TRANSFORMATION OF THE SPECTRUM OF COUPLED NONLINEAR OSCILLATORS DURING TRANSITION TO THE CHAOTIC DYNAMIC

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Transition to the chaotic dynamic in the system of two coupled conservative nonlinear oscillators has been studied. Oscillators are represented by conservative LC circuits with nonlinear capacitors. The system exhibits chaotic behavior in some range of initial conditions. We focus on the spectra analysis of the oscillations that correspond to qualitatively different phase trajectories. Certain correspondence with the signal modulation has been found.

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INTRODUCTION

Two coupled nonlinear oscillators represent one of the simplest systems which manifest chaotic dynamic set up. A number of more complicated problems (including parametric waves' interaction) can be simplified to the aforementioned system. In the present paper we study transformation of the intrinsic oscillations` spectrum in the system of two coupled LC circuits with cubic-nonlinear capacitors (Fig.1) during the transition to chaotic dynamic.

We rely on the previous studies in which the chaotic oscillations in similar system were studied in detail [3, 4]. It was found that current through the coupling capacitor takes the shape of short narrow impulses when the total energy in the considered system (due to the initial conditions) is focused mostly in one circuit and outreaches some critical value. As the result under certain circumstances behavior of another circuit becomes chaotic. Described result has some resemblance with the well-known Chirikov–Taylor standard map [1].

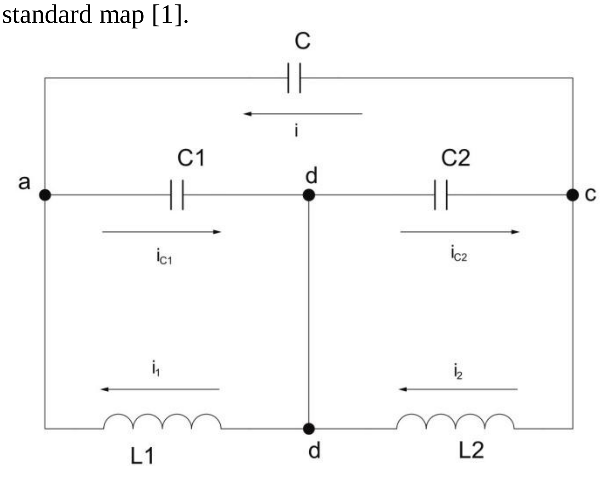

Fig. 1. Two coupled LC circuits

1. SYSTEM UNDER STUDY

Let us consider the system of two coupled LC circuits. Coupling is performed by the linear capacitor. Capacitors in LC circuits are nonlinear. Considered system is shown in Fig. 1. Capacitors` nonlinearity is defined as

$$C_{1,2}(U) = C_{10,20}\left(1+\alpha U^2\right), \quad (1)$$

where U is the voltage drop on capacitor. System behavior is described by a set of equations obtained from Kirchhoff`s circuit laws. Performing simple transformations and leave out current trough the coupling capacitor C and voltage drops on capacitors C1 and C2 one can obtain the set of differential equations for currents trough the inductors L1 and L2:

$$\begin{cases} i_1 + L\left\{C + C_{10}\left[1+3\alpha L^2\left(\dfrac{d\,i_1}{d\,t}\right)^2\right]\right\}\dfrac{d^2 i_1}{d\,t^2} + LC\dfrac{d^2 i_2}{d\,t^2} = 0; \\ i_2 + L\left\{C + C_{20}\left[1+3\alpha L^2\left(\dfrac{d\,i_2}{d\,t}\right)^2\right]\right\}\dfrac{d^2 i_2}{d\,t^2} + LC\dfrac{d^2 i_1}{d\,t^2} = 0. \end{cases} \quad (2)$$

We assume $C_{10} = C_{20} = C_0$, $L_1 = L_2 = L$. Since currents trough inductors and their time derivatives are the phase variables (2) that describe behavior of the studied system.

2. PRELIMINARIES

Transition to the chaotic dynamics in the system of two coupled oscillators can be explained in terms of intrinsic nonlinear resonance. Thus we can write down the condition for nonlinear resonance:

$$k\omega_1\left(\vec{I}\right) = l\omega_2\left(\vec{I}\right), \quad (3)$$

where $\omega_{1,2}\left(\vec{I}\right)$ are intrinsic frequencies of the oscillators which depend on the action variables $\vec{I} = \left\{I_1, I_2\right\}$, $l,k = 1,2,3...$ are arbitrary natural numbers. Equation (3) means that oscillations in the one oscillator are anharmonic due to nonlinearity of the oscillator and their k-order harmonic excites l-order subharmonic that is intrinsic for the second oscillator. In our case this type of dynamics presumes that amplitude of oscillations in the first circuit is much greater than in

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the second circuit. However the latter cannot be linear since there is chaotic oscillations and resonance overlapping (which namely is global chaos) just in the second circuit.

Spectra of quasi-periodic oscillations in circuits are shown in Fig. 2 (the lower spectrum corresponds to the first circuit, the higher to the second) .The first circuit oscillations` spectrum comprises of its intrinsic frequency (line 1) and its higher odd harmonics (lines 2, 3). The second circuit oscillations` spectrum includes in particular the intrinsic frequency of the second circuit. Line 4 corresponds to the process:

$$\omega_2 + \omega_2 \to (\omega_2 + \Delta) + (\omega_2 - \Delta),\ \Delta = 3\omega_1 - \omega_2 .$$

The consecutive process

$$3\omega_1 + 3\omega_1 \to (3\omega_1 + \Delta) + (3\omega_1 - \Delta)$$

excites line 5 and so on.

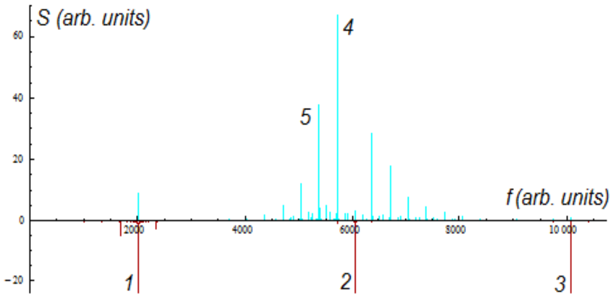

Fig. 2. Quasiperiodic spectra of oscillations

In the conservative system spectrum depends on the initial condition; moreover not only it depends on the system energy value but on the location of the phase point on the isoenergetic manifold in the phase space. Initial position of the phase point can either situate in the regular motion region or in the chaotic layer.

Poincare-Birkhoff theorem states that in the cross-section $\{I_1, \theta_1\}$ (or $\{I_2, \theta_2\}$) of the resonant torus in case of near integrable system only $2kn$ (correspondingly $2\,km$, $k = 1, 2, 3...$) invariant points are conserved and elliptic and hyperbolic points alternate. There is a system of second order tori near phase trajectory which correspond to elliptic points and they include their own fixed points. Corresponding phase trajectories are trend lines to higher order tori that represent higher order nonintegrable perturbations. In the cross-section $\{I_1, \theta_1\}$ (or $\{I_2, \theta_2\}$) they form stability islands of higher orders.

3. NUMERICAL RESULTS

System (2) was numerically integrated with Wolfram Mathematica program. Then for Poincare section $i_1=0$ on the plane of the phase variables of the 2nd oscillator $\{i_2, \frac{di_2}{dt}\}$ Poincare map is built. Also spectra of i_1 and i_2 were calculated.

Let us consider the qualitatively different types of dynamic that are observed in the area of nonlinear resonance. Numerical integration of motion equations was performed for various but isoenergetic initial conditions. We mean that total energy of the system in all initial cases must be the same value (and since the system is conservative for every time point).

Poincare maps for the 4 different phase trajectories in the nonlinear resonance area are shown in Fig. 3. Here all initial conditions are isoenergetic to the case when voltage on the capacitor in one circuit set to 7.5 Volts and other initial conditions set to zero. Given points of the Poincare maps are located mostly near six islands of stability.

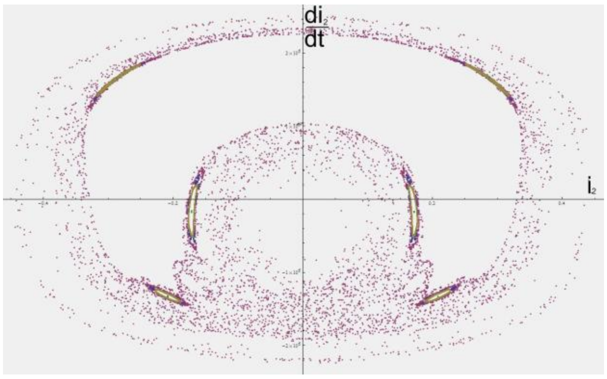

Fig. 3. Poincare maps for qualitatively different phase trajectories

One of such islands is shown in Fig. 4. Here point A corresponds to the elliptic points. Line B corresponds to first order island of stability. Point C and other similar points correspond to third order stability islands. Point D belongs to the stochastic layer.

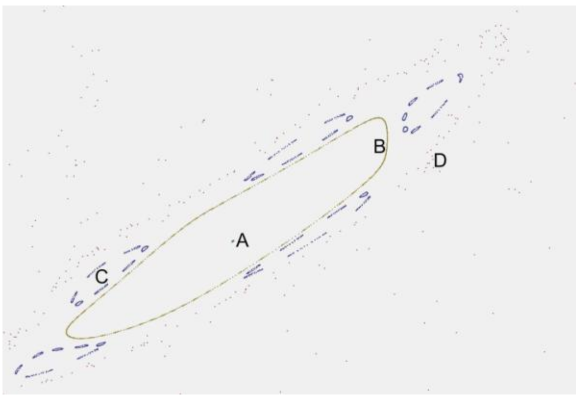

Fig. 4. Poincare maps for qualitatively different phase trajectories (zoomed part of Fig. 3)

For each phase trajectory type spectra of i_1 and i_2 were calculated on the limited time span. For initial conditions set on the A-type phase trajectory spectra of the currents in the leading (red, lower half) and driven (cyan, upper half) circuits are shown in Fig. 5. The current spectrum in the leading circuit consists primarily of its intrinsic frequency and its higher uneven harmonics. Current spectrum in the driven circuit comprises linear combinations of the intrinsic frequencies of the leading and the driven circuit. There are harmonics which coincide with the intrinsic harmonic of the leading circuit and other harmonics that are exited in the processes similar to the described in the previous section. The latter represent the modulation which is performed through the leading circuit influence on the driven one. In this case modulation is equivalent to the anharmonic phase and amplitudial modulation on the same frequency.

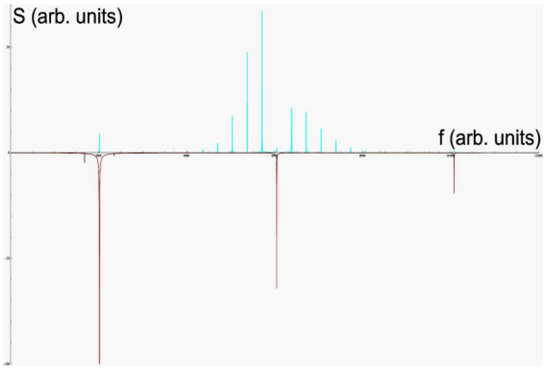

Fig. 5. Spectra for two circuits corresponding to the elliptic point

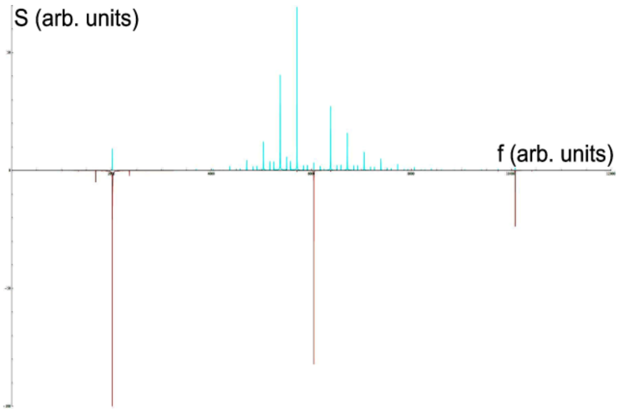

Fig. 6. Spectra that corresponds to first order islands of stability

Spectra of i_1 and i_2 that correspond to the B-type phase trajectories are shown in Fig. 6. The overall picture remains the same with exception that sidebands of the harmonics in the driven circuit are exited. This effect indicates that modulation signal is modulated in its own turn (but with the frequency that doesn't match the original modulation frequency).

Comparison of the spectra that correspond to the first (red, lower half) and third (cyan, upper half) order islands of stability are shown in Fig. 7. For the upper spectrum modulation series continue, and spectrum comprises exited sidebands of the sidebands unlike the lower spectrum. Sidebands of the higher orders become difficult to distinguish.

Spectra of the D-type dynamic mode that correspond to the stochastisity layer are shown in Fig.8. Considerable section of the driven circuit oscillations is continuous. The latter demonstrates that motion corresponding to this initial conditions is irregular.

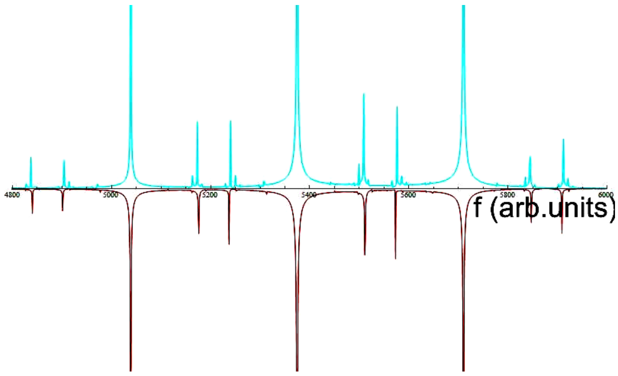

Fig. 7. Comparsion of the spectra in the driven circuit that correspond to the first and the third order islands of stability

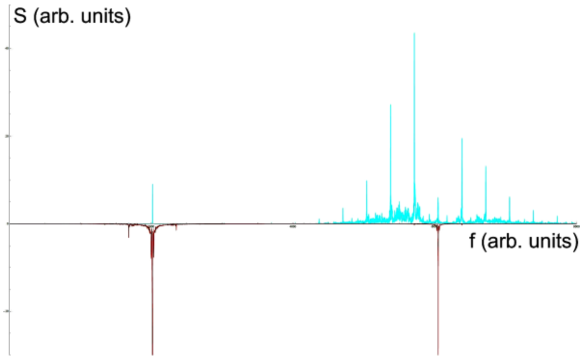

Fig. 8. Spectra that correspond to stochastisity layer

4. DISCUSSION

In the previous section we considered spectra for various phase trajectories that are located in the nonlinear resonance area. Obtained results can be explained in the following way.

For the integrable Hamiltonian system it is possible to introduce canonical variables action-angle. Actions for the system of two cubic-nonlinear oscillators can be represented as some sum of terms linear by the arbitrary (observed) canonical phase variables with the fractional powers. In the region of the nonlinear resonance for the simplest case action variables perform so called phase oscillations: $\frac{dI}{dt} \sim \cos\psi$ where $\psi = m\theta_1 - n\theta_2$, θ_i are angles, m, n are resonant numbers. ψ can be found as the solution of the nonlinear pendulum equation:

$$\frac{d^2\psi}{dt^2} + \Omega_0^2 \sin\psi = 0 .$$

Arbitrary canonical phase variables are periodical functions of angles and their amplitudes depend on the action value. So this phase variables undergo amplitude and phase modulation.

The phase trajectory corresponding to the elliptic point is resonant (not perturbed), and anharmonic amplitude modulation is caused by the periodical dependence of the phase variables on the canonical angles. When phase trajectories correspond to the stability island of the first order, action variables are no longer conserved. And since angles depend on the action, they oscillate with the phase oscillations' frequency. So amplitude and phase modulation appears for the observed phase variables. For the phase trajectories corresponding to the stability islands of higher orders, modulated signal becomes modulated in its own turn.

CONCLUSIONS

Spectra of the conservative set of two coupled nonlinear oscillators were studied.

If the initial conditions correspond to the first-order elliptic point, then oscillations` spectra comprise only ω_1, ω_2 frequencies and their linear combinations [1]. Modulation in this case resembles the anharmonic amplitude modulation.

If the initial conditions are shifted, then oscillations` spectra comprise sidebands of ω_1, ω_2 frequencies and their linear combinations. Modulation in this case resembles to the anharmonic amplitude and phase modulation.

Furthermore if initial phase point is located on the second order stability island then modulation signal is modulated in its own turn.

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ТРАНСФОРМАЦИЯ СПЕКТРА СОБСТВЕННЫХ КОЛЕБАНИЙ ДВУХ СВЯЗАННЫХ НЕЛИНЕЙНЫХ КОНСЕРВАТИВНЫХ ОСЦИЛЛЯТОРОВ В ПРОЦЕССЕ УСТАНОВЛЕНИЯ СТОХАСТИЧЕСКИХ КОЛЕБАНИЙ

С.С. Сябер, И.А. Анисимов

Исследуется переход к хаотической динамике в системе двух связанных консервативных нелинейных осцилляторов. Осцилляторы являются колебательными контурами с нелинейными конденсаторами. Для некоторых начальных условий в системе наблюдается хаотическая динамика. Основное внимание уделяется анализу спектров, отвечающих качественно отличающимся фазовым траекториям. Обнаружена некоторая схожесть с процессами модуляции сигналов.

ТРАНСФОРМАЦІЯ СПЕКТРА ВЛАСНИХ КОЛИВАНЬ ДВОХ ЗВ’ЯЗАНИХ НЕЛІНІЙНИХ КОНСЕРВАТИВНИХ ОСЦИЛЯТОРІВ У ПРОЦЕСІ ВСТАНОВЛЕННЯ СТОХАСТИЧНИХ КОЛИВАНЬ

С.С. Сябер, І.О. Анісімов

Досліджується перехід до хаотичної динаміки в системі двох зв’язаних консервативних осциляторів. Осцилятори є коливальними контурами з нелінійними конденсаторами. Для деяких початкових умов у системі спостерігається хаотична динаміка. Основна увага приділяється аналізу спектрів, що відповідають якісно відмінним фазовим траєкторіям. Знайдена певна схожість із процесами модуляції сигналів.