RADIATION FROM RELATIVISTIC POSITRONS DURING PLANAR CHANNELING IN CRYSTALS

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We present a detailed calculation of channeling radiation of planar-channeled positrons from crystal targets in the framework of our approach, which was proposed recently.

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1. INTRODUCTION

Theoretical studies of the radiation from planar-channeled electrons and positrons are due to M.A. Kumakhov and R. Wedell [1], N.K. Zhevago [2], A.I. Akhiezer, I.A. Akhiezer and N.F. Shul`ga [3,4].

Experimentally the channeling radiation (CR) of positrons was observed by different groups [5,6] demonstrating strong and sharp peaks in the spectrum.

The purpose of the present work is to calculate the spectral-angular distribution of the channeling radiation intensity emitted from positrons in the framework of approach, which was proposed recently [7].

2. GENERAL REMARKS ON CR AND CALCULATION

We consider a relativistic charged particle incident onto a crystal at a small angle to a crystal planes. In the planar channeling case for positively charged positrons, the channel is between the crystal planes. This channel is the source of a potential well in the direction transverse to particle motion giving rise to transversely bound states for the particle. Transitions to lower-energy states lead to the phenomenon known as channeling radiation (CR). Calculations of the CR process are carried out by using the rules of quantum electrodynamics [1,2]. The Doppler formula for the energy of photon emitted is derived using the energy and momentum conservation laws. For this energy one gets

$$\omega = \frac{\omega_{nn'}}{1 - \beta_{//}\cos\theta} \tag{1}$$

where $\omega_{nn'} = \varepsilon_n - \varepsilon_{n'}$; ε_n and $\varepsilon_{n'}$ are the discrete energy levels of the transverse oscillations of the positron in the channel before and after radiation, respectively; $\beta_{//} = \frac{|\vec{p}_{//}|}{E}$, E and $p_{//}$ are the energy and the longitudinal momentum of the positron, and θ is the angle of radiation emission relative to the direction of motion of the channeled positron. For positrons of not too high transverse energies, a good approximation (see e.g., Ref. [1,2]) is the harmonic potential leading to equidistant energy levels

$$\varepsilon_n = \Omega\,(n + 1/2) \tag{2}$$

where $\Omega = \frac{2}{d_p}\sqrt{\frac{2U_0}{E}}$, d_p is the distance between planes in the corresponding units, and U_0 is the depth of the potential well. Since $\frac{|\vec{p}_{//}|}{E} \approx (1 - \frac{1}{2}\gamma^{-2})$ (where $\gamma = \frac{E}{m}$), $\cos\theta \approx (1 - \frac{1}{2}\theta^2)$ and taking into account Eq. (2), Eq. (1) can be expressed as

$$\omega = 2\gamma^2 \frac{\Omega\,(n - n')}{(1 + \theta^2\gamma^2)} \tag{3}$$

It follows from Eq. (3) that the radiation of a maximum frequency

$$\omega = 2\gamma^2\Omega\,(n - n') \tag{4}$$

is emitted in the forward direction (at $\theta = 0$). The case $n - n' = 1$ corresponds to the peak values of the experimental channeling radiation spectra [6], being the first harmonic with the photon energy $\omega = 2\gamma^2\Omega$. As it follows from Eq. (3), photons emitted via positron transition from any initial level n to the final level $n - 1$ are identical (i.e., have the same energies for the same emission angles). This means that the resulting amplitude should be given by an additive superposition of amplitudes of all such transitions. The positron state outside the crystal $(z < 0)$ is a plane wave, whereas inside the crystal $(z > 0)$, the part of its wave function corresponding to the transverse motion is a superposition of the harmonic oscillator eigenvectors. Factors c_n describing transitions from the initial state to states with the transverse energy levels n can be found using boundary conditions set upon the wave function at the crystal boundary $(z = 0)$. Then, a transition to the closest lower level $n-1$ occurs with emission of a photon having energy ω. One may expect that the total amplitude of the transition from the initial to final state accompanied by the photon emission is determined by products of the amplitudes c_n and $M_{n,n-1}$. Following the rules of the quantum mechanics, we express this amplitude as

$$A \propto \sum_n c_n M_{n,n-1}\,, \tag{5}$$

were summation is performed over all the harmonic oscillator levels. Also we must find an additive superposition of amplitudes of all transitions $n \to n-2$, $n \to n-3, \ldots$ etc. Taking into account these considerations, we can write the transition matrix element in the form

$$\left|M_{if}\right|^2 = \sum_j \left|\sum_n c_n M_{n,n-j}\right|^2, \tag{6}$$

where $j = n - n'$. It is well known from the quantum electrodynamics that the transition matrix element is given by

$$\left|M_{if}\right|^2 = \frac{2\pi \cdot e^2}{\omega \cdot V}\left|J_\lambda\right|^2, \tag{7}$$

where $e^2 = \frac{1}{137}$, $V = L^3$ is the normalization volume, $\lambda = 1, 2$ indicates the linear photon polarization, and

$$J_\lambda = \int \Psi'^{\dagger} \alpha_\lambda e^{-ik\cdot r} \Psi\, d^3 r\,, \tag{8}$$

with $\alpha_\lambda = \vec{\alpha} \cdot \vec{\varepsilon}_\lambda^{\,*}$. The prime indicates the final state. As we mentioned above, the wave function is the solution of the time-independent Dirac equation for a relativistic particle moving with momentum $p_{//} = (0, p_y, p_z)$ in a one-dimensional planar potential $U(x)$ periodic in the x direction (which is normal to the channeling planes)

$$(i\alpha \cdot \nabla + E - \beta m)\Psi = U(x)\Psi\,, \tag{9}$$

where m and E are the particle's mass and energy, α and β are the Dirac matrices. Separating the wave function Ψ into large and small components,

$$\Psi = \begin{pmatrix} \Psi_a \\ \Psi_b \end{pmatrix} \tag{10}$$

and using the standard representation for the Dirac matrices, leads to a Pauli-type equation for the large components,

$$\vec{\sigma} \cdot \vec{\nabla}\,(E - U(x) + m)^{-1}\,\vec{\sigma} \cdot \vec{\nabla}\Psi_a + (E - U(x) - m)\Psi_a = 0$$

Since a potential $U(x)$ is independent of y and z, the solution of last equation is a plane wave in the yz plane

$$\Psi_a \propto \exp[i(p_z z + p_y y)]\varphi(x)\chi\,. \tag{11}$$

This allows us to transform a Pauli-type equation into a one-dimensional, relativistic Schrödinger equation for the transverse motion

$$-\frac{1}{2E}\frac{d^2\varphi(x)}{dx^2} + U(x)\varphi(x) = \varepsilon\varphi(x)\,, \tag{11a}$$

where

$$\varepsilon = \frac{E^2 - m^2 - p_z^2 - p_y^2}{2E}. \tag{12}$$

In a given potential, the latter will assume certain bound-state eigenvalues $\varepsilon_n < 0 \quad (n = 0, 1, 2, \ldots)$, with corresponding eigenfunctions $\varphi_n(x)$. The wave function of Eq.(10) is finally obtained in the form

$$\Psi = \frac{N}{L}\begin{pmatrix} \vec{\chi} \\ \dfrac{-i\vec{\sigma} \cdot \nabla}{E + m}\chi \end{pmatrix} \exp(i\vec{p}_{//}\vec{r}_{//})\varphi_n(x)\,, \tag{13}$$

where L^2 being the two-dimensional normalization volume for the plane waves, $N = \sqrt{\frac{E+m}{2E}}$, and χ being a two-component spinor which is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ when the particle spin points in the $+z$ or the $-z$ direction in the rest frame, respectively. For the harmonic potential $U(x) = U_0 x^2$, it is well known that the corresponding eigenfunctions being given by

$$\varphi_n(x) = \sqrt[4]{\frac{E\Omega}{\pi}}\frac{1}{\sqrt{2^n n!}}\exp(-E\Omega x^2/2)H_n(\sqrt{E\Omega}\,x),$$

where H_n are the Hermite polynomials. According to Eq. (8), we find, using Eq. (13) and last expression for the wave functions, the first-order matrix element corresponding to the $n \to n-j$ transverse transition

$$J_\lambda = (2\pi)^2\delta(\vec{p}_{//} - \vec{p}'_{//} - \vec{k}_{//})NN'\chi_2^{*}\vec{\varepsilon}_\lambda^{\,*}(\vec{A} + i[\vec{B}\vec{\sigma}])\chi_1,$$

where

$$\begin{aligned}
A_x &= -iI_2\left(\frac{1}{E+m} + \frac{1}{E-\omega+m}\right), \\
\vec{A}_{//} &= I_1\vec{p}_{//}\left(\frac{1}{E+m} + \frac{1}{E-\omega+m}\right), \\
B_x &= iI_2\left(\frac{1}{E-\omega+m} - \frac{1}{E+m}\right) + \frac{k_x}{E-\omega+m}I_1, \\
\vec{B}_{//} &= I_1\left(\frac{\vec{p}_{//}}{E+m} - \frac{\vec{p}'_{//}}{E-\omega+m}\right),
\end{aligned} \tag{14}$$

with

$$\begin{aligned}
I_1 &= \int \exp(-ik_x x)\varphi^*_{n-j}(x)\varphi_n(x)dx, \\
I_2 &= \int \exp(-ik_x x)\varphi^*_{n-j}(x)\frac{d\varphi_n(x)}{dx}dx,
\end{aligned} \tag{15}$$

Then we calculate the matrix element and the differential intensity, and after summation over polarization of emitted photon and the positron we find by integrating over $d^2 p'_{//}$

$$\frac{d^2 I}{d\omega\, do} = \frac{e^2 \omega^2}{2\pi} \sum_j \left| \sum_n c_n M_{n,n-j} \right|^2 \times$$
$$\times\, \delta(\omega - \omega\beta_{//}\cos\theta - \omega_{n,n-j}) \qquad (16)$$

where the factors c_n are, in the case of the parabolic potential and when the initial positron is a plane wave, given by

$$c_n = \frac{i^n}{\sqrt{2^{n-1} n!}} \sqrt[4]{\frac{\pi}{E\Omega}} \exp\left(-\frac{p_x^2}{2E\Omega}\right) H_n\left(\frac{p_x}{\sqrt{E\Omega}}\right). \qquad (17)$$

3. CONCLUSIONS

Following N.K. Zhevago [2], the spectral-angular distribution of emitted photons is represented as

$$\frac{d^2 I}{d\omega\, do} \propto \sum_f \left| M_{if} \right|^2 \qquad (18)$$

The sum entering Eq. (18) is the one over the quantum numbers *f* of the transverse motion of the particle. Then, the probability of having a definite transverse energy is taken into account by multiplying each term of this sum by a corresponding factor. In our consideration, discrete levels of the transverse motion in the harmonic oscillator potential refer to the intermediate state of the particle. Accordingly, the contribution to the intensity due to transitions, e.g., between the closest levels is determined by the square of the absolute value of Eq. (5)

$$\frac{d^2 I^{(1)}}{d\omega\, do} \propto \left| \sum_n c_n M_{n,n-1} \right|^2. \qquad (19)$$

In other words, unlike Ref. [2], we get an expression that contains interference terms mixing amplitudes of photon emission from different equidistant levels. We would like to note that dynamics of channeling electron in a crystal differs from that of the positron case. The transverse potential well for the electron does not give rise to equidistant energy levels for transverse particle motion. Therefore, there are no interference contributions to the photon emission intensity similar to those present in Eq. (19). In our opinion, this could explain the greater intensity in case of channeling positron compared to that for the electron observed in experiment [6]. Corresponding numerical calculations will be given in a subsequent publication.

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ИЗЛУЧЕНИЕ РЕЛЯТИВИСТСКИХ ПОЗИТРОНОВ ПРИ ПЛОСКОСТНОМ КАНАЛИРОВАНИИ В КРИСТАЛЛАХ

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Представлены подробные вычисления излучения позитронов при плоскостном каналировании в кристаллической мишени в рамках предложенного ранее подхода.

ВИПРОМІНЮВАННЯ ВІД РЕЛЯТІВІСТСЬКИХ ПОЗІТРОНІВ ПРИ ПЛОЩИННОМУ КАНАЛІРУВАННІ В КРИСТАЛАХ

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Представлено докладні обчислювання випромінювання при каналіруванні площинно-каналіруючих позітронів із кристаличної мішені за нашим підходом, який було запропоновано раніше.