THEORETICAL STUDIES OF THE RESONATOR CONCEPT OF DIELECTRIC WAKEWIELD ACCELERATOR

N.I. Onishchenko, G.V. Sotnikov
National Science Center "Kharkov Institute of Physics and Technology"
Kharkov, Ukraine
E-mail: sotnikov@kipt.kharkov.ua

Theoretical studies and numerical simulation of wakefield excitation in planar dielectric resonator by regular sequence of relativistic electron bunches are carried out. It is shown, that in resonator case at conservation of field peaking, i.e. maintenance of multimode condition, the number of bunches, participating in wakefield summing, is increased. As a result the amplitude of the field, used for charged particle acceleration, essentially increases in comparison with a section of the dielectric waveguide of the same length. Thus, legitimacy of the resonator concept for wakefield method of charged particle acceleration is proved. Numerical calculations are carried out for planned experiment on accelerator "Almaz-2" on wakefield excitation in the rectangular dielectric resonator by a sequence of bunches with charge 0.32 nC and energy 4 MeV each.

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1. INTRODUCTION

Is dielectric wakefield accelerator (DWA) a tribute to the modern style, does it arouse academic interest or can it compete with other new and traditional methods of acceleration? In favour of positive answer to the last question it can be said, that numerous theoretical investigations of acceleration with use of wakefields in dielectric structures pass in a stage of the experimental realization [1-3].

Acceleration with use of wakefields in dielectric structures is one of varieties of two-beam methods of acceleration in which for creating of intensive longitudinal electrical field the high-current beam (or a beam called driving) is used. Thus the necessity for use of external source of microwave field takes out. The second beam (or the beam called driven) is accelerated in the field of the first beam. For experiments in Brookhaven National Laboratory (BNL) [1] for excitation of accelerating field the sequence of supershort (with duration of 3.5 fs) electron bunches with energy 500 MeV is used, and in experiments in Argonne National Laboratory (ANL) [3] – the sequence of short (with duration of 6.7 ps) electron bunches with energy 15 MeV is used.

The attractiveness of dielectric wakefield accelerator consists of the following:

- simplicity of its prototype creation (section of waveguide) and of the further testing of main principles;
- stability and easy controllability of slowing medium parameters;
- possibility of sectioning with the purpose of reaching high energies of accelerated particles.

The only essential weakness of DWA is the possibility of breakdown on dielectric surface. Use of ceramic structures [2] allows to increase considerably breakdown thresholds and to reach electric field strengths not below, than in traditional accelerators. Other possibility of breakdown effect diminution is use of dielectric structures and electron bunches with micron sizes [1].

DWA can work in single-mode [2,3] or multimode [4] conditions. Such classification is connected with number of harmonics participating in complete electromagnetic field forming. The electron bunch, passing through dielectric structure, excites a set of harmonics relevant to transversal modes of natural oscillations of the structure. Therefore at use of sequence of bunches in the first case for supporting of effectiveness of the accelerator it is necessary to suppress "parasitic" (not relevant to operation mode) oscillations, and in the second case it is necessary to choose the parameters of slowing structure so that to provide equidistance of resonant with bunch eigenfrequencies of structure. At that bunch repetition rate should be multiple to wave length of principal mode.

Wakefield strengths in multimode regime of DWA are much higher than fields in single-mode regime. However at use of cylindrical waveguide with partial dielectric filling as dielectric structure realization of multimode regime faces difficulties of maintenance of excited frequency equidistance.

In the present work we investigate multimode excitation of wakefields by a sequence of electron bunches in the planar dielectric resonator. The need for resonator concept of DWA originates in connection with necessity of elimination of undesirable effect of removal of wakefield with group velocity of excited waves from waveguide dielectric structure [5]. The first outcomes of theoretical examinations [6] and numerical calculations with numerical PIC code KARAT [7], carried out for cylindrical geometry of the dielectric resonator, showed, that strong restriction on maximum number of bunches which give the contribution to growth of amplitude of the field, can be removed in resonator concept of DWA. In an optimal condition excitation of dielectric resonator by a sequence of bunches is similar to excitation of resonator by mode-locked laser equipped with an "optical switch". At the moment when reflected from the output of the resonator short laser impulse comes to the input, the optical switch injects the next impulse into the resonator. But, as mentioned above, in cylindrical geometry it is difficult to realize multimode condition of DWA operation. Below, in planar geometry, we will consider the possibility of combination of resonator advantage with already available advantages of DWA, namely, with multimode regime and use of great number of bunches.

2. WAKEFIELD IN THE PLANAR DIELECTRIC RESONATOR

Let's obtain the expression for longitudinal electric field excited by a sequence of bunches in the dielectric resonator. Let the planar metal resonator has a cross size a ($-a/2 \le x \le a/2$) and its length is equal L. The wave guide is filled by the homogeneous dielectric with permittivity ε. Along the axis of waveguide there is a drift channel which sizes are small in comparison with cross size of the resonator, that allows to neglect shift of its eigenfrequencies in comparison with the complete filling by dielectric. We will suppose, that monoenergetic thin electron bunches are injected into the input of resonator $z=0$ and then move with a stationary velocity v_0 along the axis. The distribution of current density of a single bunch have the form of:

$$j_z = Q_b\delta\left(x-x_0\right)\delta\left(t-t_{0i}-z/v_0\right)\times [\theta(t-t_{0i})-\theta(t-t_{0i}-L/v_0)]/v_0, \quad (1)$$

where Q_b – a charge of a bunch per length unit in y direction, t_{0i} – the time of i – th bunch injection into the wave guide, x_0 – cross coordinate of a bunch, $\theta(t)$ – Heaviside function.

Having solved the wave equation taking into account vanishing of tangential components of electric field on metal walls of the resonator, we will receive the expression for longitudinal electric field:

$$E_z = E_0\sum_{i=0}^{N_b}\sum_{m=1}^{\infty}\sum_{l=0}^{\infty}\delta_l\omega_{10}\frac{\cos(k_l z)}{\omega_{ml}^2-\omega_l^2}\left\{\left[\frac{\omega_{ml}^2-k_l^2c^2/\varepsilon}{\omega_{ml}}\times \sin\omega_{ml}(t-t_{0i}) - \frac{\omega_l^2-k_l^2c^2/\varepsilon}{\omega_l}\sin\omega_l(t-t_{0i})\right]\theta(t-t_{0i}) - (-1)^l\left[\frac{\omega_{ml}^2-k_l^2c^2/\varepsilon}{\omega_{ml}}\sin\omega_{ml}(t-t_{0i}-L/v_0) - \frac{\omega_l^2-k_l^2c^2/\varepsilon}{\omega_l}\times \sin\omega_l(t-t_{0i}-L/v_0)\right]\theta(t-t_{0i}-L/v_0)\right\}G_m(x,x_0), \quad (2)$$

where

$E_0 = -8\pi Q_b v_0/aL\varepsilon\omega_{10}$ $\omega_l = k_l v_0$ $\kappa_m = \pi m/a$ $k_l = \pi l/L$;

function δ_l is equal 1 if $l=0$ and is equal 2 if $l \ne 0$;

$G_m(x,x_0) = \sin[\kappa_m(x+a/2)]\sin[\kappa_m(x_0+a/2)]$ $\kappa_m = \pi m/a$,

N_b – the number of bunches in a sequence.

Eigenfrequencies of the resonator and Cherenkov frequencies are defined by expressions:

$$\omega_{ml}^2 = (\kappa_m^2+k_l^2)c^2/\varepsilon, \ \omega_l = k_l v_0 \quad (3)$$

Apparently from (2) the complete field consists of the field of space charge (relevant to frequencies ω_l) and of the fields, excited by a bunch in the resonator on frequencies ω_{ml}. After leaving of all particles the resonator ($t > t_{0Nb} + L/v_0$) the field of space charge disappears, and expression for longitudinal electric field gets the form:

$$E_z = E_0\sum_{i=0}^{N_b}\sum_{m=1}^{\infty}\sum_{l=0}^{\infty}\delta_l\omega_{10}\frac{\cos(k_l z)}{\omega_{ml}^2-\omega_l^2}\frac{\kappa_m^2c^2/\varepsilon}{\omega_{ml}}\times \left[\sin\omega_{ml}(t-t_{0i}) - (-1)^l\sin\omega_{ml}(t-t_{0i}-L/v_0)\right]G_m(x,x_0), \quad (4)$$

Let's note, that at realization of the condition

$$\omega_{ml} = \omega_l \quad (5)$$

the relevant items in the sum (4) become dominant. The indicated condition is nothing else than as condition of Cherenkov radiation in slowing medium. Then these resonant items can be treated as Cherenkov field, accumulated in dielectric resonator, and the rest of the field as a field of transition radiation on both boundaries. We should note, that radiation of a charged particle in the vacuum rectangular resonator was considered first in paper [8], and in the cylindrical vacuum resonator in paper [9]. In these cases the requirement of Cherenkov radiation is not fulfilled. For our studies namely the resonant case is of interest.

Because of discreteness on longitudinal wave numbers k_l of oscillation spectrum, the resonant condition at optional sizes of resonator, permittivity and energy of bunch can be fulfilled only approximately and only for a finite number of harmonics. Taking into account, that we want to implement a multimode condition of DWA, it makes sense to find the relation between sizes from the resonant requirement. Let the resonant condition is fulfilled for harmonic $m=1$, $l=N$. Then from (3),(5) we obtain

$$L = Na\sqrt{\beta_0^2\varepsilon-1}, \ \beta_0 = v_0/c \quad (6)$$

i.e. the length of the resonator should be multiple half-integer of wave lengths of the basic resonant harmonic. The condition of equidistance is fulfilled automatically for harmonics $l = Nm$ ($m = 1,2,\ldots$), thus, multimode operation regime of DWA is provided.

For supporting of coherent summing of fields in the resonator it is necessary, that the resonant frequency ω_{1N} was multiple to frequency of bunch following. In the case when bunches are injected on every period of first harmonic with frequency $f = \omega_{1N}$ of the wave, this condition sets the cross size of the dielectric resonator

$$a = v_0/2f\sqrt{\beta_0^2\varepsilon-1} \quad (7)$$

Conditions (6) and (7) are the basis of the resonator concept of dielectric wakefield accelerator. In such resonator it is provided:

1. multimode regime of field excitation;
2. coherent summing of fields from bunches of sequence.

Let's explain written above at discussion of expression (4) and relations (6),(7) by graphic example.

In Fig.1 dispersion dependencies of dielectric resonator ($\varepsilon = 2.1$), excited by a sequence of electron bunches with energies $4\,MeV$, frequency of bunch repetition $f = 2.85\,GHz$ are presented. The cross size, chosen according to condition (7), is equal $a = 5,045\,cm$, and

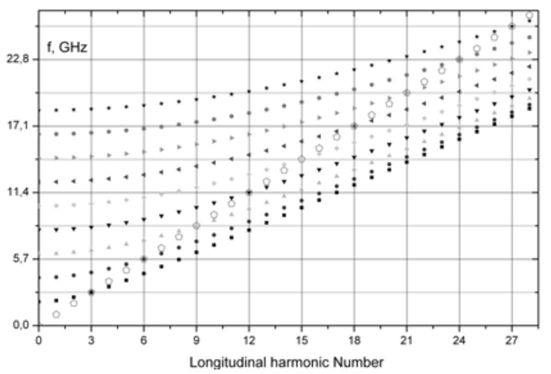

Fig.1. *Eigenfrequencies of the dielectric resonator for $N=3$. First 9 cross harmonics of the resonator are presented. Pentagons show Cherenkov frequencies of the bunch*

longitudinal size − according to requirement (6), is equal $L=15{,}677\,cm$ (at that third longitudinal harmonic is chosen as resonant, $N=3$). The horizontal grid on fig.1 is lined with interval, equal to frequency of bunch repetition, and vertical grid − with interval, equal to the number of longitudinal resonant harmonic. As Fig.1 illustrates, cross points of Cherenkov frequencies (pentagons) and eigenfrequencies of the resonator are multiple to the frequency of bunch repetition.

3. NUMERICAL CALCULATIONS

Let bunches have rectangular shape and their longitudinal size is L_b, cross size is a_b ($-a_b/2 \le x_0 \le a_b/2$). Wakefield from a sequence of bunches of finite sizes is obtained by integration of expression (2) by time of injection t_{0i} and by cross positions x_0 of microbunches:

$$\begin{aligned}E_z = {} & E_b \sum_{i=0}^{N_b}\sum_{m=1}^{\infty}\sum_{l=0}^{\infty}\delta_l \frac{\cos(k_l z)}{\omega_{ml}^2-\omega_l^2}\left\{\frac{\Omega_m^2}{\omega_{ml}^2}\Big[\theta(t_i)\big[1-\cos\omega_{ml}t_i\big]-\theta(t_i-L_b/v_0)\big[1-\cos\omega_{ml}(t_i-L_b/v_0)\big]\Big]-\theta(t_i)\big[1-\cos\omega_l t_i\big]+\right.\\ & \theta(t_i-L_b/v_0)\big[1-\cos\omega_l(t_i-L_b/v_0)\big]-(-1)^l\frac{\Omega_m^2}{\omega_{ml}^2}\Big[\theta(t_{iL})\big[1-\cos\omega_{ml}t_{iL}\big]-\theta(t_{iL}-L_b/v_0)\big[1-\cos\omega_{ml}(t_{iL}-L_b/v_0)\big]\Big]+ \\ & \left.(-1)^l\theta(t_{iL})\big[1-\cos\omega_l t_{iL}\big]-(-1)^l\theta(t_i-L_b/v_0)\big[1-\cos\omega_l(t_{iL}-L_b/v_0)\big]\right\}G_m(x,a,a_b),\end{aligned} \quad (8)$$

where: $t_i=t-(i-1)T$, $t_{iL}=t_i-L/v_0$,
$\Omega_m^2=\kappa_m^2 v_0^2/(\beta_0^2\varepsilon-1)$, $E_b=-16Q_b(\beta_0^2\varepsilon-1)c^2/a_b L L_b \varepsilon^2$,
$G_m^b=\sin^2(\pi m/2)\sin(\pi m a_b/2a)\cos(\pi m x/a)$.

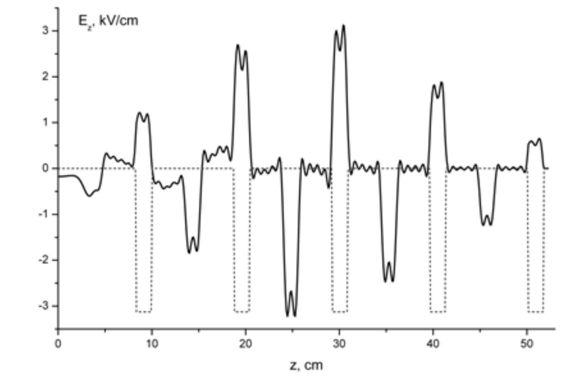

Fig.2. Longitudinal distribution of electric field E_z in the centre of resonator ($x=0$) at time $t=1{,}738\,ns$ after injection of 5 bunches. Dashed line shows the shape and position of bunches

Let's choose the following parameters for numerical calculations: $L_b=1{,}7\,cm$, $a_b=0{,}1a$, $N=10$, $N_b=101$, $Q_b=0.32\,nC/cm$, the rest of parameters − the same as on Fig.1. In Fig. 2 and Fig.3 distributions of wakefield for two moments are presented: $t=1{,}738\,ns$ (5 bunches are injected into the resonator) and $t=35{,}639\,ns$ (the last bunch of sequence is injected into the resonator).

In initial stage of injection (Fig.2) the amplitude of the field grows from the head of a sequence to the location of the group wavefront, excited by the first bunch, and then decreases to the input of the resonator.

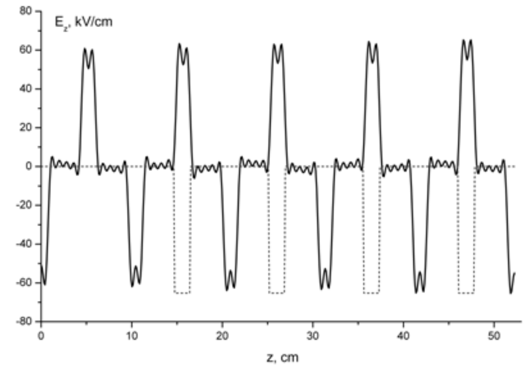

Fig.3. The same, that on Fig.2 at instant $t=35{,}639\,ns$ after injection of 101 bunches

Wakefield in the resonator before leaving of the first bunch qualitatively and quantitatively coincides with the field in semi-infinite waveguide. The shape of wakefield impulses and their duration approximately repeats the shape and duration of bunches. In later times, after all bunches are injected into the resonator, almost homogeneous distribution of field amplitude is formed in it.

In contrast to considered case of the resonator, in the semi-infinite waveguide increasing from input to output distribution of field amplitude [5] is formed. At that the number of bunches working on build up of maximum amplitude, is much lower, than in the resonator. For the dielectric waveguide with the same length, cross size and permittivity, as for the considered resonator this number is equal 6. From comparison of Fig.2 and Fig.3 it follows, that all bunches of sequence equally contribute to the forming of amplitude of longitudinal electric field. I.e. it is possible to excite wakefield in the resonator with the amplitude considerably exceeding amplitude of field in the semi-infinite waveguide. At that the regularity of oscillations is kept.

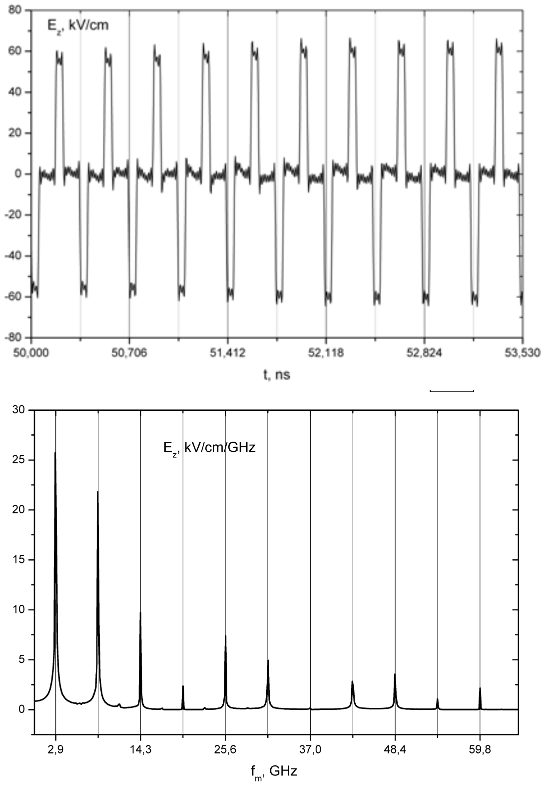

Fig.4. Time diagram (upwardly) of longitudinal electric field $E_z(x=0, z=0)$ and its spectral density (below) in the dielectric resonator after injection of 101 bunches. Parameters are the same, as on Fig.3

In Fig.4 time diagram of wakefield in the dielectric resonator (above) after injection of 101st bunch is presented, last bunch left the resonator at time $t \approx 38\,ns$. Wakefield at input of the resonator has a shape of rectangular pulses sequence with period equal to period of bunch repetition rate. The amplitude of pulses weakly varies with time. In the bottom part of Fig.4 the spectrum of longitudinal electric field is presented. It is seen, that only odd resonant frequencies constitute wakefield. Non-resonant harmonics almost do not contribute to amplitude of wakefield.

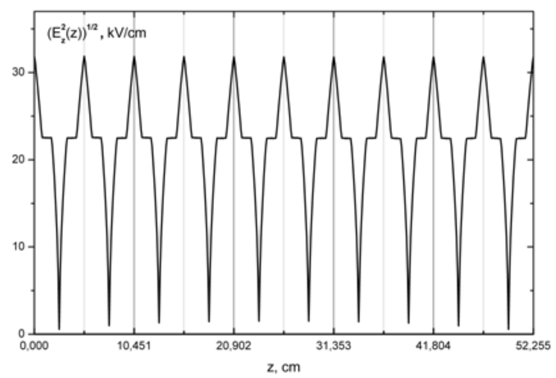

Fig.5. Time averaged on time ($50\,ns \leq t \leq 85\,ns$) longitudinal distribution of wakefield in the centre of resonator ($x = 0$). Parameters are the same, as on Fig.4

In Fig.5 longitudinal distribution of wakefield averaged over time is presented. It is seen, that on average over time, the standing wave is formed after leaving of all bunches from the resonator. The narrow peaks, formed by a great number of resonant harmonics, have mean amplitude approximately $33\,kV/cm$ and the distance between them is equal to period of bunch repetition.

For investigation of acceleration of driven bunch electrons we took an axial bunch with duration and energy, equal to duration and energy of driving bunches. In Fig.6 the gain of energy of electrons, accelerated along the resonator, is shown.

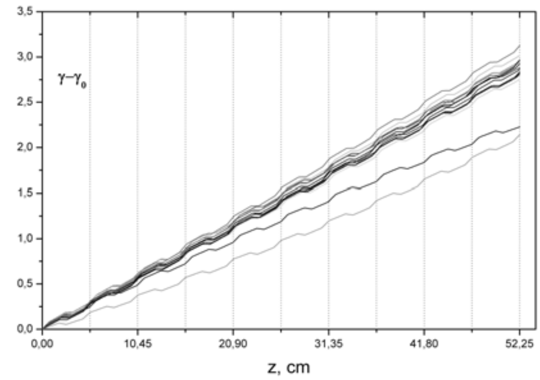

Fig.6. Gain of energy of electrons, accelerated along the resonator. Parameters of the resonator and sequence of bunches are the same, as on Fig.4

The gain of energy of at most accelerated electrons is equal 1,6 MeV, that is approximately equal to half of energy gain of a particle in the constant field, equal to half of wakefield amplitude, excited in the resonator. In Fig.7 the value of longitudinal electric field, accelerating a particle along the resonator, is presented. From comparison of Fig.6 and Fig.7 it follows, that acceleration gradient is equal to mean of wakefield on particle trajectory. Peaks of wakefield have the shape of bunches and follow with period twice greater, than period of wakefield. Such periodicity of field distribution on particle trajectory is typical for a resonator system.

Thus, carried out studies showed realizability of the resonator concept of dielectric wakefield accelerator in planar geometry.

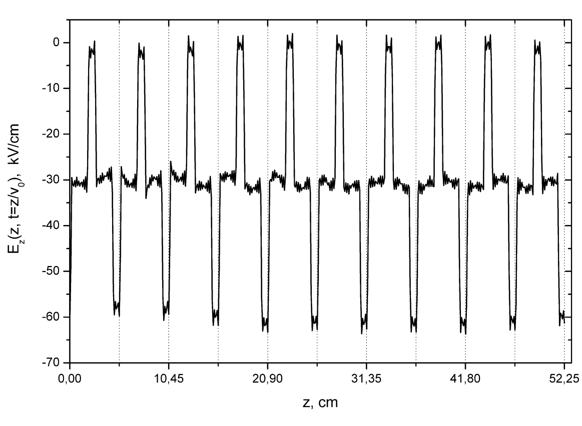

Fig.7. Longitudinal electric field strength on the trajectory of the most accelerated particle. Parameters of the resonator and sequence of bunches are the same, as on Fig.4

4. ACKNOWLEDGMENTS

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ТЕОРЕТИЧЕСКИЕ ИССЛЕДОВАНИЯ РЕЗОНАТОРНОЙ КОНЦЕПЦИИ УСКОРИТЕЛЯ НА КИЛЬВАТЕРНЫХ ПОЛЯХ В ДИЭЛЕКТРИКЕ

Н.И.Онищенко, Г.В.Сотников

Проведено теоретическое исследование и численное моделирование возбуждения кильватерного поля в плоском диэлектрическом резонаторе регулярной последовательностью сгустков релятивистских электронов. Показано, что в резонаторном случае при сохранении пикирования поля, т.е. многомодового режима, увеличивается количество сгустков, участвующих в суммировании кильватерного поля. В результате амплитуда поля, используемая для ускорения заряженных частиц, существенно возрастает по сравнению с отрезком диэлектрического волновода такой же длины. Таким образом, в работе доказана правомерность резонаторной концепции для кильватерного метода ускорения заряженных частиц. Численные расчеты выполнены для планируемого эксперимента на ускорителе “Алмаз-2” по возбуждению кильватерного поля в прямоугольном диэлектрическом резонаторе последовательностью сгустков с зарядом 0.32 нК и энергией 4 МэВ каждый.

ТЕОРЕТИЧНІ ДОСЛІДЖЕННЯ РЕЗОНАТОРНОЇ КОНЦЕПЦІЇ ПРИСКОРЮВАЧА НА КІЛЬВАТЕРНИХ ХВИЛЯХ У ДІЄЛЕКТРИКУ

М.І.Онищенко, Г.В.Сотніков

Проведено теоретичне дослідження та чисельне моделювання збудження кільватерного поля у плоскому діелектричному резонаторі регулярною послідовністю згустків релятивістських електронів. Показано, що у резонаторному випадку при збереженні пікірування поля, тобто багатомодового режиму, збільшується кількість згустків, що приймають участь у підсумовуванні кільватерного поля. В результаті амплітуда поля, що використовується для прискорення заряджених часток, істотно зростає у порівнянні з відрізком діелектричного хвилеводу такої самої довжини. Таким чином, у роботі доведена правомірність резонаторної концепції для кільватерного методу прискорення заряджених часток. Чисельні розрахунки виконані для запланованого експерименту на прискорювачі “Алмаз-2” по збудженню кільватерного поля у прямокутному діелектричному резонаторі послідовністю згустків з зарядом 0.32 нК та енергією 4 МеВ кожний.