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RELATIVISTIC ELECTRON BUNCH EXCITATION OF WAKE FIELDS IN A MAGNETOACTIVE PLASMA

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We have been demonstrated that a multiple excess of the accelerated bunch energy ε_{max} over the energy of the exciting. REB is possible in a magnetoactive plasma at a certain relationship between the parameters of the "plasma-bunch-magnetic field" system owing to a hybrid volume-surface character of REB-excited wake fields, even without using the REBs contoured in the longitudinal direction.

The ideas of using collective fields for acceleration in the plasma and noncom-pen-sated charged beams were stated as early as in 1956 by V.I. Veksler, G.I. Budker and Ya.B. Fainberg [1-3]. The appearance and development of new powerful energy sources such as lasers, high-current relativistic electron beams, superhigh-power microwave generators, gave another impetus to the development of the collective methods of charged particle acceleration. As a result, in 1979 [4] and 1985 [5], there appeared new modifications of the method of charged particle acceleration in a plasma by charge density waves (ref. [3]), where it was proposed that the accelerating fields should be excited by laser pulses and relativistic electron bunches.

In our opinion, the excitation of accelerating fields in a plasma by an individual relativistic electron bunch (REB) appears most preferable, because it is nonresonant in character, and therefore, is little sensitive to the longitudinal plasma density inhomogeneity observed in experiment. Besides, to preclude the electromagnetic filamentation slipping of instabilities, etc. [6-9], it is reasonable to use the stabilizing external longitudinal magnetic field [10]. Aside from stabilization, the magnetic field also gives rise to a multitude of new wave branches, and this, as will be shown below, essentially extends the potentialities of the wake-field method of charged particle acceleration.

The report presents the results from theoretical studies into the processes of REB excitation of wake fields in a magnetoactive plasma, both the cases of an unbounded plasma and the waveguide with a partial plasma filling.

1. Let us define the wake-field by an axially symmetric REB moving in the magnetoactive plasma along the axis *z*. We neglect the thermal motion of electrons, and assume ions to be immobile. The REB current density is written as

$$\vec{j}_{ext} = -I_0 \frac{\psi(r)}{2\pi} T\left(t - \frac{z}{V_0}\right)\vec{e}_z, \quad (1)$$

where I_0 is the peak value of the total current of bunch, $\psi(r)$ is the function describing the current distribution in the bunch cross section, the function T describes the longitudinal profile of the bunch (maxT = 1), *t* is the time, z and r are, respectively, the longitudinal and radial coordinates, V_0 is the bunch velocity, e_z is the unit vector in the direction of bunch motion and an external magnetic field $H_0 = H_0 e_z$. Formula (1) describes the REB current density in the approximation of assigned motion, i.e., the "rigid" bunch model is used. The function $\psi(r)$ satisfies the condition

$$\int_0^{r_b} \psi(r) r dr = 1,$$

where r_b is the maximum radius of REB.

We express the electric field E, the magnetic field H and the current density of REB (1) in terms of the Fourier integral, then the set of Maxwell equations takes the form

$$kE_{\omega\varphi} = k_0 H_{\omega r}, \qquad kH_{\omega\varphi} = k_0\left(\varepsilon_1 E_{\omega r} + i\varepsilon_2 E_{\omega\varphi}\right),$$

$$\frac{1}{r}\frac{d}{dr} rE_{\omega\varphi} = ik_0 H_{\omega z},$$

$$\frac{dH_{\omega r}}{dr} - ikH_{\omega r} = ik_0\left(\varepsilon_1 E_{\omega\varphi} - i\varepsilon_2 E_{\omega r}\right), \quad (2)$$

$$\frac{dE_{\omega z}}{dr} - ikE_{\omega r} = ik_0 H_{\omega\varphi},$$

$$\frac{1}{r}\frac{d}{dr} rH_{\omega\varphi} = ik_0\varepsilon_3 E_{\omega z} - \frac{2I_0}{c}\psi(r)T(\omega),$$

where $k = \omega/V_0$, $k_0 = \omega/c$,

$$\varepsilon_1 = 1 - \frac{\omega_{pe}^2(\omega + i\nu)}{\omega\left[(\omega + i\nu)^2 - \omega_{He}^2\right]},$$

$$\varepsilon_2 = \frac{\omega_{pe}^2 \omega_{He}}{\omega\left[(\omega + i\nu)^2 - \omega_{He}^2\right]},$$

$$\varepsilon_3 = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\nu)},$$

ω_{pe}, ω_{He} are, respectively, the Langmuir and Larmor frequencies of plasma electrons, ν is the effective collision frequency, φ is the azimuthal coordinate.

The set of first-order differential equations (2) is conveniently reduced to coupled second-order differential equations for longitudinal components of electric and magnetic fields

$$\frac{1}{r}\frac{d}{dr}r\frac{dH_{\omega z}}{dr}+p_H^2H_{\omega z}=-ikk_0\frac{\varepsilon_2\varepsilon_3}{\varepsilon_1}E_{\omega z}-$$
$$-k\frac{\varepsilon_2}{\varepsilon_1}\frac{2I_0}{c}\psi(r)T(\omega)\quad ,$$

$$\frac{1}{r}\frac{d}{dr}r\frac{dE_{\omega z}}{dr}+p_E^2E_{\omega z}=ikk_0\frac{\varepsilon_2}{\varepsilon_1}H_{\omega z}+$$
$$+i\frac{\left(k_0^2\varepsilon_1-k^2\right)}{k_0\varepsilon_1}\frac{2I_0}{c}\psi(r)T(\omega)\quad ,(3)$$

where $p_H^2=k_0^2\frac{\varepsilon_1^2-\varepsilon_2^2}{\varepsilon_1}-k^2$,

$$p_E^2=k_0^2\varepsilon_3-k^2\frac{\varepsilon_3}{\varepsilon_1}.$$

The solution to the nonuniform set of equations has the form

$$H_{\omega z}=i\pi\frac{I_0}{c}T(\omega)\frac{k\varepsilon_2}{\varepsilon_1}\times$$
$$\times\int\begin{bmatrix}A_1G(\lambda_1r_0,\lambda_1r)-\\-A_2G(\lambda_2r_0,\lambda_2r)\end{bmatrix}r_0dr_0\psi(r_0)\quad ,$$

$$E_{\omega z}=-\pi\frac{I_0}{c}T(\omega)\frac{1}{k_0\varepsilon_3}\times$$
$$\times\int\left[B_1G(\lambda_1r_0,\lambda_1r)-B_2G(\lambda_2r_0,\lambda_2r)\right]r_0dr_0\psi(r_0)\quad ,(4)$$

where $A_i=\frac{\lambda_i^2}{\lambda_1^2-\lambda_2^2}$, $\quad B_i=\frac{\lambda_i^2\left(p_H^2-\lambda_i^2\right)}{\lambda_1^2-\lambda_2^2}$,

$\lambda_{1,2}$ are the transverse wave numbers of ordinary and inordinary waves, respectively, and are the roots of the biquadratic equation

$$\lambda^4-\lambda^2\left(p_E^2+p_H^2\right)-k^2k_0^2\varepsilon_3\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2+$$
$$+p_E^2p_H^2=0\quad ,(5)$$

They can be written as:

$$\lambda_{1,2}=\frac{p_E^2+p_H^2}{2}\pm\sqrt{\frac{\left(p_E^2-p_H^2\right)^2}{4}+k^2k_0^2\varepsilon_3\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2}\,,$$

$$G(\lambda_ir_0,\lambda_ir)=\begin{cases}H_0^{(1)}(\lambda_ir)J_0(\lambda_ir_0), & r>r_0\\ H_0^{(1)}(\lambda_ir_0)J_0(\lambda_ir), & r<r_0\end{cases}$$

where $J_0(\lambda_ir)$, $H_0^{(1)}(\lambda_ir)$ are the Bessel functions and Hankel functions, respectively.

In the ultrarelativistic case, we can put $V_0 = c$. Then, instead of eq.(5) we have

$$\lambda_{1,2}^2=\frac{k_0^2}{\varepsilon_1}\left[\varepsilon_3(\varepsilon_1-1)\pm\varepsilon_2\sqrt{\varepsilon_3}\right]\,. \quad (6)$$

In the frequency range $\omega^2 < \omega^2_{pe}$ the transverse wave numbers are complex.

In the limiting case of a strong magnetic field, $\omega^2_{He} >> \omega^2_{pe}$, the expression for transverse wave numbers takes the form

$$\lambda_1^2=k_0^2-k^2,\qquad \lambda_2^2=\left(k_0^2-k^2\right)\varepsilon_3.$$

For simplicity sake we consider an infinitely thin annular bunch of radius r_b

$$T(\omega)=1.\qquad \psi(r)=\frac{\delta(r-r_b)}{r_b}\quad (7)$$

Then, for the electric field on the axis of the system $r = 0$ we obtain the following integral representation

$$E_z=-\frac{1}{2}\frac{Q_0}{V_0^2\gamma_0^2}\int_{-\infty}^{\infty}\exp(-i\omega t)H_0^{(1)}(k_\perp r_b)\omega d\omega,\quad (8)$$

where Q_0 is the total charge of bunch, γ_0 is the relativistic factor, $k_\perp^2 = (k_0^2 - k^2)\varepsilon_3$. Note that in the frequency range $\omega < \omega_{pe}$ the bunch emits an electromagnetic field in the radial direction, because $k_\perp^2 > 0$.

The argument of the Hankel function has two branch points $\omega = \pm\omega_{pe} - i\nu/2$ lying in the lower half-plane of the complex variable ω. Let us draw a cut along the line of segment connecting the branch points, and close the integration contour in the lower half-plane over the semi-circle of infinite radius. As a result, the initial integral (8) reduces to the integral over the cut shores. Then, using the path-tracing rule for the Hankel function, we obtain the following expression for the longitudinal component of the electric wake field

$$E_z=-i\frac{Q_0\omega_{pe}^2}{V_0^2\gamma_0^2}\int_{-1}^{1}\begin{matrix}\exp(-is\tau)\times\\ \times J_0\left(\mu\sqrt{1-s^2}\right)sds\end{matrix}\,,\quad (9)$$

where $\tau=\omega_{pe}\left(t-\frac{z}{V_0}\right)$, $\quad \mu=\frac{\omega_{pe}r_b}{V_0\gamma_0}$.

After calculation of the integral we obtain from (9)

$$E_z = \frac{2Q_0\omega_{pe}^2}{V_0^2\gamma_0^2}\frac{\tau}{\tau^2+\mu^2}\times$$

$$\times\left(\frac{\sin\sqrt{\tau^2+\mu^2}}{\sqrt{\tau^2+\mu^2}} - \cos\sqrt{\tau^2+\mu^2}\right) \quad (10)$$

At large distances behind the bunch, the wake-wave field decreases as $1/\tau$. This is due to the fact that the oscillations in the plasma placed in a strong magnetic field have the finite group velocity. The radiation of plasma waves from the near-axis region causes the wake field to decrease in the longitudinal direction.

2. Let us consider the plasma waveguide with a partial plasma filling in the external magnetic field, i.e., the waveguide, where between the plasma boundary $r = a$ and the conducting housing $r = b$ there is a vacuum gap.

To derive a dispersion equation of eigenwaves of this plasma wave guide, it is necessary to find the electromagnetic fields in the vacuum gap $b<r<a$ and join them with the plasma fields through the use of boundary conditions on the plasma surface. The boundary conditions are standard, i.e., the continuity of electromagnetic-field tangential components. As a result, we obtain the dispersion equation, which can be conveniently written as a determinant

$$\mathrm{Det}\, A = 0, \quad (11)$$

where the matrix A has the following components

$$A_{11}=1,\ A_{12}=1,\ A_{13}=-1,\ A_{14}=0,$$

$$A_{21}=\Gamma_1\frac{\omega}{c\lambda_1}\frac{J_1(\lambda_1 a)}{J_0(\lambda_1 a)},\ A_{22}=\Gamma_2\frac{\omega}{c\lambda_2}\frac{J_1(\lambda_2 a)}{J_0(\lambda_2 a)},$$

$$A_{23}=0, \qquad A_{24}=\frac{\omega}{cw},$$

$$A_{31}=\Gamma_1,\ A_{32}=\Gamma_2,\ A_{33}=0,\ A_{34}=Q(wa),$$

$$A_{41}=\varepsilon_3\frac{\omega}{c\lambda_1}\frac{J_1(\lambda_1 a)}{J_0(\lambda_1 a)},\ A_{42}=\varepsilon_3\frac{\omega}{c\lambda_2}\frac{J_1(\lambda_2 a)}{J_0(\lambda_2 a)},$$

$$A_{43}=-\frac{\omega}{cw}F_1(wa),\ A_{44}=0,$$

where

$$Q(wa)=\frac{I_0(wa)K_1(wb)+K_0(wa)I_1(wb)}{I_1(wa)K_1(wb)-I_1(wb)K_1(wa)},$$

$$\Gamma_{1,2}=\frac{1}{2\varepsilon_1}\left\{(\varepsilon_1-\varepsilon_3)\times\left(\varepsilon_1-\frac{k^2}{k_0^2}-\varepsilon_2^2\pm\left[\left((\varepsilon_1-\varepsilon_3)\left(\varepsilon_1-\frac{k^2}{k_0^2}\right)-\varepsilon_2^2\right)^2+4\varepsilon_2^2\varepsilon_3\frac{k^2}{k_0^2}\right]^{1/2}\right)\right\}$$

$$\lambda_{1,2}=\frac{k_0}{2\varepsilon_1}\left\{(\varepsilon_1+\varepsilon_3)\left(\varepsilon_1-\frac{k^2}{k_0^2}\right)-\varepsilon_2^2\pm\left[\left((\varepsilon_1-\varepsilon_3)\left(\varepsilon_1-\frac{k^2}{k_0^2}\right)-\varepsilon_2^2\right)^2+4\varepsilon_2^2\varepsilon_3\frac{k^2}{k_0^2}\right]^{1/2}\right\}$$

$$F_1=\frac{I_0(w_p r)\Delta_0(wr_b,wb)}{I_0(w_p a)\Delta_0(wa,wb)};\quad w_p=(k^2-k_0^2\varepsilon_3)^{\frac{1}{2}};$$

$$w^2=k^2-k_0^2;$$

$$\Delta_0=I_0(wr)K_0(wb)-I_0(wb)K_0(wr);$$

Character of field distribution in the cross section for the waveguide is determined by the transverse wave numbers. If $\lambda_{1,2}^2 > 0$, then the wave is three-dimensional (volumetric). However, if $\lambda_{1,2}^2 < 0$, then the wave is pertai pertaining to surface. And finally, if $\lambda_{1,2}^2$ are the complex variables, then the wave is hybrid. The boundaries of the region, where $\lambda_{1,2}^2$ become complex, are defined by the inequalities

$$\omega_1 > \omega > \omega_2,$$

where

$$\omega_{1,2}=kc\times\frac{2\omega_{pe}^2+\omega_{He}^2\pm\left(\omega_{pe}^4+\omega_{He}\omega_{pe}^2-\omega_{He}^2k^2c^2\right)^{1/2}}{\omega_{He}^2+4k^2c^2} \quad (12)$$

To excite the REB of the hybrid wave, the relativistic factor must satisfy the following condition

$$\gamma_0 > \frac{\omega_{He}}{2\omega_{pe}}. \quad (13)$$

The field pattern and the frequency of the hybrid wave being in synchronism with the bunch were found by the numerical methods.

We attained the characteristic radial distribution of the longitudinal electric-field component at the following plasma and waveguide parameters: $\frac{\omega_{He}}{\omega_{pe}} = 6.3$, $\frac{\omega_{pe} a}{c} = 23.3$, $\frac{b}{a} = 2.4$, $\gamma_0 = 4.6$. The wake hybrid wave frequency is here equal to 0.35 ω_{pe}. It is shown that for the radius $\frac{r}{a}$ = 0.8 the magnitude of the longitudinal electric-field component has a deep maximum corresponding to the energy transformation coefficient

$$R_E = \left| \frac{E_{z\max}}{E_{z(r=0)}} \right| = 37 .$$

Note that a great value of the transformation coefficient corresponds to a significant (R_E times) excess of the maximum energy obtained by the accelerated bunch as compared to the energy of the bunch exciting the wake field, because the energy transformation coefficient R_E is equal to the ratio of the electric field amplitude accelerating the guided bunch to the electric field amplitude decelerating the bunch that excites the accelerating wake field.

So, it has been demonstrated that a multiple excess of the accelerated bunch energy ε_{max} over the energy of the exciting. REB is possible in a magnetoactive plasma at a certain relationship between the parameters of the "plasma-bunch-magnetic field" system owing to a hybrid volume-surface character of REB-excited wake fields, even without using the REBs contoured in the longitudinal direction, namely:

$$\varepsilon_{\max} = mc^2 \left(R_E \gamma_0 - 1 \right). \quad (14)$$

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