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# IONS CROSS-B COLLISIONAL DIFFUSION AND ELECTROMAGNETIC WAVE SCATTERING

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The calculation is presented of the averaged quadratic displacement of a collisional charged particle in a magnetic field. This calculation is used to obtain the statistical presentation of the electromagnetic field scattered by these particles. These results extend the previous calculations that were restricted to non-magnetized particles (Ornstein equation, Einstein diffusion, etc.). In addition this calculation foresees effects that are absent of the Ornstein equation: a modulation of the averaged quadratic displacement function at the cyclotron frequency and a maximum of the Cross-B diffusion coefficient when the cyclotron frequency is equal to the collision frequency (Bohm diffusion).

## Introduction

Collective scattering (CS) is a common diagnostics of plasma turbulence that provides information on plasma irregularities and motions. This information is especially valuable for magnetized ionosphere as well as for fusion plasmas. In some cases, plasma cross-B diffusion coefficients can be obtained from the CS signal spectrum analysis (1). The information given by the CS signal, however, is constituted not only of macroscopic plasma parameters but also of some particle macroscopic parameters at a given spatial scale. These macroscopic features are expected when the observed scale is of the order of the ion Larmor radius.

This analysis is the objective of the present contribution. A theoretical examination of the CS physical process leads to a calculation of the scattering signal issued from a magnetized plasma with taking into account effects of collisions with neutrals.

## Magnetized ion motion and the scattering signal

If we have a set of particles exposed to an incident electromagnetic field, the scattered electromagnetic field is sum of individual field scattered from each elementary particles. The scattering experiment geometry defines a scale. When this scale is larger then the Langmuir length $\lambda_D$ the elementary building blocks are the dressed ions. The total scattered field is the sum of fields scattered by all of observed dressed particles:

$$\vec{E}_s(R,t)=\frac{r_0}{R}\vec{E}_i(0,t)\sum_{j=1}^{N}e^{i\vec{k}\cdot\vec{r}_j(t)}, \qquad (1.1)$$

where $\vec{E}_i(0,t)$ is the incident field complex amplitude at a reference origin near to the position of the scattering particle $\vec{r}_j$, $r_0$ is the classical Thomson radius, $R$ is the distance between scattering position and observer, and $\vec{k}$ is the analyzing wave-vector which appear as difference between scattered wave-vector $\vec{k}_i$ and incident wave-vector $\vec{k}_s$:

$$\vec{k}=\vec{k}_s-\vec{k}_i\,. \qquad (1.2)$$

We assumed $k\lambda_D \ll 1$. To form the field at time $t+\tau$, the particle trajectories are spit in two terms: the mean fluid displacement $\vec{\Delta}(\vec{r}(t),\tau)$ (defined as the displacement averaged over a small scale cloud of particles) and the additional individual particle displacement $\vec{\delta}_i(t,\tau)$ (relative to the center of mass of our local "cloud"):

$$\vec{r}_j(t+\tau)=\vec{r}_j(t)+\vec{\Delta}(\vec{r}_j(t),\tau)+\vec{\delta}_j(t,\tau) \qquad (1.3)$$

The product of both fields at time $t+\tau$ and at initial time $t$ provides the scattered field correlation function. If statistical independence is assumed between density, macroscopic and microscopic displacements, the correlation function will take the following form:

$$C(\tau)\cong\left|\frac{r_0}{R}\vec{E}_i(0,t)\right|^2 NS(\vec{k})\left\langle e^{i\vec{k}\cdot\vec{\Delta}_\tau}\right\rangle\left\langle e^{i\vec{k}\cdot\vec{\delta}_\tau}\right\rangle \qquad (1.4)$$

where $S(\vec{k})$ is the “form factor”.

$$S(\vec{k})=\frac{1}{N}\left\langle\left|\sum_{j=0}^{N}e^{-i\vec{k}\vec{r}_j}\right|^2\right\rangle \qquad (1.5)$$

The next two terms are the “statistical characteristics” of the macroscopic displacement $\vec{\Delta}_\tau$ and of the microscopic random displacement, both occurred during time $\tau$ along $\vec{k}$. If these displacements are gaussian random processes:

$$\left\langle e^{-i\vec{k}\cdot\vec{\Delta}_\tau}\right\rangle=e^{-\frac{1}{2}k^2\left\langle\Delta^2_{\vec{k},\tau}\right\rangle}$$
$$\left\langle e^{-i\vec{k}\cdot\vec{\delta}_\tau}\right\rangle=e^{-\frac{1}{2}k^2\left\langle\delta^2_{\vec{k},\tau}\right\rangle} \qquad (1.6)$$

The macroscopic random mean square (turbulent motion) can usually be described by the Ornstein dispersion (see e.g. (2)):

$$\left\langle\Delta^2_{\vec{k},t}\right\rangle=2D_M\tau_c\left(\frac{t}{\tau_c}-1+e^{-\frac{t}{\tau_c}}\right), \qquad (1.7)$$

where $D_M$ and $\tau_c$ are respectively the turbulent fluid diffusion coefficient and correlation time.

For large time $t$, the Ornstein dispersion tends to the Einstein diffusion, proportional to the time.

$$\left\langle \Delta^2 \right\rangle \simeq 2D_M t \tag{1.8}$$

Consequently the "characteristic" (1.6) is a decreasing exponential at the inverse time rate $k^2 D_M$.

## Ion motion in a plane perpendicular to a uniform magnetic field

The scattering electric field time correlation (1.4) is thus made of the product of two different "statistical characteristics". One corresponds to the mean motion, and the second one to the microscopic motion. The microscopic motion is the cyclotron motion at angular frequency $\omega$, with random ion-neutral collisions at frequency $\nu$. Using an appropriate microscopic, individual particle model, the microscopic diffusion can be calculated (3).

The motion of a charged particle in a magnetic filed between two collisions is along a Larmor circular orbit (See Figure 1). A sample trajectory in Figure 1 is shown for a positively charged particle. The mean square displacement does not depend on particle charge.

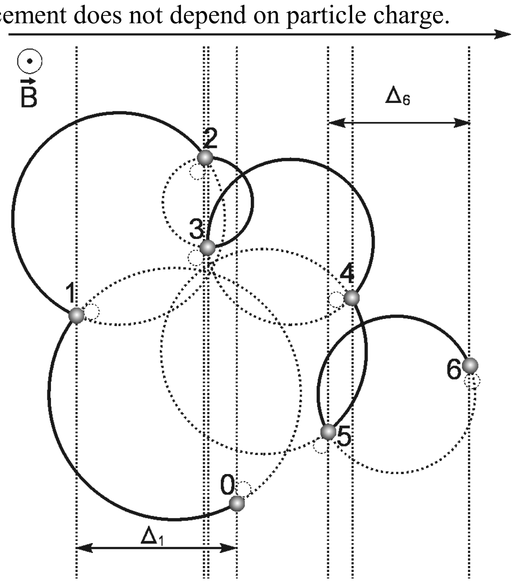


*Figure 1. Projection of a charged particle sample trajectory on a surface perpendicular to the magnetic field. Bold line — trajectory; positions 0,1,2,3,4,5,6 — show points of collision of our particle with neutrals.*

For the uniform magnetic filed, charge particle trajectory is circle of Larmor radius:

$$\rho = \frac{u}{\omega}, \tag{1.9}$$

where $u$ is the particle velocity and $\omega$ is the cyclotron frequency.

The cyclotron motion is interrupted by collision with a neutral atom. After collision the particle continues a rotation at cyclotron frequency but around a new center of rotation and with a different radius.

The particle motion between collisions $i-1$ and $i$ is shown in Figure 2. It is a circular motion of cyclotron radius $\rho_i$. The projection of this displacement along a given direction (perpendicular to the magnetic field) from position $i-1$ to position, can be written as:

$$\begin{aligned}\delta_i &= \rho_i \cos(\varphi_i) - \rho_i \cos(\varphi_{i-1}) \\ &= \rho_i \left[ \cos\left(\varphi_{i-1} - \omega_C (t_i - t_{i-1})\right) - \cos(\varphi_{i-1}) \right]\end{aligned} \tag{1.10}$$

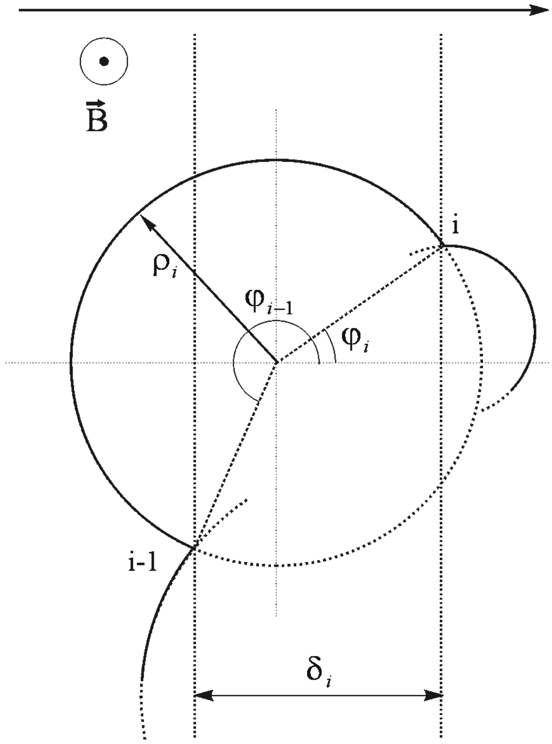


*Figure 2. Sample of trajectory (projection on face perpendicular to magnetic filed) between two collisions.*

The total displacement after $N$ collision is the sum of elementary displacements:

$$\Delta_N = \sum_{i=1}^{N+1} \delta_i, \tag{1.11}$$

when the number of collisions is large, a statistical average has to be made over the velocity probabilities distribution (or equivalently over the Larmor radius $\rho_i$ probability distribution) and over the collision phase angle $\varphi_i$. We assume that radius length before and after collision not correlated and that after collision phases probability distribution $\varphi_i$ is uniform between $0$ and $2\pi$.

Then the mean square of (1.11) can be written as:

$$\begin{aligned}\left\langle \Delta_N^2 \right\rangle &= 4\rho^2 \sin^2\left(\frac{\omega_C (t - t_N)}{2}\right) + \ldots + \\ &+ 4\rho^2 \sin^2\left(\frac{\omega_C (t_2 - t_1)}{2}\right) + 4\rho^2 \sin^2\left(\frac{\omega_C t_1}{2}\right).\end{aligned} \tag{1.12}$$

The average over the different possible number of collisions can be calculated using the probability function of a number $n$ of collision in a time interval $t$:

$$P_n = \nu^n e^{-\nu t}, \tag{1.13}$$

where $\nu$ is the collision frequency.

The mean of collision number takes the following form:

$$\left\langle \Delta^2 \right\rangle_t = 4\rho^2 \sum_{N=0}^{\infty} v^N e^{-vt} \int_0^t dt_N \ldots \int_0^{t_2} dt_1 \sum_{i=1}^{N+1} \sin^2\left( \frac{\omega_C \left(t_i - t_{i-1}\right)}{2} \right) \quad (1.14)$$

After integration and summation, the microscopic mean square random motion (diffusion) normalized to the root mean square ion cyclotron radius $\rho$ across the magnetic field is found as:

$$\frac{\left\langle \Delta^2 \right\rangle_t}{\rho^2} = 2\frac{\frac{\omega_C^2}{v^2}}{1+\frac{\omega_C^2}{v^2}} \left( \left(1+vt\right) - \frac{2}{1+\frac{\omega_C^2}{v^2}} + e^{-vt} \frac{\left(1-\frac{\omega_C^2}{v^2}\right)\cos\left(\omega_C t\right) - 2\frac{\omega_C}{v}\sin\left(\omega_C t\right)}{1+\frac{\omega_C^2}{v^2}} \right) \quad (1.15)$$

The resulting diffusive motion (normalized to the mean cyclotron radius) is a function of the time $t$, of the cyclotron frequency $\omega_C$ and of the collision frequency $v$. This diffusive motion contains a modulated part at the cyclotron frequency that is dumped at the collision rate.

Variation of this mean square motion as a function of time (normalized to the collision frequency) and of the collision rate $v$ (normalized to the collision frequency) is shown in Figure 3.

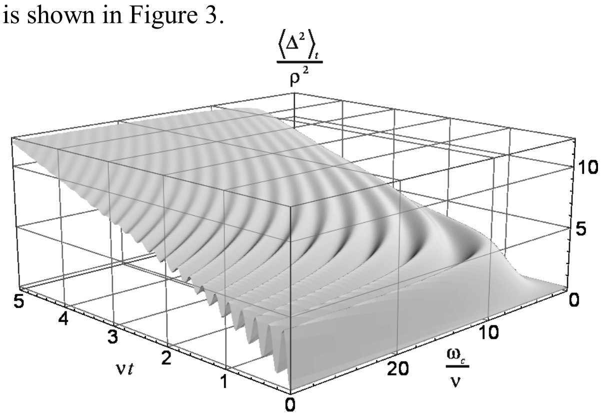


*Figure 3. Variation of mean square motion as a function of time and magnetic field intensity. Both variables: time $t$ and cyclotron frequency $\omega_c$ (magnetic field) are normalized to the collision frequency $v$*

This figure clearly shows that for fixed value of magnetic field the initial part of time variation is significantly modulated at the cyclotron frequency. This modulation dumped as the time increases above the collision time ($vt > 1$) and at longer time the mean square motion recovers the classical behavior of linear incense at the Einstein's rate for Brownian motion:

$$\left\langle \Delta^2 \right\rangle_t = 2\frac{\sigma_v^2}{v} t = 2Dt \quad (1.16)$$

Another presentation of function (1.15) is shown in Figure 4 where time and collision frequency are normalized to the cyclotron. This normalization used in Figure 4 is especially appropriate for auroral plasmas. In this case, the magnetic field is approximately uniform, while the collision frequency is a rapidly varying function of altitude. In looking at Figure 4, the collision frequency $v$ can be thought as the inverse altitude (large collision frequency are down earth, low to zero collision frequency are in the high ionosphere). In laboratory plasma instead the collision frequency is uniform but not the magnetic field and this is why the normalization choice of Figure 3 might be more appropriate.

In the no-collision case, the diffusion is seen to oscillate and come back to zero each cyclotron period. This modulation is kept as long as the collision rate is small. For large collision frequency instead, the diffusion behavior is that of Brownian motion, linearly increasing with time.

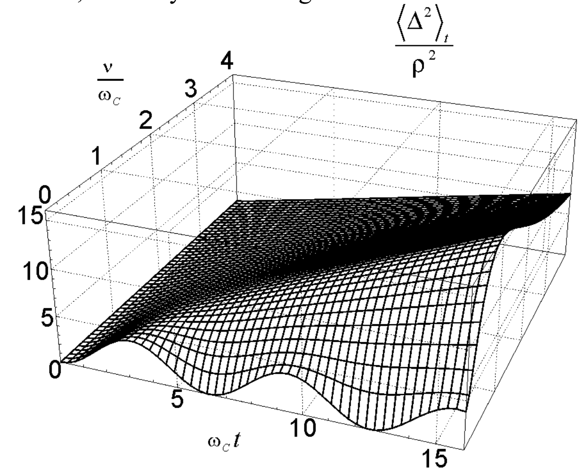


*Figure 4. Variation of diffusion as a function of time and collision frequency. Both time $t$ and collision frequency $v$ variable are normalized to the cyclotron frequency $\omega_c$*

A maximum of the displacement rate of increase with time is observed for the case where the collision frequency $v$ is equal to the cyclotron angular frequency $\omega_C$: $v = \omega_C$. This rate of increase is known as the Bohm diffusion.

The similar result could be obtained with a more elaborate Fokker-Plank model of collision.

## Microscopic "characteristic function" and scattered electric field time correlation

The result (1.15) obtained for microscopic mean square random motion could be used in (1.6) to obtain the "characteristic function" and thus the scattered electric field time correlations functions. This time correlation is:

$$\left\langle e^{-i\vec{k}\cdot\vec{\Delta}_r(t)}\right\rangle = \exp\left[-k^2\rho^2\frac{\frac{\omega_C^2}{\nu^2}}{1+\frac{\omega_C^2}{\nu^2}}\left((1+\nu t)-\frac{2}{1+\frac{\omega_C^2}{\nu^2}}+e^{-\nu t}\frac{\left(1-\frac{\omega_C^2}{\nu^2}\right)\cos(\omega_C t)-2\frac{\omega_C}{\nu}\sin(\omega_C t)}{1+\frac{\omega_C^2}{\nu^2}}\right)\right] \quad (1.17)$$

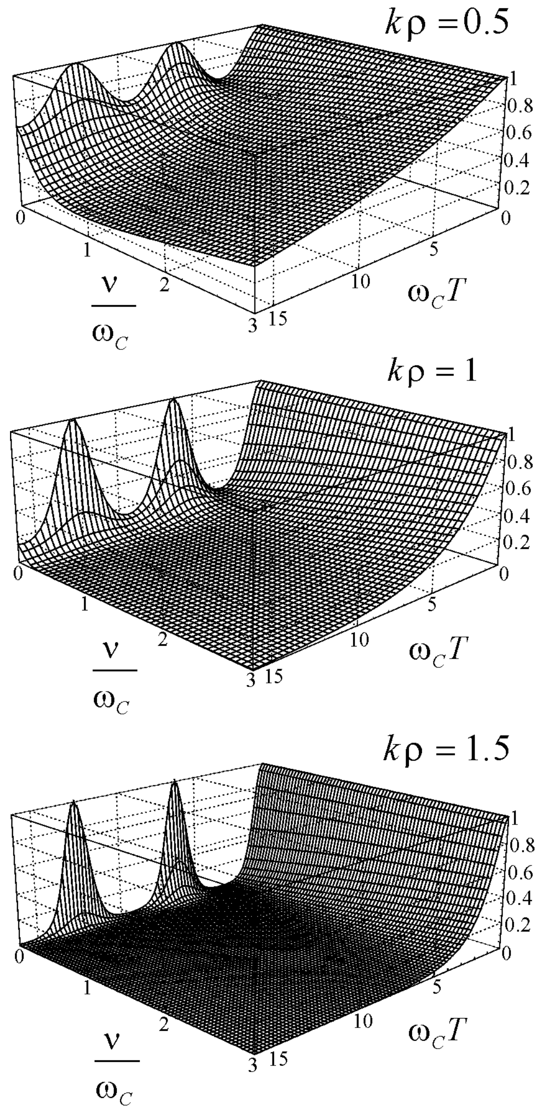


*Figure 5. Variation of microscopic part of the scattered electric field correlation function with time and collision frequency (both normalized to the cyclotron frequency) for different observed scale* $k\rho$

This calculated time correlation functions is shown in Figure 5 for several different value of $k\rho$, as a function of time $\omega_C t$ and of the collision frequency $\nu/\omega_C$, for cases where $k\rho = 0.5;1;1.5$, i.e. when the observed scale is of the order of $2\pi$ times the cyclotron radius. These parameters are in the range observation done by HF radar, of the backscattered electric field in auroral ionosphere. In these plasmas the atomic nitrogen Lamoure radius is about 3m and the scattering wavelength is 10 to 15 meters. The height at witch the collision ion – neutral is equal to the angular ion-cyclotron frequency is 100km, at the upper border of the E-region of ionosphere.

For low collision rate, the correlation function is seen to be modulated at the cyclotron frequency and the modulation depth is larger for large $k\rho$.

For higher collision frequencies particle forget rapidly about its cyclotron characteristics and the modulation disappears.

The fastest time decrease of the correlation function is observed when $\nu = \omega_C$.

We should notice again this is not the "complete" scattered electric field correlation function but only the "microscopic" part of it. The other components in equation (1.4) should also contribute. However some of microscopic function properties should still hold sins they may apply for different characteristic time and scale. Namely, experiment done for different values of the observation scale $k\rho$ might show the modulation at the cyclotron frequency (Figure 5).

The modulation amplitude of the characteristic function should be especially sensible for large $k$, where $k\rho > 1$.

## Conclusion

The microscopic mean square random motion of a charged particle in a uniform magnetic field was calculated and used to obtain the microscopic part of the scattered electric field time correlation function. The scattered electric field is a common observations in natural (ionosphere) and fusion plasmas. Since the calculated microscopic time correlation function is a multiplier of the total correlation function, one might expect that some of its features could be observed experimentally. The cyclotron modulation should especially be expected when frequency is small, i.e. for fusion plasma or for high altitude ionosphere. It also predict a maximum of the diffusion coefficient in the ionosphere at the E-layer were $\nu = \omega_C$.

## References


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