BREMSSTRAHLUNG IN ALPHA-DECAY: ANGULAR ANALYSIS OF SPECTRA

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A quantum mechanical method of calculation of bremsstrahlung spectra in alpha-decay of heavy nuclei with taking into account an angle between directions of the alpha-particle motion and the photon emission is presented. Dependence between the bremsstrahlung spectrum and the angle is obtained in a simple analytical form. The method can be used for a comparative analysis of experimental data, obtained at different angles.

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1. INTRODUCTION

Research of bremsstrahlung in α-decay of heavy nuclei has caused an increased interest last years. We note main purposes for study of this phenomenon:

- study of properties of the α-decay dynamics on the basis of analysis of the experimental spectra of the bremsstrahlung; development of a non-stationary model of detailed description of this process (including a space barrier region), tested on the basis of the experimental data;
- investigation of the *subbarrier bremsstrahlung in the alpha-decay*, i.e. the photons emission during tunneling of the alpha-particle through decay barrier;
- construction of a method of tunneling time determination of the alpha-particle through the decay barrier (which approximately equals to nuclear times values $10^{-20}\ldots10^{-24}$ sec) on the basis of analysis of the experimental spectra of the bremsstrahlung.

For successful realization of all these researches the theoretical model is needed, which allows describing the alpha decay with bremsstrahlung and calculating their main characteristics, and which is tested by experimental data.

The experiments were fulfilled with such nuclei: ^{210}Po, ^{214}Po, ^{226}Ra and ^{244}Cm. Note, that in the behavior of the experimental spectra for the nucleus ^{210}Po, obtained by Russian-Italian group [1] and Japanese group [2] independently, there is a qualitative difference, which had caused discussions in some papers. These experiments were fulfilled for the different values of the angle between the directions of the alpha-particle motion and the photon emission (which equal to 90° and 25°) and, perhaps, by this fact one can explain the difference between their spectra. But we note that more accurate analysis can be fulfilled on the basis of the unified model which allows calculating the bremsstrahlung spectra with taking into account the different values of such angle, and this analysis had not done else.

Constructed theoretical models are differed also in their description of the bremsstrahlung spectra. Here, the instant accelerated model [3], developed on the basis of classical electrodynamics and used such characteristics as a velocity of the alpha-particle during its leaving outside from the barrier region, gives enough good description of the experimental bremsstrahlung spectra. But one can consider these characteristics as additional parameters, which introduction allows moving the calculated bremsstrahlung spectrum curve near the experimental data. Quantum mechanical models, proposed both by T. Papenbrock and G.F. Bertsch [4], and by E.V. Tkalya [5], do not use such additional parameters and their descriptions of experimental spectra are less satisfactory (from our point of view).

However, the models, constructed on the basis of quantum electrodynamics and without semi-classical approach, are the most effective in study of the alpha-decay dynamics in the space barrier region and also for detailed study of the subbarrier bremsstrahlung effect in the alpha-decay. And the model, in which the method of calculation of the bremsstrahlung spectra takes into account the angle value, will be useful for analysis of the experimental spectra, obtained for different angles (at present, the proposed models [2-5] are isotropic).

2. SHORT REVIEW OF QUANTUM MECHANICAL MODELS

In paper [6] we proposed the multipolar quantum mechanical model of the alpha-decay with the bremsstrahlung, which allows calculating the bremsstrahlung spectra in dependence on the angle value between directions of the alpha-particle motion and the photons emission. In accordance with this model, the probability of the spontaneous photon emission in the alpha-decay is ($\omega_{fi} = E_i - E_f$):

$$\frac{dW}{d\Omega_{\nu}} = \frac{Z_{eff}^{2}\, e^{2} k_{f} \omega_{fi}}{(2\pi)^{4} m} \left|p(k_i, k_f)\right|^{2},$$

$$k_{i,f} = \frac{1}{\hbar}\sqrt{2mE_{i,f}}\,, \tag{1}$$

where $p(k_i,k_f)$ has the following form:

$$p(k_i, k_f) = \sum_{\alpha=1,2} e^{(\alpha)*} \int_{0}^{+\infty} dr \int d\Omega\, r^{2} \psi_{f}^{*}(\vec{\boldsymbol{r}})\, e^{-i\vec{\boldsymbol{k}}\vec{\boldsymbol{r}}} \frac{\partial}{\partial \vec{\boldsymbol{r}}} \psi_{i}(\vec{\boldsymbol{r}}). \tag{2}$$

Here Z_{eff} is the effective charge, m is reduced mass of the composite system (the alpha-particle and the daughter nucleus), E_i and E_f are the total energy of the system in initial *i*-state (i. e. the state of the system before the photon emission) and final *f*-state (i.e. the state of the system after the photon emission), k_i and k_f are the wave vector of the system in the initial *i*- and final *f*-states, $\psi_i(\boldsymbol{r})$ and $\psi_f(\boldsymbol{r})$ are the wave function of the system in the initial *i*- and final *f*-states, $e^{(\alpha)}$ is the unit polarization vector of the emitted photon, *k* is the wave vector of the photon, $w_{i,f} = k = |k|$. Vector $e^{(\alpha)}$ is perpendicular to *k* in Coulomb calibration. We use the unit system when $\Box = 1$ and $c = 1$.

Main difference between the methods, in which the calculation of the bremsstrahlung spectrum is based on quantum electrodynamics, consists in different approaches for calculation of the value $p(k_i,k_f)$ (on our view, the best review of such methods is present in [5]). Here, we use the multipolar expansion of the vector potential of electromagnetic field of the daughter nucleus (see [7], p.57, 51, 49). Our result is:

$$\begin{aligned}
&p(k_i,k_f) = \sqrt{2\pi} \\
&\times \sum_{\boldsymbol{l}=1} \left(\sqrt{2l+1}(-i)^l[p^{\boldsymbol{Ml}}(k_i,k_f) - ip^{\boldsymbol{El}}(k_i,k_f)]\right), \\
&p^{\boldsymbol{Ml}}(k_i,k_f) = I_1 J_{\boldsymbol{l}}(l), \\
&p^{\boldsymbol{El}}(k_i,k_f) = -\sqrt{\frac{l+1}{2l+1}} I_2 J_{\boldsymbol{l}}(l-1) + \sqrt{\frac{l}{2l+1}} I_3 J_{\boldsymbol{l}}(l+1)
\end{aligned} \tag{3}$$

where

$$J_{\mathbf{l}}(n) = \int_0^{+\infty} r^2 \psi^*_{f,\mathbf{l}}(r) \frac{d\psi_i(r)}{dr} j_n(kr)\, dr \tag{4}$$

$$\begin{aligned}
I_1 &= \sum_{\mu=-1}^{1} \mu\, \Box_\mu \int Y^*_{LM}(\vec{\mathbf{n}}^f_r)\mathbf{T}_{01,0}(\vec{n}^i_r)\mathbf{T}^*_{1l,\mu}(\vec{n}_v)\, d\Omega \ , \\
I_2 &= \sum_{\mu=-1}^{1} \mu^2 \Box_\mu \int Y^*_{LM}(\vec{\mathbf{n}}^f_r)\mathbf{T}_{01,0}(\vec{n}^i_r)\mathbf{T}^*_{1l-1,\mu}(\vec{n}_v)\, d\Omega \ , \\
I_3 &= \sum_{\mu=-1}^{1} \mu^2 \Box_\mu \int Y^*_{LM}(\vec{\mathbf{n}}^f_r)\mathbf{T}_{01,0}(\vec{n}^i_r)\mathbf{T}^*_{1l+1,\mu}(\vec{n}_v)\, d\Omega
\end{aligned} \tag{5}$$

and $j_n(kr)$ is the spherical Bessel function of order *n* (*n* is a natural number), $Y_{LM}(\mathbf{n}^f_r)$ are the normalized spherical functions, $\mathbf{T}_{\mathbf{l}m,\boldsymbol{\mu}}(n)$ are the vector spherical harmonics (see [7], p. 45).

Calculations of the bremsstrahlung spectrum in the alpha-decay of the nucleus ^{210}Po on the basis of such a model give enough good description of the experimental data [1], obtained by Russian-Italian group for the angle 90°. Evaluation of the spectrum at angle 25° on the basis of this models gives a monotonous behavior (without appearance of the "hole" like experimental data [2]).

However, there are some problems in the described model, concerned with the restriction of calculations accuracy of wave functions in the asymptotic area in the states before and after the photon emission. As the subintegral expression in the integral (4) is the oscillated and slowly damped function, that calculations convergence of the spectra is limited. As a result, the numerical calculation of the spectra becomes difficult sufficiently and in this sense the model [6] is not suitable enough. The convergence of the spectra calculation by the models [4, 5] is higher and these models are not exposed to necessity of calculations of the wave functions in the asymptotic area with high precision. But, probably, they give more approximated calculation of the bremsstrahlung spectra (see [6]).

The second important point in the analysis of quantum mechanical models is their possibility to calculate the bremsstrahlung spectra in dependence on the value of the angle between directions of the alpha-particle motion and the photons emission, and, as result, a possibility to fulfill a comparative analysis of the experimental data for the nucleus ^{210}Po [1, 2], obtained for the angles 90° and 25°. Here, the model [6] allows doing this at the first time. In this model the dependence of the bremsstrahlung spectra on the angle value between directions of the alpha-particle motion and the photon emission is in the angular integrals (5). However, such angular dependence is not suitable for the speed angular qualitative analysis of experimental spectra.

Further, we present a new alternative approach for the calculation of the bremsstrahlung spectra, where such angular dependence is shown more obviously and more simple.

3. SIMPLIFIED APPROACH FOR THE ANGULAR CALCULATIONS OF THE SPECTRA

Let's consider $\mathbf{k}$ and $\mathbf{r}$. Vector $\mathbf{k}$ is the photon impulse and points to a direction of the photon motion. Vector $\mathbf{r}$ is the radius-vector, which points to a space position of the alpha-particle relatively to the center of mass of the daughter nucleus. We suppose, that as the mass of the daughter nucleus is sufficiently more then the alpha-particle mass, then the direction of the radius-vector of the alpha-particle position coincides with the direction of alpha-particle velocity. Then the angle between the vectors $\mathbf{k}$ and $\mathbf{r}$ is the angle between the directions of the alpha-particle motion and the photon emission. One can write:

$$\exp(-i\mathbf{kr}) = \exp(-ikr \ \cos\beta) \tag{6}$$

where $k = |\boldsymbol{k}|$, $r = |\boldsymbol{r}|$, β is the angle between the direction of the alpha-particle motion $\mathbf{k}/k$ and the direction of the propagation of the emitted photon $\mathbf{r}/r$.

Find the expression for the bremsstrahlung spectrum. Let's write polarization vectors e^α in terms of circular polarization vectors ξ with opposite directions of rotation (see [7], p. 42):

$$\xi_{-1} = \frac{1}{\sqrt{2}}(e^1 - ie^2), \quad \xi_{+1} = -\frac{1}{\sqrt{2}}(e^1 + ie^2) \tag{7}$$

We obtain:

$$\begin{aligned}
p(k_i,k_f) &= \sum_{\mu=-1,+1} h_\mu \xi^*_\mu \int_0^{+\infty} dr \int d\Omega\, r^2 \times \\
&\times \varphi^*_f(\mathbf{r}) e^{-i\mathbf{kr}} \frac{\partial}{\partial \mathbf{r}} \varphi_i(\mathbf{r}),
\end{aligned} \tag{8}$$

where

$$h_{-1} = (1-i)/\sqrt{2}, \quad h_1 = (-1-i)/\sqrt{2}. \tag{9}$$

We use the following property (see [7], p. 44-46; [6]):

$$\frac{\partial}{\partial \vec{\boldsymbol{r}}}\psi_i(\vec{\boldsymbol{r}}) = -\frac{d\psi_i(r)}{dr}\boldsymbol{T}_{01,0}(\vec{\boldsymbol{n}}_r^i),$$

$$\boldsymbol{T}_{01,0}(\vec{\boldsymbol{n}}_r^i) = \sum_{\mu=-1}^{1}(110|-\mu\,\mu\,0)Y_{1,-\mu}(\boldsymbol{n}_r^i)\xi_\mu,$$

$$(110|1,-1,0) = (110|-1,1,0) = \sqrt{1/3}, \qquad (10)$$

where $(110|-\mu\ \mu\ 0)$ are the Clebsch-Gordan coefficients. Taking into account Eq. (10), (6) and the orthogonal property of the polarization vectors ξ_{-1} and ξ_1, we find:

$$p(k_i,k_f) = -\sqrt{1/3}\sum_{\mu=-1,+1}h_\mu\int_0^{+\infty}drr^2\varphi_f^*(r)\frac{\partial\varphi_i(r)}{\partial r} \qquad (11)$$
$$\times\int d\Omega\, Y_{l'm'}^*(\boldsymbol{n}_r^f)Y_{1,-\mu}(\boldsymbol{n}_r^i)e^{-ikr\cos\beta}.$$

Further, we suppose that the process of the photon creation does not change the direction of the alpha-particle motion, i.e.:

$$\mathbf{n}_r^i = \mathbf{n}_r^f \qquad (12)$$

Taking into account this approximation, and also the orthogonal property of the functions $Y_{lm}(\mathbf{n}_r)$, we obtain the following expression for $p(k_i,k_f)$:

$$p(k_i,k_f) = -\sqrt{\frac{1}{3}}\sum_{\mu=-1,+1}h_\mu\int_0^{+\infty}r^2e^{-ikr\cos\beta}\times \qquad (13)$$
$$\times\varphi_f^*(r,l=1,m=-\mu)\frac{\partial\varphi_i(r,l=m=0)}{\partial r}dr,$$

Note, that we have the quantum numbers $l=1,\ m=\mu$ for final state and $l=m=0$ for initial state. Such determination follows from the supposition (12). From here we can also conclude, that in multipolar approach of quantum mechanical calculation of the bremsstrahlung spectra the E1 multipole gives the most important contribution into the total spectrum (such idea of estimation of multipole contributions into a total bremsstrahlung spectrum is obtained at the first time).

In such a form of $p(k_i,k_f)$ the dependence of the bremsstrahlung spectrum on the angle is more obvious. In new approach one can calculate the bremsstrahlung spectrum on the basis of (1), where one can use Eq. (13) for $p(k_i,k_f)$ value. Note that the model [6] allows calculating the bremsstrahlung spectra only with taking into consideration of the selected electrical and magnetic multiples, whereas the method, proposed in this paper, gives the calculation of the bremsstrahlung spectrum as a whole.

4. CONCLUSIONS

We propose new approach for calculation of the bremsstrahlung spectra in alpha-decay, where the angle between the directions of the alpha-particle motion and the photon emission is taken into account, and the dependence of the bremsstrahlung spectra on the values of such angle is more obvious and simple then in model [6]. This method can be useful for a comparative analysis of the experimental data [1,2], obtained for different angles.

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ТОРМОЗНОЕ ИЗЛУЧЕНИЕ ПРИ АЛЬФА-РАСПАДЕ: УГЛОВОЙ АНАЛИЗ СПЕКТРОВ

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Представлен квантово-механический метод расчета спектров тормозного излучения при альфа-распаде тяжелых ядер, учитывающий величину угла между направлениями распространения альфа-частицы и излучения фотона. Зависимость спектра от величины угла получена в явном аналитическом виде. Метод может быть использован для сравнительного анализа экспериментальных данных, полученных для разных величин угла.

ГАЛЬМІВНЕ ВИПРОМІНЮВАННЯ ПРИ АЛЬФА-РОЗПАДІ: КУТОВИЙ АНАЛІЗ СПЕКТРІВ

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Представлено квантово-механічний метод розрахунку спектрів гальмівного випромінювання при альфа-розпаді важких ядер, що враховує величину кута між напрямками поширення альфа-частинки і випромінювання фотона. Залежність спектра від величини кута отримано у явному аналітичному виді. Метод може бути використаний для порівняльного аналізу експериментальних даних, отриманих для різних величин кута.