega' \sim \Omega$), for arbitrary strength of magnetic and radial electric fields, azimuth wave number m and longitudinal wave vector k_z. As seen from (3), in the strong electric field ($\Omega >> \omega_{ci}$) the normalized frequency spectrum in the ion rotating frame of reference ω'/Ω does not depend on ion mass, i.e. the spectrum ω'/Ω is universal.

The dispersion equation for the plasma waves has the form $\det(L_{kl} - b^2 A_{kl}) = 0$. It was solved numerically for the waves with various azimuth wave numbers m.

4. SPECTRA of PLASMA OSCILLATIONS

The obtained spectra of oscillations (ω'/Ω -in the rotating and ω/Ω - in the laboratory frames of reference) are presented in Fig. 1 depending on parameter b^2 ($10^{-5} \le b^2 \le 5 \cdot 10^{-1}$) for ions of atomic nitrogen at density $N = 10^7\,cm^{-3}$, $B_0 = 200\,Gs$, $k_z = 1cm$, $a = 1cm$, $\Psi(a) = 1V$. The chosen value of electric potential $\Psi(a)$ corresponds to a strong electric field for ions of the atomic nitrogen ($\omega_{ci}/\Omega \approx 0.18$).

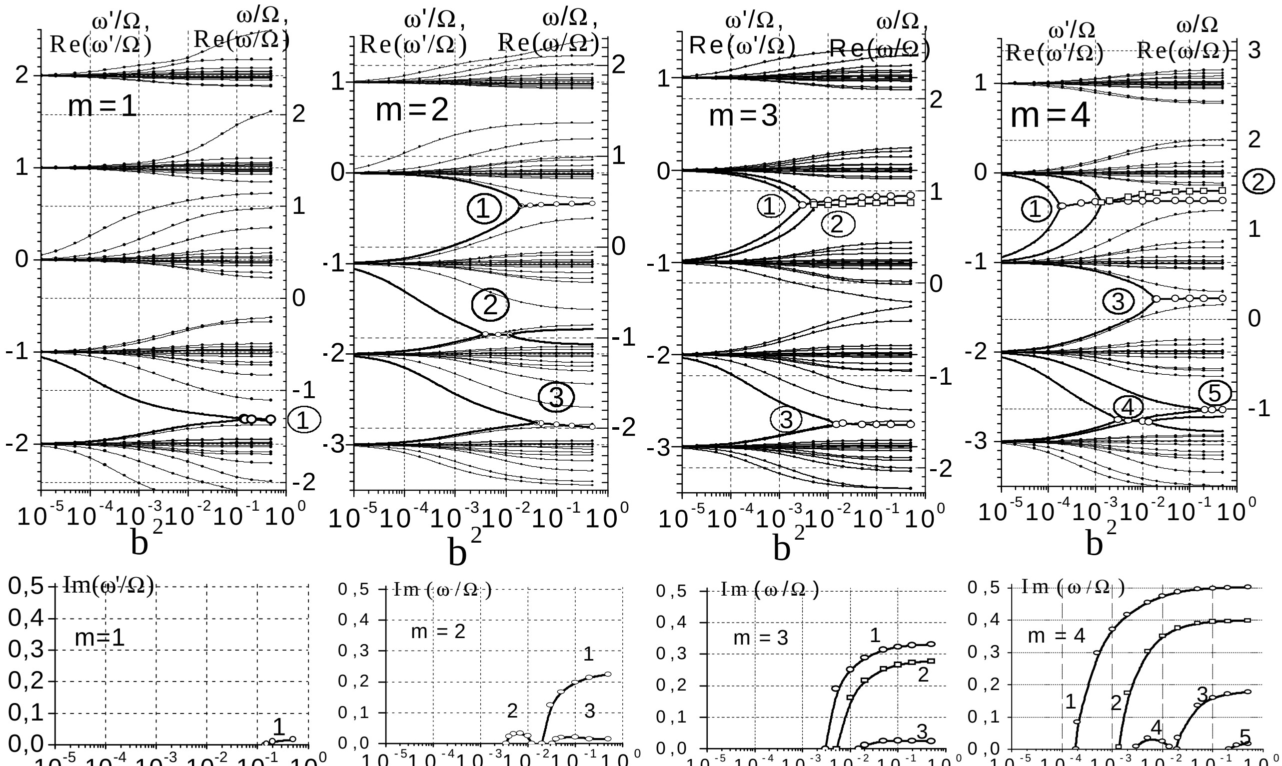

Fig.1. The spectra and growth rates of plasma oscillations depending on parameter b^2 for azimuth wave numbers $m = 1, 2, 3, 4$. The unstable sections of spectra are plotted by thick lines with circles and are marked with numerals

The curves form families located close to the ion "modified" cyclotron frequency (both above and below it) and its harmonics including zero-order one. Different curves inside the family correspond to different radial modes. The lowest radial modes are located further from integer value of "modified" cyclotron frequency. The curves, belonging to the adjacent families, approach each other with the parameter b^2 increasing. Some lowest radial modes (zero-order and less often first-order) can intersect. These modes become unstable. There is the instability of the "negative mass" type [6]. It is characteristic of oscillators, by what the ions of plasma essentially are. The behaviour of curves in fig. 1 is similar to behaviour of eigen waves of electron unbounded homogeneous plasma with distribution function of oscillator-type ([7], chapter 7, 9).

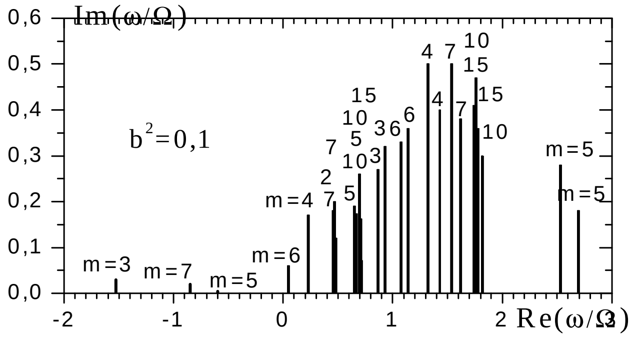

Fig.2. The frequencies ($\mathrm{Re}(\omega/\Omega)$ *) and growth rates (* $\mathrm{Im}\,\omega/\Omega$ *) of all unstable modes in laboratory frame of reference. The parameters are the same as in fig.1. Figures above columns specify azimuth wave numbers*

The maximum growth rates achieve some tenth parts (up to 0.5) of the ion "modified" cyclotron frequency Ω. As a rule, one - two lowest radial modes with large growth rates ($\mathrm{Im}\,\omega/\Omega \sim 0.2-0.5$) are present in a spectrum at $m \geq 2$. In the latter case the frequencies of two unstable modes are very close to each other. On changing parameter b^2 over some orders of magnitude the real part of normalized frequency of unstable modes ($\mathrm{Re}(\omega'/\Omega)$) change insignificantly, less than by 0.1. It's interesting that in the laboratory frame of reference nearly all unstable waves rotate in a positive direction ($\mathrm{Re}\,\omega/m > 0$) and their frequencies are of the order of $\mathrm{Re}\,\omega \sim \Omega$ (Fig.2).

At $m = 0$ all radial modes are stable. At $m = 1$ only one mode is unstable with small growth rate ($\mathrm{Im}\,\omega/\Omega \sim 0.02$, $\mathrm{Re}\,\omega/\Omega \approx -1.3$).

Calculations were carried out also at $N = 10^6\,cm^{-3}$ and $\Psi(a) = 452V$. This potential is created on the bound of the electron plasma with density $n_e = 10^9\,cm^{-3}$. In this case $\omega_{ci}/\Omega \approx 0.0087$. The obtained spectra of plasma oscillations are stable. The curves are located close to the integer values of ω'/Ω.

REFERENCES

1. V.G. Dem'yanov, Yu.N. Yeliseyev, Yu.A. Kirochkin, A.A. Luchaninov, V.I. Panchenko, K.N. Stepanov //*Fizika plasmy*. 1988, v. 14, №7, p. 840.
2. Yu.N. Yeliseyev, Yu.A. Kirochkin, K.N. Stepanov // *Fizika plasmy*. 1991, v. 17, №9, p.1072-1082 .
3. Yu.N. Yeliseyev, Yu.A. Kirochkin, K.N. Stepanov // *Fizika plasmy*. 1992, v. 18, №12, p. 1575-1583.
4. Yu.N. Yeliseyev // *Fizika Plasmy*. 2006, v.32, №10, p.1-10.
5. D.V. Chibisov, V.S. Mikhailenko, K.N. Stepanov // *Plasma Phys. Contr. Fusion*. 1992, v. 34, p. 95-117.
6. *Plasma Electrodynamics*/ ed. by A.I. Akhiezer, Moscow: "Nauka", 1975.
7. G. Bekefi / *Radiation Processes in Plasmas*, Moscow: "Mir", 1971.

РАВНОВЕСИЕ И УСТОЙЧИВОСТЬ ЦИЛИНДРИЧЕСКОЙ ПЛАЗМЫ, СОСТОЯЩЕЙ ИЗ НЕЗАМАГНИЧЕННЫХ ИОНОВ, РОДИВШИХСЯ В СОСТОЯНИИ ПОКОЯ, И ЗАМАГНИЧЕННЫХ ЭЛЕКТРОНОВ, В СКРЕЩЕННЫХ ПОЛЯХ

Ю.Н. Елисеев

Рассматриваются равновесие и нелокальная задача об устойчивости цилиндрической плазмы, состоящей из замагниченных электронов и незамагниченных ионов, которые родились в состоянии покоя при ионизации остаточного газа. Используется равновесная функция распределения таких ионов. Дисперсионное уравнение собственных спиральных плазменных волн получено аналитически и решено численно для различных азимутальных волновых чисел. Для низших радиальных мод возможна неустойчивость типа «отрицательной массы» с частотой порядка частоты радиальных осцилляций иона и максимальным инкрементом до 0,5 этой частоты.

РІВНОВАГА ТА СТІЙКІСТЬ ЦИЛІНДРИЧНОЇ ПЛАЗМИ, ЩО СКЛАДАЄТЬСЯ З НЕЗАМАГНІЧЕНИХ ИОНІВ, ЯКІ НАРОДИЛИСЯ У СТАНІ СПОКОЮ, ТА ЗАМАГНІЧЕНИХ ЕЛЕКТРОНІВ, У СХРЕЩЕНИХ ПОЛЯХ

Ю.М. Єлісеєв

Розглядається рівновага та нелокальна задача стійкості циліндричної плазми, що складається з замагнічених електронів та незамагнічених іонів, які народились в стані спокою при іонізації залишкового газу. Використовується рівноважна функція розподілу таких іонів. Дисперсійне рівняння власних спіральних плазмових хвиль одержане аналітично та розв'язане чисельно для різних азимутальних хвильових чисел. Для нижчих радіальних мод можлива нестійкість типу «негативної маси» з частотою порядку частоти радіальних осциляцій іона та максимальним інкрементом до 0,5 цієї частоти.