PHOTOPRODUCTION OF THE ELECTRON-POSITRON PAIR WITH PHOTON EMISSION KINEMATICS IN STRONG MAGNETIC FIELD

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The process of the photoproduction of the electron-positron pair with photon emission in strong magnetic field is studied. A kinematics of the process is investigated and threshold values of energies and momenta of particles are found.

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1. INTRODUCTION

Study of the quantum-electrodynamic (QED) processes in the presence of strong magnetic field close to the critical value of about 10^{13} Gs is important for exploration of astronomical objects such as pulsars. These sources of radiation, it is well known, are connected to neutron stars, where near to stars' surface, the magnetic field has value of such an order. The first theoretical works for study of the process of electron-positron pair photoproduction in magnetic field were performed in the middle of the last century yet [1]. This process in the field of more complicated configuration (magnetic field plus plate wave along field) is studied in paper [2]. Later two-photon electron-positron pair production in the magnetic field was studied [3]. However, it should be noted that similar quantum-electrodynamic processes could be accompanied by emitting of additional photon. This work is devoted to the study of such a process. In this work we use the relativism system of units: $\hbar = 1,\ c = 1$.

2. KINEMATICS OF THE PROCESS

The examined process is described by the Feynman diagrams represented on Fig. 1, where wavy lines are photons and the continuous lines correspond to electrons (positrons) with 4 momenta $\mathrm{p} : (\varepsilon, 0, 0, p),\ \mathrm{p}^+ : (\varepsilon^+, 0, 0, p^+)$ $\mathrm{p} : (\varepsilon, 0, 0, p),\ \mathrm{p}^+ : (\varepsilon^+, 0, 0, p^+)$ respectively. The wave function of electron has the form [4,5]:

$$\Psi^- = \frac{1}{\sqrt{S}} e^{-i(\varepsilon t - p_y \cdot y - p \cdot z)}\ \Psi^-(\zeta),$$

$$\zeta = \sqrt{hm}(x + p_y / hm^2),\quad h = H / H_0 = eH / m^2, \qquad (1)$$

where $\Psi^-(\zeta)$ is a bispinor, which is expressed through Hermits functions. For the electron and positron the following laws of dispersion are performed in the magnetic field:

$$\varepsilon = \sqrt{\tilde{m}^2 + p^2},\quad \varepsilon^+ = \sqrt{m^{*2} + (p^+)^2},$$

$$\tilde{m}^2 = m^2 + 2l^- hm^2,\quad m^{*2} = m^2 + 2l^+ hm^2, \qquad (2)$$

where l^-, l^+ are Landau levels of electron and positron.

The function (1) has the form of a flat wave in relation to three variables namely t, y, z, therefore amplitude of the process contains three Dirac delta functions, which correspond to the laws of conservation of following form:

$$\begin{cases} \omega = \omega' + \varepsilon + \varepsilon^+, \\ k_z = k_z' + p + p^+, \\ k_y = k_y' + p_y + p_y^+. \end{cases} \qquad (3)$$

The analysis of these expressions determines the kinematics of the process. For this purpose it is convenient to define the function *f(p)*, which for the real processes applies in a zero:

$$f(p) = \omega - \omega' - \sqrt{\tilde{m}^2 + p^2} - \sqrt{m^{*2} + (p + \omega' u - \omega \cdot v)^2},$$

$$v = \cos\theta,\quad u = \cos\theta'. \qquad (4)$$

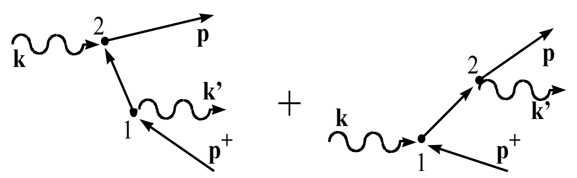


Fig. 1. *Feynman diagrams for the process of electron-positron pair photoproduction with emitting of a photon*

Dependence of this function f on the longitudinal momentum of electron p is represented on Fig. 2.

As follows from a picture, the process of electron-positron pair production by a photon can take place only at energies of a photon higher than some threshold value. This value depends on frequency of a final photon and is defined by following expression: $\partial f / \partial p = 0,\ f = 0$.

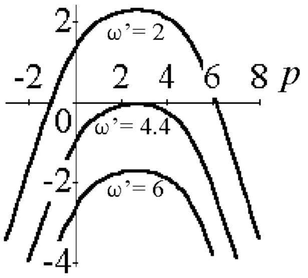


Fig. 2. *Dependence f(p) at the different values of frequency of a final photon for h = 0.3, $\bar{l}$ = 2, $\overset{+}{l}$ = 1, w = 10, v = 0.5, u = 0*

The thresholds values of energies and momenta of particles are equal thus:

$$\varepsilon_m = \frac{\tilde{m}}{\sqrt{1-\beta^2}} = \frac{\tilde{m}}{\tilde{m}+m^*}W, \qquad p_m = \beta\cdot\varepsilon_m,$$

$$\overset{+}{\varepsilon}_m = \frac{m^*}{\sqrt{1-\beta^2}} = \frac{m^*}{\tilde{m}+m^*}W, \qquad \overset{+}{p}_m = \beta\cdot\overset{+}{\varepsilon}_m, \tag{5}$$

where $W \equiv \omega - \omega'$, $\beta = \dfrac{k_z - k'_z}{\omega - \omega'}$.

The values of energies and momenta of an electron (positron) above threshold have the form:

$$\varepsilon_{1,2} = \frac{a \pm b\cdot\beta}{2W(1-\beta^2)}, \quad p_{1,2} = \frac{a\cdot\beta \pm b}{2W(1-\beta^2)},$$

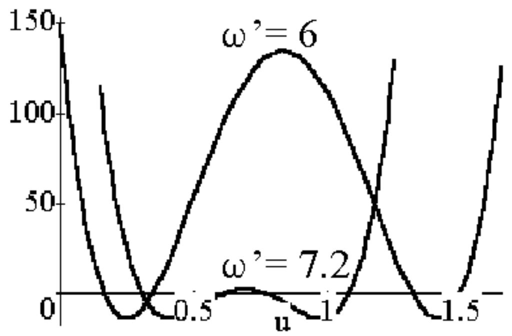
$$\overset{+}{\varepsilon}_{1,2} = \frac{a^+ \mp b\cdot\beta}{2W(1-\beta^2)}, \quad \overset{+}{p}_{1,2} = \frac{a^+\cdot\beta \mp b}{2W(1-\beta^2)}, \tag{6}$$

where

$$a = W^2(1-\beta^2) + \tilde{m}^2 - m^{*2},$$

$$a^+ = W^2(1-\beta^2) - \tilde{m}^2 + m^{*2},$$

$$b^2 = a^2 - 4\tilde{m}^2W^2(1-\beta^2).$$



Puc. 3. *Dependence b(p) at the different values of frequency of a final photon for h = 0.3, $\bar{l}$ = 2, $\overset{+}{l}$ = 1, w = 10, v = 0.5*

The final photon is radiated with frequency in interval $0 \le \omega' \le \omega - (\tilde{m} + m^*)$, $(\beta = 0)$, besides the angle of radiation of a photon is limited by an interval which is defined by following condition $p_1 = p_2$ or b(u)=0. The dependence of b(u) is presented on Fig. 3. Crossing of lines with abscises axis on Fig. 3 corresponds to the limit values of cosines of angles, that have the form:

$$u_{\min} = \frac{\omega\ v}{\omega'} - \frac{1}{\omega'}\sqrt{W^2 - (\tilde{m} + m^*)^2},$$

$$u_{\max} = \frac{\omega\ v}{\omega'} + \frac{1}{\omega'}\sqrt{W^2 - (\tilde{m} + m^*)^2}. \tag{7}$$

In common case a final photon is radiated under an angle $u_{\min} \le u \le u_{\max}$, *e.* $u_{\min} > -1$, $u_{\max} < 1$. In the examined process the impulse of the electron also depends on the direction of movement of the photon. Such dependence is represented on Fig. 4.

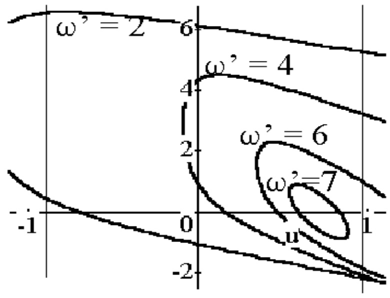


Fig. 4. *Dependence of momentum of electron on the direction of movement of the photon at the different values of frequency of final photon for h = 0.3, $\bar{l}$ = 2, $\overset{+}{l}$ = 1, w = 10, v = 0.5*

3. PROBABILITY OF THE PROCESS

The wave functions of the electron and the photon have normalized constants *L*x, *L*y, *L*z:

$$\Psi \sim \frac{1}{\sqrt{S}} = \frac{1}{\sqrt{L_yL_z}}, \quad A \sim \frac{1}{\sqrt{V}} = \frac{1}{\sqrt{L_xL_yL_z}}. \tag{8}$$

These values come into amplitude and probability of unity state too:

$$A_{if} = M\frac{1}{SV}\delta^3, \quad W_{if}1 = |M|^2\frac{1}{S^2V^2}\cdot\frac{\delta^3 TS}{(2\pi)^3}, \tag{9}$$

where δ€ are three Dirac delta functions (3).

The number of final states is

$$dN = \frac{Sd^2p\cdot Sd^2p^+\cdot Vd^3k'}{(2\pi)^7}. \tag{10}$$

Product of the probability (9) and the number of final states (10) gives differential probability.

It should be noted that the physical values don't must have any nonphysical constants such as *L*x, *L*y, *L*z. It has place for differential probability if it takes into account, that

$$L_x = 2|x_0| = 2|p_y|/hm^2. \tag{11}$$

Differential probability of process in time unit it is possible to present through amplitude *M* in a next form [6,7]:

$$\frac{dW_{if}}{T} = hm^2|M|^2\delta(\varepsilon)\frac{dp\cdot d^3k'}{2(2\pi)^{10}}. \tag{12}$$

Then total probability is equal to

$$W_{if} = \int_0^{\omega'_{max}} d\omega' \int_{u_{min}}^{u_{max}} du \left(\frac{dW_{if}(p_1)}{d\omega' du} + \frac{dW_{if}(p_2)}{d\omega' du} \right). \quad (13)$$

This probability at resonance conditions can exceed the probability of photon pair production without emitting of a photon.

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КИНЕМАТИКА ФОТОРОЖДЕНИЯ ЭЛЕКТРОН-ПОЗИТРОННОЙ ПАРЫ С ИСПУСКАНИЕМ ФОТОНА В СИЛЬНОМ МАГНИТНОМ ПОЛЕ

П.И. Фомин, Р.И. Холодов

Рассматривается процесс фоторождения электрон-позитронной пары с испусканием фотона в сильном магнитном поле. Исследована кинематика процесса и найдены пороговые значения энергий и импульсов частиц.

КІНЕМАТИКА ФОТОНАРОДЖЕННЯ ЕЛЕКТРОН-ПОЗИТРОННОЇ ПАРИ З ВИПРОМІНЮВАННЯМ ФОТОНА В СИЛЬНОМУ МАГНІТНОМУ ПОЛІ

П.І. Фомін, Р.І. Холодов

Розглядається процес фотонародження електрон-позитронної пари з випромінюванням фотона в сильному магнітному полі. Досліджено кінематику процесу і знайдено порогові значення енергій та імпульсів часток.