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ABOUT GENERATION OF SUBHARMONICS AND STOCHATIC REGIMES ARISING IN PERIODIC PLASMA-FILLED WAVEGUIDES EXCITED BY CHARGED BEAM

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Various mechanisms for generation of subharmonics in periodic plasma-filled waveguides excited by charged beam are considered. It is shown that beam particles seized by the excited electromagnetic wave can demonstrate under certain conditions the complex dynamics that results in generation of subharmonics and cause the transition to the stochastic regime of the instability.

1. Introduction

A variety of nonlinear phenomena in slowing electrodynamical plasma-filled structures and waveguides, exciting by charged beams, was discussed in many papers (see e.g. [1–3]).

However, a diversity of realizations and conditions for exciting of oscillations in such systems can be explained by a variety of physical mechanisms leading to complication of particle dynamics in a rather wide band of excited oscillations. Below we shall discuss some aspects of this problem and concentrate our attention on particular features of nonlinear mechanisms of excitation of periodic waveguides by charged beams. The interest to these systems is conditioned by great perspectives in their application [4–6].

Below we shall consider the high-quality beams (in terms of [1]), which are characterized by low velocity spread.

2. Consequences of satellite instability

It is known that the slowly varying (in comparison with the frequency of oscillations) factor in front of the field amplitude can result in a well-known phenomenon of period doubling and further complication of phase dynamics of particles, seized by the potential well. Really, the particles, seized by the field of the wave, move along the finite trajectories in a phase plane (e.g. in the space "coordinate - velocity" in the frame of references, where the beam is in rest) with a mean trembling frequency Ω_{tr}.

Due to development of the satellite instability (see e.g. [2]) the field amplitude oscillates in the beam rest reference frame with a frequency Ω_μ and can be represented as a sum of three terms:

$$A = A_0(1+\mu\cos\Omega_\mu t)\sin kz = \\ = A_0 + \frac{1}{2}\mu A_0 \sin(kz-\Omega_\mu t) + \frac{1}{2}\mu A_0 \sin(kz+\Omega_\mu t). \quad (1)$$

Following to [7], note that in addition to the main band of separatrix cells with seized particles two extra identical bands appear in this case, sliding relative to each other (and to the central band). They are more narrow than the central band and one of them is shifted upwards and the second – downwards along the velocity axis relative to the central band on the value Ω_μ / k. The particles of the beam fall into potential wells of central, upper, or lower bands depending on their velocities.

In a vicinity of the separatrices the so-called stochastic layer appears. The behavior of a large part of particles in this layer significantly depends on their initial state and, generally speaking, can be considered as random from a physical point of view. Just in this sense it is necessary to understand the term "weak chaos" [7]. If the separatrix bands are rather far apart in a phase space, they do not exchange the particles. [7]. If the stochastic layers of different separatrix bands are superimposed, a probability of particle exchange between them becomes finite and groups of seized particles appears, which period of revolution on the complicated trajectories can be doubled. The doubling is promoted by a synchronizing role of relative motion of the separatrix bands. The question is how large a part of these particles and what a degree of their influence on the growth of field subharmonics. With further converging of separatrix layers (that takes place in regime of saturation of above discussed instabilities, when $\Omega_\mu \sim \Omega_{tr}$) the motion of a noticeable part of particles becomes complicated that leads to generation of next subharmonics, and then to further chaotization of dynamics.

The authors of many works [8, 10, 11] noticed that with increase of interaction length or beam duration a more and more complicated dynamics was observed for transition from the stationary regime to a regime with gradual generation of subharmonics with lower and lower frequencies (usually under the scenario of period doubling), and, at last, to a regime with pronounced stochastic origin. Really, the beam particles, seized to the potential well of oscillations, drifting with the beam, are located in the interaction region during a finite time. Therefore, the greater the interaction region, the greater time for complex quasiperiodic motion in pulled together and sliding relative to each other potential wells, which have appeared because of modulation of oscillation amplitude.

Preliminary modulation of the beam (see e.g. [8]) not only accelerates the instability, but also determines a regime of instability development, having an influence on the frequency and wave number shifts (velocity of envelope drift). It is important that the beam modulation can have an influence on the frequency of field modulation in the beam rest reference frame and thereby on the bifurcation thresholds and conditions of transition to the stochastic regime [9].

However the regular mechanism of subharmonics generation is also possible. The analysis of dynamics of test particles in the field of spatial modes under consideration has shown that when potential wells moves relatively to each other with the velocity Ω_{tr}/k, there is always exist a "resonance" beam particle located in a vicinity of separatrix of the main mode. It moves with a spatial period, which is two times greater than the wavelength of the main mode and this motion remains stable during a long time. This can result in excitation of subharmonics, which frequency is a half of the main mode frequency. The subharmonic field of finite amplitude can capture the particles located in a phase vicinity of the "resonance" particle. Thus the phase volume of beam particles interacting with the subharmonic is finite.

3. About influence of beam spatial charge on generation of subharmonics and transition of instability to the stochastic regime

The inclusion of beam spatial charge leads to modification of the expression for increment of the beam-plasma instability. It is known that for low-density beams the instability increment is proportional to $n_b^{1/3}$ and for dense beams the increment is proportional to $n_b^{1/2}$ (in a case of non-uniform beams the effective, e.g. mean density $\overline{n}_b$ should be assumed). With increase of beam density the process of excitation of induced oscillations in the waveguide becomes similar to the non-resonance instability. As this takes place, the motion of seized particles in the potential well separates on two contradirectional flows due to small phase shift of the excited wave during the instability development (in the case of "pure" non-resonance instability the phase shift is insignificantly small and may be neglected). Then, two bunches of particles (two quasi-particles) are formed revolving in opposite directions in the phase plane. Generally speaking, their volume and speed of rotation (frequency of oscillation in the potential well or trembling frequency Ω_{tr}) may take different values. Recall that in the case of low-density beams there is only single quasi-particle, which motion goes on at the initial moment of the instability in the slowing phase of oscillations. The increase in beam density leads to appearance of the second quasi-particle at the initial moment of the instability, which motion goes on in the accelerating phase of oscillations.

The presence of several bunches of seized particles (quasi-particles) in the potential well of the main wave, which frequencies in the reference frame, moving with a phase velocity of the wave, are different, can result in the case of multiple frequencies in generation of subharmonics. This happens due to synchronism between spatial period of oscillations of "resonance" beam particles seized by the fields and subharmonic wavelengths. Note that the oscillations with frequencies less than the frequencies of proper waves, excited resonantly by the beam in the waveguide system, are also unstable. Because of this, the oscillations of seized particles result in appearance of perturbations with frequencies $\omega \pm \Omega_{tr}$ and development of low-frequency spectrum band of non-resonance oscillations [2]. In more general case of aliquant frequencies Ω_{tr}, corresponding to different quasi-particles, the spectra of non-resonance oscillations may become practically continuous. Then, the particle motion becomes very intricate and can't be distinguished in practice from random.

4. About spatial spectrum of oscillations excited in periodic waveguide systems

Consider some particular features of nonlinear regimes of beam instabilities in periodic waveguide systems. The rigorous treatment of wave propagation in periodic waveguides allows the conclusion to be made that the number of independent oscillations with different frequencies ω in the waveguide corresponds to the number of full modulation periods of such waveguide parameters as shape, size, etc. (needless to say that it concerns only those oscillations, which are affected by this modulation). However the oscillations having the same frequency possess due to a periodicity an infinite number of spatial modes, which longitudinal wave-numbers are equal to $k \pm nk_0$, where k is the longitudinal wave-number of the mode having a largest amplitude at this frequency, $k_0 = 2\pi/L$, L is the spatial period of waveguide parameter modulation, $n \in N$. With growth of n the oscillation amplitude decreases as α^n, where α is the parameter proportional to the modulation depth (usually, $\alpha \ll 1$). As a rule, the beam can be synchronized only with satellites $k + k_0$ or $k + 2k_0$ since the phase velocity ω/k of proper modes (corresponding to proper modes of the analogous smooth waveguide) often exceed the light velocity in vacuum. The resonance interaction with the satellite leads to the growth of as its amplitude as amplitudes of coupled modes, including the main mode, which contains the main power of the electromagnetic field. However, the series expansion of the field in spatial modes by using the small parameter α seems to be incorrect owing to the poor convergence of the series. In this case other techniques for analysis of oscillations in periodic waveguides should be used. In our opinion, the most perspective method for this purpose was developed in Kharkov National University (Ukraine).

5. About a possibility to increase the effectiveness of generation in periodic waveguides

In the regime of instability saturation the beam particles seized by the field of the satellite ($k + k_0$) are located in the phase space "coordinate-velocity" in the velocity range from $-\Omega_{tr}/(k + k_0)$ to $\Omega_{tr}/(k + k_0)$ (in the reference frame where the beam is in rest). It is obvious that beside the main band of separatrix cells with particles seized by the satellite field there is a lot of analogous bands of different widths sliding relatively to each other and displaced upward and downward with respect to the band of the satellite ($k+k_0$) along the velocity axis. We are interesting in the band of the main mode (which amplitude exceeds the amplitude of the satellite by the factor $1/\alpha$) located in the velocity range from $vk_0/k-\Omega_{tr}/\alpha(k+k_0)$ to $vk_0/k+\Omega_{tr}/\alpha(k+k_0)$. If these bands will verge towards each other with increase in the field amplitude at the frequency ω then the drifting particles and a part of particles seized by the satellite can occur in the field of the main mode. Under some conditions, beside of the dynamics complication for this group of particles and generation of subharmonics due to appearance of closed periodic trajectories (see above) the regime of so-called supercritical excitation of the main mode can be realized. The growth of the main mode will result in further overlapping of the separatrix bands and reseizing of most of the beam particles by the main mode that will provoke its further growth conditioned by this mechanism. Obviously, that this mechanism manifests itself in the case of periodic waveguide systems with low modulation depth and excited by non-relativistic charged beams.

6. About influence of non-sinusoidal parameter modulation in periodic waveguides

Other mechanism for complication of dynamics of seized particles and transition to stochastic regimes of the instability is the resonance excitation of oscillations in a vicinity of upper harmonics of the main mode [9]. Really, the periodic system (a corrugated waveguide) contains only one spatial harmonic as was assumed above. When the depth of the corrugated waveguide is increased on retention of a period it is necessary to take into account the upper spatial harmonics. Here we restrict our consideration by a role of the second spatial harmonic $A_2\cos(2k_0z)$, where A_2 is its amplitude.

Obviously, the charged beam will interact effectively with the first side mode, which longitudinal wave number is shifted on $2k_0$ with respect to the longitudinal wave number of the main mode with higher frequency. The frequency of the latter almost twice exceeds the frequency of oscillations considered above. Foregoing analysis, the expressions obtained for instability increments, as well as equations for the field turn out to be valid for oscillations with higher frequency, which effective (even resonance) excitation becomes possible due to presence of the second spatial harmonic in the spectrum of the periodic structure. To make sure in this, it is sufficiently to substitute α and k_0 for α_2 and $2k_0$ accordingly. Equation of motion for beam particles in the case of resonance instability in this case takes the different form (details see in [9]).

Now particles of the beam occur affected by the field of two modes, synchronized with the beam and which potential wells are differ on depth and longitudinal size. The ratio of frequencies and longitudinal wave-numbers are differ from 2 and some values are listed in the table:

d	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
β_1	1.24	1.25	1.26	1.27	1.28	1.29	1.31	1.32	1.34	1.36	1.37
β_2	1.05	1.05	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06
ω_2/ω_1	1.86	1.85	1.85	1.84	1.83	1.83	1.82	1.81	1.80	1.79	1.79
k_{z1}/k_{z2}	2.18	2.19	2.20	2.21	2.22	2.24	2.25	2.26	2.28	2.29	2.31

where $\beta_i=v_{\mathrm{ph},i}/\mathrm{c}$, $v_{\mathrm{ph},i}$ are the phase velocities of modes under consideration.

The linear rates of the instability for these resonance oscillations and also their amplitudes can be also estimated [9].

The presence of two synchronized with the beam periodic potential wells of different amplitudes and having, generally speaking, aliquant spatial periods can be a reason (together with the above-described mechanism) of noticeable complication of dynamics of seized particles (see, e.g. [7]).

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