

ELECTRON RADIATION IN THE FIELD OF RUNNING LINEARLY POLARIZED ELECTROMAGNETIC WAVES

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The results of integration of the Lorentz equation for a relativistic electron in the field of the sum of linearly polarized electromagnetic waves with different frequencies running in the same direction are presented. It was shown that electron velocity is almost periodic function of time when electron is moving in the field of running linearly polarized electromagnetic waves. Expansion of the field radiated by the electron in the generalized Fourier series in calculating the spectral-angular distribution of the radiation intensity was used. Expressions for frequency and intensity of direct and back Compton radiation on the combinative harmonics of the external field were obtained.

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1. INTRODUCTION

The methods of production of the short-wave electromagnetic radiation on interaction of an intensive laser beam and a beam of relativistic electrons are moving in opposite directions to each other, are widely discussing in the scientific literature [1,2].

Electromagnetic field of a laser beam near the maximum of the radiation transverse distribution one can consider as a plane linearly polarized electromagnetic wave. One and more waves with frequencies are different from the frequency of the principal mode can be presented in the laser beam. In this connection it is very interesting to analyze motion and radiation of an electron in the field of the sum of running plane linearly polarized electromagnetic waves by methods of classical electrodynamics.

The results of integration of the Lorentz equation for a relativistic electron are shown in the presented work. Exact solutions of Lorentz equation were obtained. It was shown that the components of the electron velocities are rational functions of a function, which satisfies the one-dimensional wave equation.

As it was proved in [3] the solution of the wave equation is almost-periodic function of time. As a result of the statement expansion in the generalized Fourier series can be and was used for calculations of spectral-angular distribution of the electron radiation intensity [4].

Formulas for spectral-angular distribution of the electron radiation intensity were obtained. The probability of the direct and back Compton radiation generation (according to the initial electron direction of motion with respect to direction of waves motion) on the combinative harmonics of the external field was shown.

2. ELECTRON TRAJECTORIES IN THE FIELD OF LASER WAVES

The Lorentz equation of motion for relativistic electron in electromagnetic field

$$\frac{d}{dt} m \vec{v} = c \vec{E} + \frac{e}{c} [\vec{v} \vec{H}], \quad (1)$$

where $m = m_0 / (1 - \beta^2)^{1/2}$, $\beta = v/c$, m_0 is the rest mass of an electron, c is light velocity, e is charge of an electron, \vec{v} is vector of electron velocity, t is time,

\vec{E}, \vec{H} is the vector of electric and magnetic field, respectively.

The electric component of the sum of electromagnetic field one can write as the following

$$E_z = \sum_m E_{v_m} \cos \left[2\pi v_m \left(t - \frac{x}{c} \right) + \delta_{v_m} \right], \quad (2)$$

where v_m is frequency, δ_{v_m} is initial value of a wave phase, E_{v_m} are projections of the field electric component on z axis. Assuming that normal vectors to the front of each running waves n have the same directions with the direction of axis x projections $E_x \equiv 0, E_y \equiv 0$.

Vector of the magnetic field

$$H = [n E], \quad (3)$$

$n = i$, i is unit vector of the axis x .

Projecting (1) to the coordinate axes, we can obtain:

$$\frac{1}{c} \frac{d}{dt} (\xi \beta_{zt}) = e(1 - \beta_{xt}) \times \sum_m E_{v_m} \cos \left[2\pi v_m \left(t - \frac{x}{c} \right) + \delta_{v_m} \right], \quad (4)$$

$$\frac{1}{c} \frac{d}{dt} (\xi \beta_{xt}) = e\beta_z \sum_m E_{v_m} \cos \left[2\pi v_m \left(t - \frac{x}{c} \right) + \delta_{v_m} \right], \quad (5)$$

$$\frac{1}{c} \frac{d}{dt} (\xi \beta_{yt}) = 0, \quad (6)$$

where ξ is the electron energy, and

$$\beta_{xt} = \frac{1}{c} \frac{dx}{dt}, \quad \beta_{zt} = \frac{1}{c} \frac{dz}{dt}, \quad \beta_{yt} = \frac{1}{c} \frac{dy}{dt}.$$

Integrating each equations (4-6) once and solving obtained equations relatively β_{xt} , β_{zt} , β_{yt} we obtain:

$$\beta_{xt} = \frac{1 + F^2 + a^2 - B^2}{1 + F^2 + a^2 + B^2}, \quad (7)$$

$$\beta_{zt} = \frac{2BF}{1 + F^2 + a^2 + B^2}, \quad (8)$$

$$\beta_{yt} = \frac{2KF}{1 + F^2 + a^2 + B^2}, \quad (9)$$

where

$$F = \sum_m \frac{eE_{v_m}}{m_0 c (2\pi v_m)} \times \quad (10)$$

$$\times \sin \left[2\pi v_m \left(t - \frac{x}{c} \right) + \delta_{v_m} \right] + N_1,$$

$$N_1 = \frac{\beta_{zt}(t_0)}{\sqrt{1 - \beta^2(t_0)}} - \sum_m \frac{eE_{v_m}}{m_0 c (2\pi v_m)} \times \quad (11)$$

$$\times \sin \left[2\pi v_m \left(t_0 - \frac{x(t_0)}{c} \right) + \delta_{v_m} \right],$$

$$a = \frac{\beta_{yt}(t_0)}{\sqrt{1 - \beta^2(t_0)}}, \quad (12)$$

$$B = \frac{1 - \beta_{xt}(t_0)}{\sqrt{1 - \beta^2(t_0)}}. \quad (13)$$

Integrating (5) we use the formula of the energy variation:

$$\frac{d\xi}{dt} = e \frac{dz}{dt} \sum_{v_m} E_{v_m} \cos \left[2\pi v_m \left(t - \frac{x}{c} \right) + \delta_{v_m} \right], \quad (14)$$

t_0 is the initial point of time.

In integrating equations (7-9), we introduce new variable

$$s = t - \frac{x(t)}{c}. \quad (15)$$

By differentiating (15) on t and introducing:

$$\frac{1}{c} \frac{dx}{ds} = \beta_{xs}, \quad \frac{1}{c} \frac{dz}{ds} = \beta_{zs}, \quad \frac{1}{c} \frac{dy}{ds} = \beta_{ys},$$

we obtain

$$\beta_{xt} = \beta_{xs} / (1 + \beta_{xs}), \quad \beta_{xt} = \beta_{xs} / (1 + \beta_{xs}), \quad (16)$$

$$\beta_{yt} = \beta_{ys} / (1 + \beta_{xs}).$$

Index "s" points to the value depending on s.

Substituting (16) to (7-9) and solving obtained equations relatively β_{xt} , β_{zt} , β_{yt} , and also using (10) and (15), we obtain:

$$\beta_{ys} = \frac{a}{B} = \Lambda_y = \frac{\beta_{yt}(t_0)}{1 - \beta_{xt}(t_0)}, \quad (17)$$

$$\beta_{zs} = \sum_m P_m \sin[2\pi v_m s + \delta_{v_m}] + \Lambda_z, \quad (18)$$

$$\begin{aligned} \beta_{xs} = & -\frac{1}{4} \sum_m P_m^2 \cos[2\pi v_m s + \delta_{v_m}] - \\ & -\frac{1}{4} \sum_m \sum_n P_m P_n [\cos[2\pi(v_m + v_n)s + \\ & + (\delta_{v_m} + \delta_{v_n})] - \cos[2\pi(v_m - v_n)s + (\delta_{v_m} - \end{aligned}$$

$$- \delta_{v_n})] + \Lambda_z \sum_m P_m \sin[2\pi v_m s + \delta_{v_m}] + \Lambda_x, \quad (19)$$

$$\text{where } P_m = \frac{eE_m}{m_0 c (2\pi v_m) B},$$

$$\Lambda_z = \frac{\beta_{zt}(t_0)}{1 - \beta_{xt}(t_0)} - \sum_m P_m \sin[2\pi v_m s_0 + \delta_{v_m}],$$

$$\Lambda_x = \frac{\beta_{xt}(t_0)}{1 - \beta_{xt}(t_0)} + \frac{1}{4} \sum_m P_m^2 -$$

$$- \frac{1}{2} \left(\sum_m P_m \sin[2\pi v_m s_0 + \delta_{v_m}] \right)^2 -$$

$$- \Lambda_z \left(\sum_m P_m \sin[2\pi v_m s_0 + \delta_{v_m}] \right).$$

After integrating (17-19):

$$\frac{1}{c} y(s) = \Lambda_y s + b, \quad b = \frac{1}{c} y(s_0) - \Lambda_y s_0, \quad (20)$$

$$\frac{1}{c} z(s) = \psi_z(s) + \Lambda_z s + N_2, \quad (21)$$

$$N_2 = \frac{1}{c} z(s_0) + \sum_m \frac{P_m}{2\pi v_m} \cos[2\pi v_m s_0 + \delta_{v_m}] -$$

$$- \left[\frac{\beta_{xt}(t_0)}{1 - \beta_{xt}(t_0)} - \sum_m P_m \sin[2\pi v_m s_0 + \delta_{v_m}] \right] s_0,$$

$$\psi_z = - \sum_m \frac{P_m}{2\pi v_m} \cos[2\pi v_m s + \delta_{v_m}], \quad (22)$$

$$\frac{1}{c} x(s) = \psi_x(s) + \Lambda_x s + A_x, \quad (23)$$

$$\psi_x(s) = -\frac{1}{4} \sum_m \frac{P_m^2}{2(2\pi v_m)} \sin[2(2\pi v_m s + \delta_{v_m})] -$$

$$- \frac{1}{4} \sum_m \sum_n \left\{ \frac{P_m P_n}{2\pi(v_m + v_n)} \sin[2\pi(v_m + v_n)s + \right.$$

$$\left. + (\delta_{v_m} + \delta_{v_n}) \right\} - \frac{P_m P_n}{2\pi(v_m - v_n)} \times \quad (24)$$

$$\times \sin[2\pi(v_m - v_n)s + (\delta_{v_m} - \delta_{v_n})] -$$

$$- \Lambda_z \sum_m \frac{P_m}{2\pi v_m} \cos[2\pi v_m s + \delta_{v_m}],$$

$$A_x = \frac{1}{c} x(s_0) - \Lambda_x s_0. \quad (25)$$

3. RADIATION SPECTRUM

From the formulas (7)-(9) it is follows, that components of an electron velocity in the field of light wave are rational functions of F , each item of which satisfies the wave equation.

In [3] it is shown that functions, which satisfy the wave equation, are almost periodic functions of t . In this connection we make the evident assumption, that rational functions of almost periodic functions are also almost periodic functions. So, for calculating spectral-angular distribution of intensity of radiation one can use the expansion of the field radiated by the electron in the generalized Fourier series [4].

Magnetic component of the field of the electron radiation great distance apart (the distance is much greater than the wave length of the radiation field) one can write in the form:

$$H = \sum_f H_{\Omega_f} e^{-i\Omega_f t}, \quad (26)$$

where, according to [4] and [5],

$$H_{\Omega_f} = \frac{2e i \Omega_f e^{i k R_0}}{c^2 R_0} \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T e^{i \Omega_f \left(t - \frac{m r_0}{c} \right)} [m v(t)] dt. \quad (27)$$

Here R_0 is the distance from the beginning of coordinate system to the point of observation, m is unit vector in the same direction, Ω_f is radiation frequency, $k = \frac{\Omega_f}{c} m$.

For computation H_{Ω_f} in the formula (27) it is necessary to change over from time t to the parameter s , since r_0 and v , which obtained earlier, are functions of s .

Changing over in (27) from integrating on t to integrating on s , we obtain:

$$H_{\Omega_f} = \frac{2e i \Omega_f e^{i k R_0}}{c^2 R_0} e^{i \Omega_f M} \lim_{S \rightarrow \infty} \frac{1}{S} \int_{S_0}^S e^{i \Omega_f [\Lambda s + \Psi(s) + M]} [m v(s)] ds, \quad (28)$$

where

$$M = (1 - \cos \gamma_1) A_x - N_2 \cos \gamma_2 - b \cos \gamma_3, \quad (29)$$

$$\cos^2 \gamma_1 + \cos^2 \gamma_2 + \cos^2 \gamma_3 = 1,$$

$$m = i \cos^2 \gamma_1 + k \cos^2 \gamma_2 + j \cos^2 \gamma_3,$$

$$\Psi(s) = \psi_x (1 - \cos \gamma_1) - \psi_z \cos \gamma_2, \quad (30)$$

$$\Lambda = 1 + \Lambda_x (1 - \cos \gamma_1) - \Lambda_z \cos \gamma_2 - \Lambda_y \cos \gamma_3. \quad (31)$$

Projections H_{Ω_f} on the coordinate axes are equal to:

$$(H_{\Omega_f})_x = \Omega_f \eta_f \lim_{S \rightarrow \infty} \frac{1}{S} \int_{S_0}^S [\beta_{y_s} \cos \gamma_2 - \beta_{x_s} \cos \gamma_3] e^{i \Omega_f (\Lambda s + \Psi(s))} ds, \quad (32)$$

$$(H_{\Omega_f})_z = -\Omega_f \eta_f \times \lim_{S \rightarrow \infty} \frac{1}{S} \int_{S_0}^S [\beta_{y_s} \cos \gamma_1 - \beta_{x_s} \cos \gamma_3] e^{i \Omega_f (\Lambda s + \Psi(s))} ds, \quad (33)$$

$$(H_{\Omega_f})_y = \Omega_f \eta_f \times \lim_{S \rightarrow \infty} \frac{1}{S} \int_{S_0}^S [\beta_{z_s} \cos \gamma_1 - \beta_{x_s} \cos \gamma_2] e^{i \Omega_f (\Lambda s + \Psi(s))} ds, \quad (34)$$

$$\text{where } \eta_f = \frac{e 2 i e^{i k R_0} e^{i \Omega_f M}}{c R_0}.$$

In computation $(H_{\Omega_f})_x, (H_{\Omega_f})_z, (H_{\Omega_f})_y$ with the help of (32-34) (for the wide region of parameters of an electron and electromagnetic waves) value $e^{i \Omega_f \Psi(s)}$ can be replaced by the series with the finite number of terms:

$$e^{i \Omega_f \Psi(s)} = 1 + i \Omega_f \Psi(s) - \frac{\Omega_f^2}{2} \Psi^2(s) + \dots$$

As an example we examine radiation of an electron, which moves in the plane of polarization of waves. In this case, according to (32), (33), values $(H_{\Omega_f})_x$ and

$(H_{\Omega_f})_z$ are equal to zero, while $(H_{\Omega_f})_y$ with a precision up to terms $\sim P_m P_n$ is equal to:

$$(H_{\Omega_f})_y = H_{\Omega_m} + H_{2\Omega_m} + H_{(\Omega_m + \Omega_n)} + H_{(\Omega_m - \Omega_n)}, \quad (35)$$

$$H_{\Omega_m} = \sum_m \eta_{\Omega_m} \varphi_{\Omega_m} e^{-i \frac{2\pi v_m t}{\Lambda}}, \quad (36)$$

$$H_{2\Omega_m} = \sum_m \eta_{2\Omega_m} \varphi_{2\Omega_m} e^{-i \frac{2(2\pi v_m) t}{\Lambda}},$$

$$H_{(\Omega_m + \Omega_n)} = \sum_m \sum_n \eta_{(\Omega_m + \Omega_n)} \varphi_{(\Omega_m + \Omega_n)} e^{-i \frac{2\pi (v_m + v_n) t}{\Lambda}},$$

$$H_{(\Omega_m - \Omega_n)} = \sum_m \sum_n \eta_{(\Omega_m - \Omega_n)} \varphi_{(\Omega_m - \Omega_n)} e^{-i \frac{2\pi (v_m - v_n) t}{\Lambda}}, \quad (37)$$

$$\eta_{\Omega_m} = -e(2\pi v_m) e^{i \frac{2\pi v_m R_0}{c \Lambda}} e^{i \frac{2\pi v_m A}{\Lambda}} e^{i \delta v_m} \left(\frac{P_m}{\Lambda} \right), \quad (38)$$

$$\varphi_{\Omega_m} = \left\{ \left[\cos \gamma_1 - \frac{\beta_x(t_0)}{1 - \beta_x(t_0)} \sin \gamma_1 \right] + \frac{1}{\Lambda} [\beta_z(t_0) \cos \gamma_1 - \beta_x(t_0) \sin \gamma_1] \times \right. \quad (39)$$

$$\left. \times \frac{1}{1 - \beta_x(t_0)} \left[-\frac{\beta_z(t_0)}{1 - \beta_x(t_0)} (1 - \cos \gamma_1) + \sin \gamma_1 \right] - \sum_m P_m \sin[2\pi v_m s + \delta v_m] \right\},$$

$$\eta_{2\Omega_m} = e 2 i (2\pi v_m) e^{i \frac{4\pi v_m R_0}{c \Lambda}} e^{i \frac{4\pi v_m A}{\Lambda}} e^{-2i \delta v_m} \left(\frac{P_m^2}{\Lambda} \right), \quad (40)$$

$$\varphi_{2\Omega_m} = \left\{ \frac{1}{4} \sin \gamma_1 - \frac{1}{2\Lambda} L \right\},$$

$$L = \left[-\frac{\beta_z(t_0)}{1 - \beta_x(t_0)} (1 - \cos \gamma_1) + \sin \gamma_1 \right] \times \quad (41)$$

$$\left\{ \cos \gamma_1 - \frac{\beta_x(t_0)}{1 - \beta_x(t_0)} \sin \gamma_1 \right\},$$

$$\eta_{\Omega_m + \Omega_n} = i 2\pi (v_m + v_n) \times e^{i \frac{2\pi (v_m + v_n)}{c \Lambda}} e^{i \frac{2\pi (v_m + v_n) A}{\Lambda}} e^{-i(\delta v_m + \delta v_n)} \left(\frac{P_m P_n}{2\Lambda} \right), \quad (42)$$

$$\varphi_{(\Omega_m + \Omega_n)} = \left\{ \sin \gamma_1 - \frac{(v_m + v_n)}{\Lambda v_n} \times \left[-\frac{\beta_z(t_0)}{1 - \beta_x(t_0)} (1 - \cos \gamma_1) + \sin \gamma_1 \right] \times \right. \quad (43)$$

$$\left. \times \left[\cos \gamma_1 - \frac{\beta_x(t_0)}{1 - \beta_x(t_0)} \sin \gamma_1 \right] \right\},$$

$$\eta_{\Omega_m - \Omega_n} = e i 2\pi (v_m - v_n) \times e^{i \frac{2\pi (v_m - v_n)}{c \Lambda}} e^{i \frac{2\pi (v_m - v_n) A}{\Lambda}} e^{-i(\delta v_m - \delta v_n)} \left(\frac{P_m P_n}{2\Lambda} \right), \quad (44)$$

$$\varphi_{(\Omega_m + \Omega_n)} = \left\{ \sin \gamma_1 - \frac{(v_m + v_n)}{\Lambda v_n} \right\} \times \left[- \frac{\beta_z(t_0)}{1 - \beta_x(t_0)} (1 - \cos \gamma_1) + \sin \gamma_1 \right] \times \left[\cos \gamma_1 - \frac{\beta_z(t_0)}{1 - \beta_x(t_0)} \sin \gamma_1 \right] \quad (45)$$

From the formulas given earlier it follows that in addition to radiation with frequency equal to v_m/Λ frequencies $2v_m/\Lambda$ and “combined” harmonics $(v_m + v_n)/\Lambda$, $(v_m - v_n)/\Lambda$ also can be presented in the radiation spectrum. Radiation intensity on these frequencies will be proportional to P_m^4 , $(P_m - P_n)^4$ [5], respectively.

4. CONCLUSION

According to Bohr conformity principle scattering of electromagnetic waves on an electron should be well described as well with quantum electrodynamics as with classical electrodynamics if the number of quantum « n » in oscillation mode is quite big ($n \gg 1$). In principle, an electron could absorb two, three and more quantum. This should be resulted in additional maximums in the spectrum of radiation.

Shown in the paper formulas allow to evaluate the possibility of detection of radiation modes with the frequencies $2v_m/\Lambda$ and combined harmonics with frequencies $(v_m + v_n)/\Lambda$, $(v_m - v_n)/\Lambda$ in scattered radiation spectrum of laser light with beam intensity which is achievable at the present time.

Supposing that precision of the spectrum measurement equal to about 1% is sufficient, one can calculate the value of electron energy Ω and laser with field strength E_0 and the wave length λ , with which

detection of the additional maximums in the radiation spectrum possible. The following equality has to be true:

$$\left(\frac{E_0 (B/M) \lambda (M)}{\pi \xi (MeV) \cdot 16^{16}} \right)^2 \times 100 \geq 1. \quad (46)$$

For example, when $\Omega = 5$ MeV, $\lambda = 10^{-6}$ m the detection of the additional maximums in the spectrum of radiation is possible at field strength $E_0 \approx 10^{12}$ V/m.

So, as we know, at the values Ω , E_0 , λ , which we suppose to use in X-ray generator based NESTOR on the Compton scattering [6], the spectrum of radiation will be determined by radiation on frequencies v_m/Λ .

In conclusion one can point out to the fact that obtained in this work results would be interesting for the research on cosmic radio sources.

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ИЗЛУЧЕНИЕ ЭЛЕКТРОНА В ПОЛЕ БЕГУЩИХ ЛИНЕЙНО ПОЛЯРИЗОВАННЫХ ЭЛЕКТРОМАГНИТНЫХ ВОЛН

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Приведены результаты интегрирования уравнения Лоренца для релятивистского электрона в поле, представляющем собой сумму бегущих в одном направлении линейно поляризованных электромагнитных волн разной частоты. Показано, что при движении электрона в поле бегущих линейно поляризованных волн, скорость электрона является почти периодической функцией времени. При вычислении спектрально-углового распределения интенсивности излучения использовано разложение излученного электроном поля в обобщенный ряд Фурье. Получены формулы обратного и прямого комптоновского излучения на комбинационных гармониках внешнего поля.

ВИПРОМІНЮВАННЯ ЕЛЕКТРОНА У ПОЛІ БІГУЧИХ ЛІНІЙНО ПОЛЯРИЗОВАНИХ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ

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Наведено результати інтегрування рівняння Лоренца для релятивістського електрона в полі, яке уявляє собою суму бігучих в одному напрямку лінійно поляризованих електромагнітних хвиль різної частоти. Показано, що під час руху електрона в полі бігучих лінійно поляризованих хвиль, швидкість електрона є майже періодичною функцією часу. При обчислюванні спектрально-кутового розподілення інтенсивності випромінювання використано розклад випромінюваного електроном поля в узагальнений ряд Фур'є.

Отримано формули зворотного та прямого комптонівського випромінювання на комбінаційних гармоніках зовнішнього поля.