DETERMINING THE Δ(*1232*) POLE PARAMETERS

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In the framework of a resonance model the Δ(1232) pole characteristics are determined from the data of several πN elastic scattering analyses on the P_{33}.phase shift. For the residue an approximated analytical formula is obtained is obtained that was used to analyze the connection between the background and pole parameters.
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1. INTRODUCTION

In publications on the πN elastic scattering one can observe very essential changes of the absolute value and phase responses for a residue of the P_{33} wave amplitude in the Δ(1232) excitation region ([1]). In principle, this can be explained by current changes of the initial experimental base in the process of its quantitative and qualitative improvement. Another possible reason is a model uncertainty in the analytical description of the amplitude that appears substantially when extrapolating it to the pole situated outside the physical region. To clarify these problems we have considered here in the framework of the resonance model [2,3] a relation between the residue of the full amplitude and resonance amplitude depending on the presence of a background. The comparative fitting of data on the P_{33} phase shift obtained from several elastic πN scattering analyses is also fulfilled with the aim of determining the resonance and pole characteristics corresponding to the Δ(1232) resonance.

2. RESIDUE IN THE RESONANCE MODEL

Description of the P_{33} πN scattering amplitude is rendered easy by taking it purely elastic in the first resonance region. In a resonance model dependent on the full energy W (s. c. m) the S- matrix element has the following factorized form:

$$S(W) = S_B(W)S_R(W), \tag{1}$$

where S_B and S_R correspond to the potential background and the resonance. In physical region they are determined by corresponding real phase shifts δ, δ_B,δ_R:

$$S = e^{i\delta},\ S_B = e^{i\delta_B},\ S_R = e^{i\delta_R}, \tag{2}$$

and phase shift addition rule takes place:

$$\delta(W)=\delta_B(W)+\delta_R(W). \tag{3}$$

Over the whole complex plane W *the* values S_B и S_R are described by the following expressions:

$$S_B(W) = \frac{1+iB(W)}{1-iB(W)}, \tag{4a}$$

$$S_R(W) = \frac{W-M_R-i\Gamma(W)/2}{W-M_R+i\Gamma(W)/2}. \tag{4б}$$

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In the physical region $B(W)$ and $\Gamma(W)$ are real functions being related with δ_B, δ_R (M_R is the resonance mass):

$$\tan\delta_B(W) = B(W), \tag{5a}$$

$$\tan\delta_B(W) = \frac{\Gamma(W)/2}{M_R - W}. \tag{5б}$$

It is seen from formula (4b) that at W satisfying the equation

$$M_R - W - i\Gamma(W) = 0 \tag{6}$$

the function S_R has a pole. According to the scattering theory a resonance corresponds to the root of Eq. (6) $W_P=M_P-i\Gamma_P/2$, the latter situated in the forth quadrant of the complex plane W at positive M_P and Γ_P. Keeping in mind that the expansion of the denominator on the right-hand side of Eq. (3) in series on power of $(W-W_P)$ starts with a linear term and restricting ourselves with taking the latter into account, we can write Eq. (4b) in the following form:

$$S_R(W) \cong \frac{W-M_R-i\Gamma(W)/2}{(1+i\Gamma'(W_P)/2)(W-W_P)}, \tag{7}$$

where $\Gamma'(W_P)$ is the value of the derivative $d\Gamma(W)/dW$ at the pole point. The exact formula for the residue is obtained from Eq. (7):

$$\mathrm{Res}(S_R) = \frac{-i\Gamma(W_P)}{1+i\Gamma'(W_P)/2}. \tag{8}$$

Now let us use the fact that the nominator of the right-hand side of Eq. (7) goes to zero at the complex conjugated to the pole point $W_P^* = M_P + i\Gamma_P/2$ and take Taylor series at point W_P^*. Restricting ourselves with regard for the first term only and supposing that such an approximation can be extended to the pole point, we get the unipolar approximation for the resonance S matrix element

$$S_R(W) \cong \frac{1-i\Gamma'(W_P^*)/2}{1+i\Gamma'(W_P)/2}\,\frac{W-M_P-i\Gamma_P/2}{W-M_P+i\Gamma_P/2}, \tag{9}$$

From Eq. (9) the expression for the residue follows:

$$\mathrm{Res}(S_R) \cong -i\Gamma_P\frac{1-i\Gamma'(W_P^*)/2}{1+i\Gamma'(W_P)/2}. \tag{10}$$

Evidently, for the validity of Eq. (10) it is necessary, that the distance Γ_P between W_P^* and the pole would not be too large and the dependence of width on W be rather smooth. The relevant criterion can be get from Eqs. (8,10):

$$\Gamma(W_P)/\Gamma_P \cong 1 - i\Gamma'(W_P^*)/2 \quad (11)$$

In particular, for the energy independent width the relationship $\Gamma(W_P)=\Gamma_P$ takes place.

Usually the residue of the amplitude $T\equiv(S-1)/2i$ is cited in analyses. From Eq. (10) it follows

$$|\mathrm{Res}(T_R)| \cong \Gamma_P/2, \quad (12)$$

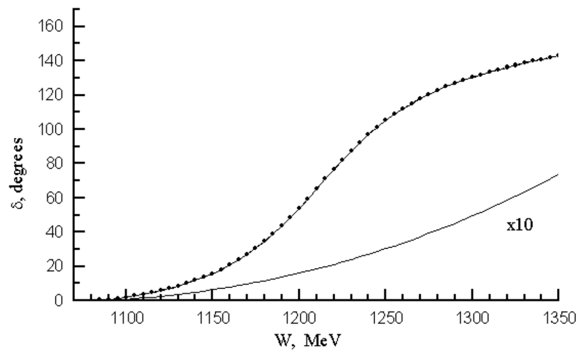

Fig. 1. *Energy dependence of δ from analysis SM99 calculated for MR=1235,14 MeV; ΓR=123,36 MeV; r=0,97520 fm; a=0,02822 fm³. The lower curve is the background phase shift multiplied by 10.*

$$\varphi(T_R) = 2\varphi_0, \quad (13)$$

$$\varphi_0 \cong \arg(1 - i\Gamma'(W_P^*)/2) = -\arg(1 + i\Gamma'(W_P)/2). \quad (14)$$

It is seen from Eq. (14,13) that the phase of the residue amounts to zero for the energy independent width.

At last, in general case with taking into account a background the residue has the following form (note that the presence of a background does not influence the pole position):

$$|\mathrm{Res}(T)| \cong |S_B(W_P)|\Gamma_P/2, \quad (15)$$

$$\varphi(T) \cong \arg(S_B) + 2\varphi_0. \quad (16)$$

3. NUMERICAL CALCULATIONS

a) Version of the resonance model. The resonance width was calculated with taking into account the threshold dependence and the factor containing a special parameter r to correct the high-energy behavior (in [4] such a factor is associated with the extent of penetration of a scattered particle over the nuclear surface):

$$\Gamma(W) = \Gamma_R\,(q/q_R)^3\,(1+q_R^2 r^2)\big/(1+q^2 r^2). \quad (17)$$

In Eq. (17) $q\equiv q(W)$ is the momentum of particles in s. c. m., q_R is its value at $W=M_R$, $\Gamma_R=\Gamma(M_R)$. The background was described by the formula

$$B(W) = aq^3(W). \quad (18)$$

Introducing an additional factor $2W_R/(W_R+W$ into the width in Eq. (17)) yields the relativistic version of the model with the following resonance amplitude:

$$T_R(W) = \frac{\Gamma(W)W_R}{W_R^2 - W^2 - i\Gamma(W)W_0}. \quad (19)$$

However, the calculations showed that transition to the relativistic version does not results in any significant changes in the outputs for the resonance and pole parameters, so we have restricted the following consideration by the nonrelativistic version of the resonance model.

The resonance and pole parameters of the P_{33} wave amplitude (our calculations)

Data	N	χ^2	M_0, MeV	Γ_0, MeV	M_P, MeV	$\Gamma_P/2$, MeV	|res.|, MeV	φ(res.), °
KP78	27	19.0	1230.99	116.00	1209.21	–50.36	52.35	–48.87
*	10	4.2	1230.91	115.44	1210.08	–49.47	50.15	–46.23
KA84	55	54.0	1231.23	118.22	1208.70	–51.36	53.75	–49.83
*	17	7.9	1231.25	116.76	1211.20	–50.72	51.31	–43.16
SM90	55	0.25	1231.29	113.91	1210.59	–49.92	51.81	–46.77
*	17	0.14	1231.28	114.05	1210.53	–49.83	51.60	–46.86
SM95	55	0.18	1231.93	113.04	1211.64	–50.09	52.37	–46.04
*	17	0.08	1231.96	113.24	1211.57	–50.18	52.51	–46.23
SM99	55	0.15	1232.52	115.23	1211.55	–50.82	53.14	–46.87
*	17	0.00	1232.54	115.41	1211.50	–50.88	53.23	–46.99
SM99s	16	65.0	1232.11	114.68	1210.89	–50.49	52.94	–47.99
SM99s,c	15	27.0	1232.08	115.00	1210.60	–50.51	52.98	–48.61
*	5	0.82	1231.89	112.84	1212.94	–48.27	47.56	–42.00

Note. *N–number of points, *–data at 1180 MeV<W<1260 MeV, c–fit without point at W =1180 MeV.*

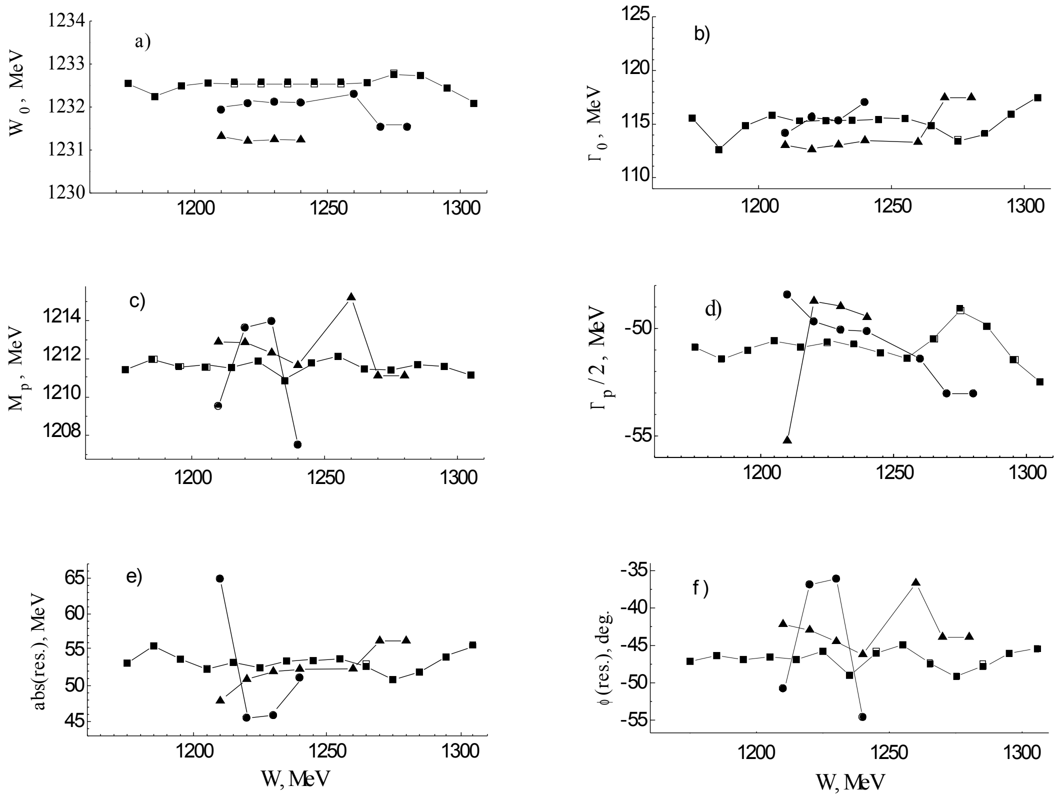

***Fig. 2**. Resonance and pole parameters determined from the data of analyses SM99 (squares), SM99s (triangles) и KA84 (circles), taken from the energy intervals of length 30, 60 и 80 MeV, respectively, centers of which are plotted on the ordinate axis.*

b) Fitting the data. Parameters of the model M_R, Γ_R, *r* and *a* were determined from the data on P_{33} phase shift in the region of W from the threshold up to 1350 MeV obtained with a step of 5 MeV by INTERNET from the SAID system (***http://said.phys.vt.edu***), named there as SM99, SM99s, SM90 and SM95 (the last phase shift analysis by Arndt et al. [5] and its preceding versions, correspondingly, letter s means the single energy version) as well as KH80 and KA84 (the analysis [6] and its subsequent "smoothed" version). In every case the resonance model enables, after fitting the parameters, a good description of the data on δ. In particular, the result of fitting to the last phase shift analysis by Arndt et al. (SM99 in Table) is represented in Fig. 1. Then we calculated the resonance parameters: "experimental mass" W_0 being a position of the point where the P_{33} phase shift comes through the value 90°, experimental width $\Gamma_0=2/(d\delta/dW)|_{W=W_0}$ as well as pole coordinates and the residue parameters.

The resonance and pole characteristics from phase shift analyses under consideration are given in Table. A special attention was paid to SM99*s* analysis (single energy) performed for the net of 16 energy points with a strong influence of the experimental material situated in vicinity of the knot values. Fitting of this data has yielded χ^2=65. But it was found that a major part of this amount is contributed from the point *W*=1180 MeV, may be due to the under estimated error (next to the last line in Table). Exclusion of this point has given much lower χ^2 value, but results for the sought parameters have changed negligibly (the last line in Table).

To evaluate the influence of separate groups of initial experimental points on the output parameters we carried out also fitting using the data on reduced energy intervals. In principle the responses obtained in such a way should be in accordance with fulfillment of the following conditions: a) availability of the high precision experimental material b) adequacy of the model used for fitting. For analysis SM99, SM99s and KA84 it is possible with choosing the minimum energy intervals of 30, 60 and 80 MeV, respectively (Fig. 2). It can be seen, that the region of about 1210−1220 MeV is the most informative. Such a test has given the best result for SM99: in this case the responses for the experimental mass, width and all pole parameters at the center of the resonance distribution proved to be the most independent on the location of the such reduced intervals with data, the errors arising as moving aside. For single energy analysis SM99s and earlier analysis KA84 the responses are rather instable even at the central region, and out of the latter they are getting quite unreasonable. So, it seems that fitting with using only the data from the central region 1180 MeV<W <1260 MeV is the most reliable when treating the whole resonance region. In Table the relevant results are given at lower lines for each analyses under consideration. One can see that in some cases they are different from the latter obtained by fitting the whole resonance region, especially for a phase of the residue.

c) The approximated formula for the residue. Calculation checking proves that approximated Eqs. (13,14) are sufficiently accurate both for the value and phase of the residue. Thus, for the solution given in Fig. 1, their derivations from the exact calculation amount 1,2% и 0,4%, respectively. This permits using Eq. (13) for ascertainment of a relation between the background parameter a and the relative value of the residue y:

$$y \equiv \frac{|\mathrm{Res}(T)|}{\Gamma_P/2} \cong \left|\frac{1+iaq^3(W_P)}{1-iaq^3(W_P)}\right| \qquad (17)$$

After solving Eq. (17) with regard to a one can calculate the background phase shift $\delta_B(W)$ using Eq. (18) and (5a). To characterize a background, result of such a calculation at W=1232 MeV is presented in Fig. 3 as a function of y. Taking y from existing phase shift analyses one can determine the corresponding values of $\delta_B(W)$ near the resonance. The latter form groups at values of about −15°, −3° and +3. However, it is necessary to put y~1,32 to get the value $\delta_B \approx +15°$ at resonance from the theory of effective Lagrangians with a number of unitarization methods [7].

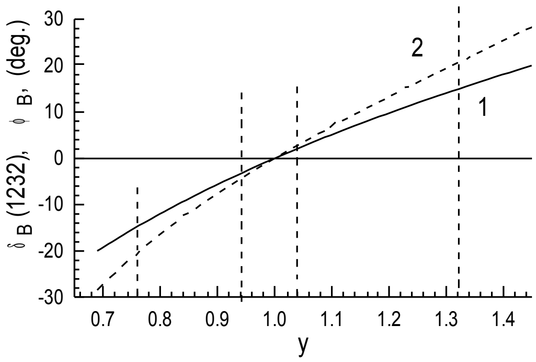

Fig. 3. *Value δB at W=1232 MeV (curve 1) and background contribution to the phase of residue (curve 2) vs y. Vertical dash lines: y≈0,76 from Arndt 95, Arndt 99a; y≈0,94 from Cutkosky 80, Arndt 99b; y≈ 1,04 from Arndt 90 ([1]) and our pole estimation on the base of SM99; y≈1,32 corresponds to δB(1232)≈15°.*

4. CONCLUSIONS

1. The approximation considered in Sect. 1 in fact is a starting point for construction of the resonance model itself beginning with the series expansion of the Jost function near the pole of amplitude, and the checking has proved this underlying thesis.

2. For the residue an approximated analytical formula is obtained that was used to analyze the connection between the background and pole parameters. In particular, from this formula it follows that the coincidence between the absolute value of a residue and the imaginary coordinate of pole points on the absence of the background.

3. At suggestion that background is described by a single parameter an equation similar to Eq. (15) allows determining in unique fashion the background phase shift using the known value of a residue and pole coordinates, and no information about $\Gamma(W)$ is necessary for this. In our case the relevant estimation of the background phase shift using data of different analyses is multiple-valued**:** at resonance it takes values from −15° up to +3°, but the value +15° from the effective Lagrangian theory [7] has not been observed.

4. Fulfilling the retrospective fitting of the data of main phase shift analyses revealed a little change in Δ (1232) pole parameters during the last couple of decades. Indirectly this can point that experimental progress in πN scattering in the region of the first resonance excitation is not so significant as it may be expected.

5. Calculation on reduced energy intervals allows estimating "the energy resolution" for different analyses. Thus on the basis of a parameterized version of Arndt analyses the resonance and pole parameters can be determined using the data from rather small energy interval of 30 MeV, and the latter can be taken in different places of the resonance region. For other analyses the minimum length of such an interval is about 60−80 MeV, and any its shift changes cruelly the output for pole parameters.

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