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SOME PROPERTIES OF NONLINEAR OSCILLATIONS IN A DIODE IN THE SPACE-CHARGE-LIMITED CURRENT REGIME

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Both the initial and late non-linear stages of the oscillation process in a beam vacuum diode in the space-charge-limited current regime are numerically investigated. Two different numerical methods are applied: Birdsall's code (B-code) and Ender-Kuznetsov code (EK-code). The capabilities of both codes are compared. The high accuracy of the EK-code gives an opportunity to build dispersion branches of instability. The domain of the parameter values, when simultaneously several stable dynamical states (limit cycles) exist, is found. A sharp decrease in the amplitude of oscillations is revealed already under a comparatively small (about 2.5%) spread in velocities of the electron beam.

1. Introduction

The non-linear oscillations arise, as a rule, when a sufficiently dense flux of electrons traverses a diode in the space-charge-limited current regime (SCLC). This process is characterized by an elaborate mode of motion of the electrons: the backward particles reflected toward the emitter, the existence of slow, long-living particles in the neighbourhood of virtual cathode (VC) oscillations. With the evolution of the oscillatory process, an intensive energy exchange between the electrons and self-consistent electric field occurs. The operation of electronic devices converting the energy of the electrons into micro-wave radiation (e.g., vircators and reditrons) is particularly based on this effect. A number of generally unsolved problems is in this domain. It deals with, e.g., such problems as follows: under which conditions does an optimum transfer of energy from the particles to the field occur, of which quality has the electron flux to be, and how many dynamical modes exist under a given external parameter?

The purpose of this paper is to investigate numerically the non-linear oscillations in a beam vacuum diode (Bursian's diode [1]) in a reflected electrons' mode. Calculations of an initial stage of the oscillation process gives an opportunity to reveal the inherent perturbation modes as well as the dependencies of the oscillation parameters on the external parameters of the diode when investigating the non-linear stage.

2. Time-independent solutions in the Bursian diode

Birdsall and Bridges studied in computer experiments the time-dependent behaviour of the short-circuited Bursian diode (see, e.g., Ref. [2]). They discovered the steady state oscillations of the constant period and amplitude (limit cycle) due to an oscillating VC at all values of the electrode gap d in the diode in the SCLC mode. In Ref. [3], these results were refined. It was shown that there are the vast d domains within which the stable time-independent states exist in such a diode. The present work continues the studies on the oscillatory process at a non-linear stage.

The investigation takes into consideration the velocity spread in the electron velocity distribution function (VDF). Further, several characteristics of the time-independent states of the diode with such a beam of the electrons are needed. Time-independent solutions of the short-circuited Bursian diode including the velocity spread in the electron VDF were investigated in details in Ref. [4]. However this work exhibits several errors in the formulas of electron density.

The following dimensionless values are involved: a coordinate $\zeta = z/\lambda_D$, time $\tau = t\omega_0$, velocity $u = v/v_0$, and potential $\eta = e\Phi/(2W)$ (here n_0 and $W = mv_0^2/2$ are density and energy of the electrons at the gap entrance, $\lambda_D = \left[W/(2\pi e^2 n_0)\right]^{1/2}$ is a beam Debye length, $\omega_0 = v_0/\lambda_D$ is a plasma frequency). To include the electron velocities spread the electron VDF at the emitter is taken as a "water-bag"

$$f_e^0(u) = (2\Delta)^{-1}\Theta\left[\Delta^2 - (u-1)^2\right] \quad (1)$$

In a SCLC mode, there is a potential barrier (VC), reflecting a fraction of the electrons toward the emitter in this diode. In the case, when f_e^0 is determined via (1), expressions for electron density arise

$$n_e = \left(\left[2\eta + (1+\Delta)^2\right]^{1/2} + \left[2(\eta-\eta_m)\right]^{1/2} - 2\left[2\eta + (1-\Delta)^2\right]^{1/2}\right)/(2\Delta), \quad \text{if } \zeta < \zeta_m, \quad -(1-\Delta)^2/2 < \eta,$$

$$n_e = \left(\left[2\eta + (1+\Delta)^2\right]^{1/2} + \left[2(\eta-\eta_m)\right]^{1/2}\right)/(2\Delta), \quad \text{if } \zeta < \zeta_m, \quad \eta_m < \eta < -(1-\Delta)^2/2,$$

$$n_e = \left(\left[2\eta + (1+\Delta)^2\right]^{1/2} - \left[2(\eta-\eta_m)\right]^{1/2}\right)/(2\Delta), \quad \text{if } \zeta > \zeta_m.$$

Here η_m and ζ_m are the potential and location of the VC. In Figs 1a,b, as an example, the functions of η_m and ζ_m on Δ are presented for the short-circuited diode with a dimensionless electrode gap $\delta = 2.0$. Time-independent solutions of the Bursian diode with $\Delta = 0$ in the electron reflection mode were investigated in details in [5,6].

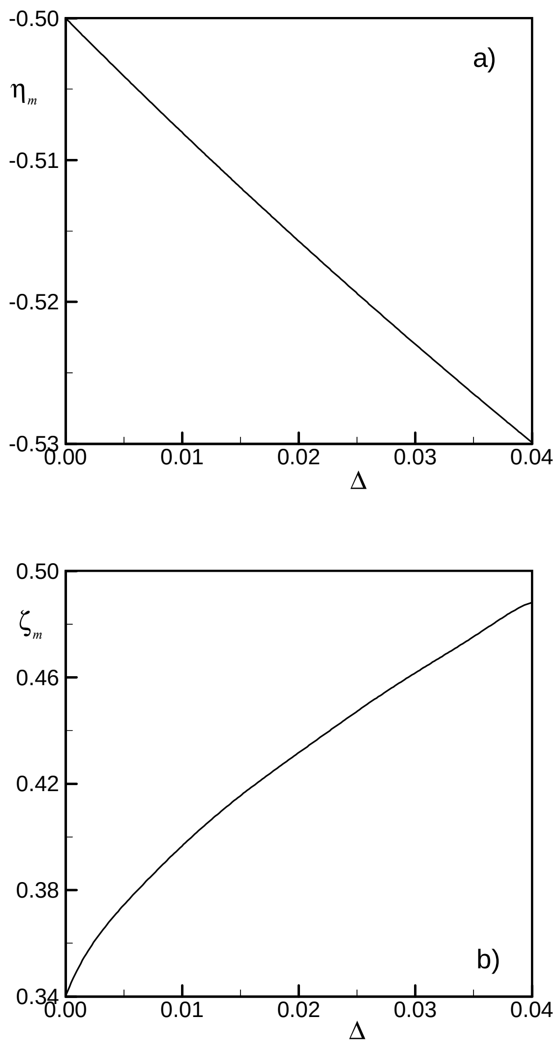

Fig. 1. Dependencies potential of the VC's top (a) and its position (b) on the velocity spread Δ *in the short-circuited Bursian diode with* $\delta = 2.0$

3. Comparison of capabilities of B-code and EK-code

At present, no analytical methods are available to study the non-linear time-dependent processes in the diode in electron reflection mode. Also, even a linear stability theory is lacking. In the present work, the EK-code [7–9] is applied to the numerical investigations of such processes. Primarily, this code was explored to study the non-linear relaxation oscillations in a Knudsen diode with a surface ionization where the VDF of the electrons and ions coming in from an emitter are the half-Maxwellian ones at the emitter's temperature. In Ref. [9], the VDF, density, and kinetic energy of the ions were calculated with high accuracy at the ionic stage of the process. This gave an opportunity to calculate such complicated phenomena as the non-linear oscillations in a Knudsen diode, to develop a theory of these processes and to reveal the new non-linear structures arising in a bounded plasma. Further, the EK-code was successfully explored calculating the time-dependent processes with a given ionic background as the uniform as nonuniform ones [8,10]. In this case, distinct from an ionic stage of an oscillatory process, ions can be treated immobile, a time-step being designated of an order of a mean free electron length via the Debye length. Especially, such a fine process as a self-consistent electron trapping within a potential well, arising in a development of an aperiodical Pierce instability, was studied and a threshold for such a trapping was revealed [10].

To calculate the VDF of the charged particles, the EK-code involves a condition of the function's conservation along a path of every particle, the latter being true in the case when the particles move in a time-dependent field with no collisions, i.e.,

$$f(\zeta_k, u, \tau^p) = f^0(u_0, \tau_0) \qquad (2)$$

Here, u is a particle's velocity when arriving a point ζ_k at a time τ^p, f^0 is the emitted particle VDF, u_0 and τ_0 are the velocity and time of particle's leaving the emitter. Calculation of each trajectory is accomplished beginning with an arrival point (ζ_k, τ^p) backward in time to the point $(0, \tau_0)$ where it intersects the emitter surface. As a result, u_0 and τ_0 are obtained, and, accordingly with (2), the particle VDF value at a given velocity u. Then, a VDF value is calculated at a new u-value and so on. Therewith, an u-step is defined in such a manner so that a difference of f^0-values for the adjacent trajectories does not exceed a prescribed magnitude. So, in the domains with the high VDF gradients and in a neighbourhood of the points of its disruption, the u-step is adjusted automatically. As a result, all the fine features of the VDF can be revealed and all its moments needed can be calculated as follows: density, particle fluxes, kinetic energy density, energy fluxes and so on. Note, that the explored algorithm gives an opportunity to investigate the characteristics even of the separate groups of the particles, e.g., a set of the particles underwent exactly n reflections.

Primarily, small perturbation development was studied from a time-independent state of the short-circuited Bursian diode (the 1-type process) [3]. Stability of the obtained results was investigated. Based on these studies, it was obtained a spatial step $\Delta\zeta$ of 0.01 and a time step $\Delta\tau$ of 0.05 for the further calculations. Then, either a perturbation decays or a process is terminated at a state with the stable non-linear oscillations (limit cycle) [3].

A comparison of the results obtained with those of other codes' application was accomplished. The code of a wide use in plasma physics under comparison was the one-dimensional code developed by the Plasma Theory and Simulation Group of Prof. Birdsall — XPDP1 (version 3.1) (B-code). There was an unique modification in the XPDP1, namely, a water-bag injection VDF (1) was inserted instead of a shifted half-Maxwellian.

As a rule, in the literature have explored the Particle Simulation codes with the number of "probe" particles $N \le 10^5$. In an earlier work [2], N did not exceed 10^4 at all, leading to erroneous results. The problem of a proper choice of $\Delta\zeta$, $\Delta\tau$, and N, when calculating the electron reflection modes in a Knudsen diode with the XPDP1 code, was studied by Kolinsky in detail in [11]. She demonstrated that $\Delta\zeta: 10^{-3}$ and $\Delta\tau: 6\cdot 10^{-4}$ were needed to obtain an accuracy similar to the EK-code, and, accordingly, a proper understanding of the process as a whole in a diode of $\delta \sim 1$ that required an essential rise in N. As a result, the CPU run time enormously increased. E.g., when calculating with the SUN SPARC Workstation, a single oscillation period calculation consumes about 20 minutes whereas a whole oscillation process consumes from 1.5 to 7.5 days. The same objection concerning an essential rise in N and consumed time when applying the Particle Simulation codes is also presented in [12].

With the above choice in $\Delta\zeta$, $\Delta\tau$ and N being accomplished via the B-code, it was revealed that in the case, when oscillations had high enough amplitudes, the spatial dependencies of the potential distribution for the same time moments obtained by two codes are closed. In particularly, time dependencies of the typical points in the potential distribution: a potential minimum η_m and its position ζ_m coincide practically. At small amplitudes of the oscillations, we had other situation. In Fig. 2, one can see that the B-code can not reveal any growth rate, but the EK-code allows to calculate the initial stage of such a process and to define the growth rate of the instability with high precision.

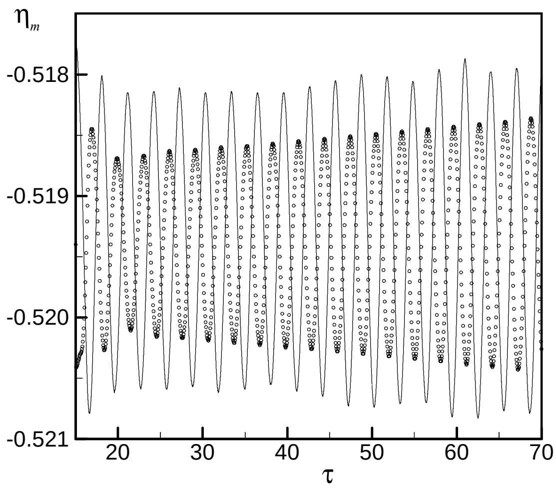

Fig. 2. Initial stage of the 1-type process. Solid line — simulations by B-code, dotted line — by EK-code

4. Some properties of non-linear oscillations

With the EK-code the dispersion curves of the diode were successfully built. With this aim, the 1-type process was calculated at a series of δ values, the $\eta_m(\tau)$ dependence was built, then, this dependence was approximated via the least squares with a function

$$F = C + \left[A\cos(\Omega\tau) + B\sin(\Omega\tau) \right] \exp(\Gamma\tau) \qquad (3)$$

and the values of a growth rate Γ and frequency Ω were determined. Fig. 3 presents a dependence Γ on δ for a velocity spread value of $\Delta = 0.02$. One can see that, in a range of $1.56 < \delta < 2.10$, the diode states with partial reflection are oscillatory unstable. Thus, in the Bursian diode, the VC oscillations are obtainable not at any gap values but within certain domains of δ. Therewith, oscillation frequency changes slightly in the obtained range: $\Omega \approx 2.1 - 2.2$ [3].

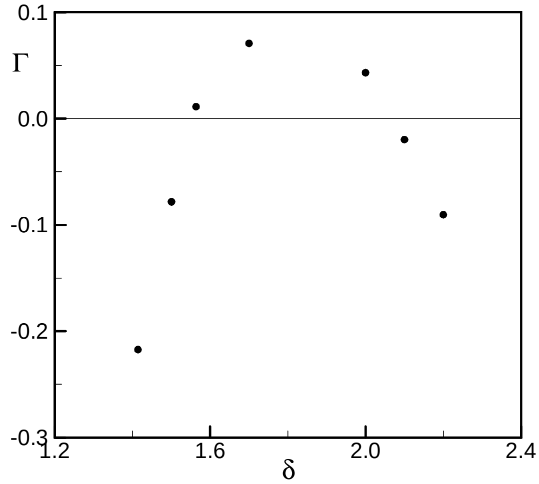

Fig. 3. Dependency of the growth rate Γ on δ for the velocity spread $\Delta = 0.02$.

Further, at the non-linear stage, the oscillatory process developed with an amplitude growth and terminated by a state of a limit cycle. The oscillations of VC height as well as its position are inherent to this dynamic state [3].

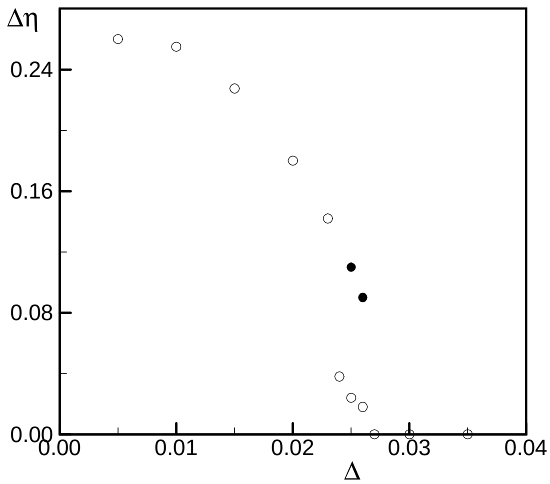

Fig. 4. The dependency of the oscillation amplitude of the VC's top potential on the velocity spread Δ in the short-circuited Bursian diode with $\delta = 2.0$.
o *— the 1-type process,* • *— the 2-type process*

A velocity spread effect on these oscillations was investigated. It was carried out the calculation of the processes of the 1-type in a short-circuited diode with $\delta = 2.0$ under Δ variations. Figure 4 shows a dependence of a twofold amplitude of oscillations ($\Delta\eta$) of a top potential of the VC on Δ. With enlarged Δ the amplitude of oscillations begins to be nearly constant, then, underwent a sharp drop and, at Δ of about 2.5%, the oscillations vanished. The calculations of the similar processes were accomplished when injection VDF is of the shifted half-Maxwellian type. The effect of a velocity spread proves to be equally strong. Moreover, if one sets $\Delta = \left(3kT / mv_0^2\right)^{1/2}$, where T is temperature of the Maxwellian and v_0 is a velocity shift, then a dependence $\Delta\eta(\Delta)$ proves to be very near to the curve in Fig. 4. Thus, the features of an oscillation amplitude as a function of a velocity spread do not depend very much on the sort of injection VDF. The revealed effect of VC oscillation drop out at a rather small value of a velocity spread is very important when working with the devices converting the energy of the electrons into micro-wave radiation.

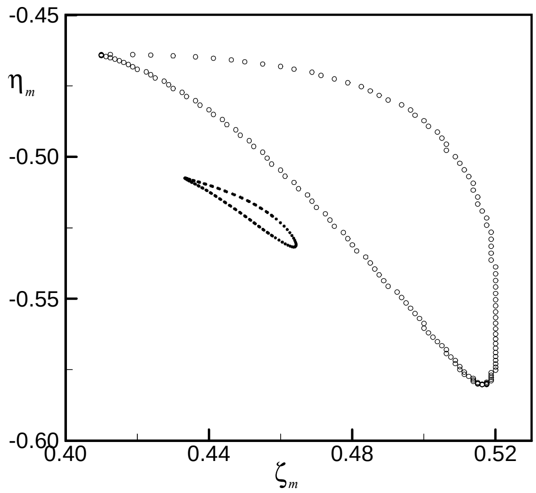

Fig. 5. Limit cycles in the short-circuited Bursian diode with $\delta = 2.0$ for $\Delta = 0.025$

In the neighbourhood of a sharp jump in oscillation amplitude (at $\Delta \approx 0.025$) another new phenomenon was discovered: simultaneous existence of the two stable limit cycles of the different amplitudes (Fig. 5 demonstrates such an example at $\Delta = 0.025$ [13]). A limit cycle with bigger amplitude A_b was obtained when calculating a transfer from a state of the steady-state oscillations at $\Delta = \Delta_1$ toward a new state after a sharp change in Δ_1 into Δ_2 (the 2-type process), and each state with a smaller amplitude A_s was the final stage of the 1-type process. Apparently, in above domain of Δ values there are the unstable oscillatory solutions of which amplitudes lie between A_s and A_b. Each stable oscillatory solution has its inherent basin of attraction, and boundaries separating stable and unstable oscillatory solutions are the two bifurcation points of the curve in Fig. 4. This is demonstrated in Fig. 6, representing a development of an instability at Δ, slightly below the left bifurcation point: initially, the process "tries" to acquire a state with a small amplitude of oscillations but the amplitude of oscillation exhibits a growth when "repelled" by an unstable limit cycle and a whole process is terminated at a state of a limit cycle with a great amplitude. Note, the 1-type process reaches a steady state oscillation mode faster: $\tau \sim 100 - 150$ when $\Delta < -0.02$, i.e., the time-independent states of the diode are relatively distinct from a domain of several limit cycles. Anomalous behaviour of oscillation amplitudes in a domain with several limit cycles is confirmed partially by nonmonotonicity in a growth rate dependence on Δ (see Fig. 7).

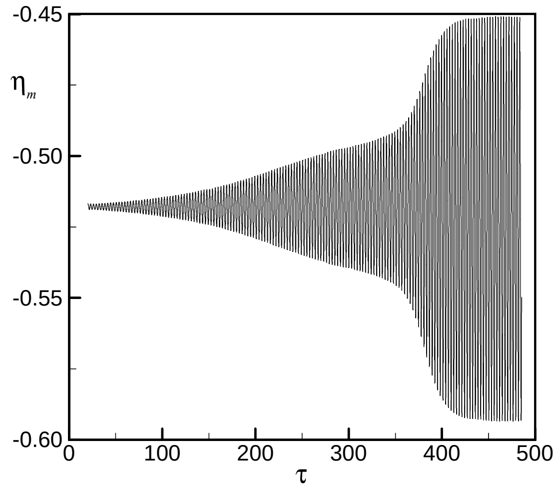

Fig. 6. Time evolution of the potential minimum η_m in the diode with $\delta = 2.0$ and the velocity spread $\Delta = 0.023$

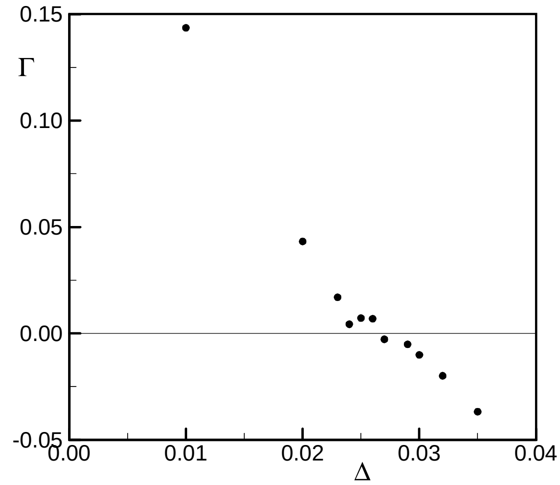

Fig. 7. Dependency of the growth rate Γ on Δ for $\delta = 2.0$

5. Summary

In the present work we analyzed both the time-independent and dynamic states of the Bursian diode with an electron beam of a small velocity spread in a SCLC mode. The EK- and B-codes were applied in these investigations making sure that reliable numerical results are obtained.

It was ascertained that the acquisition of the correct results concerning the modes under question when handling the B-code can be only approved with a drastic decrease in dimensions of the space-time cells and increase in number of particles, resulting in an essential increase in CPU time consumption when treating the problem. Hence, the results obtained with the Particle Simulation codes are to be considered with some caution. It was revealed the EK-code gains in comparison with the B-code.

The study of a stability of the time-independent solutions in the Bursian diode with electron reflection was carried out. The dispersion curves were built numerically. The domains of the stable solutions were disclosed.

The effect of an electron beam velocity spread on the on-set of the non-linear oscillations in the diode was studied. The oscillations were proved to be cancelled abruptly when increasing the spread magnitude and this disruption is of a threshold origin. Several limit cycles were discovered at the same external parameters of the diode and the basin of attraction of the stable attractors were determined.

The results obtained are of importance for electronic devices converting the energy of the electrons into micro-wave radiation.

6. Acknowledgements

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7. References

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