ELECTRODYNAMICS IN e^- LINACS

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We report a new calculation of electron beam acceleration along linac using fundamental electrodynamics as basis. Following laws are considered: increasing of work equals multiplication of force to elementary interval; power equals work growth over time interval; force acting on charge equals charge value multiplied to electric field value. Using electrodynamic characteristic, series impedance, equaled a square of electric field value divided by power, and also taking into account laws mentioned above, an equation for the electric field radiated by the beam is calculated. A transient process of electric field radiated by beam is considered.

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1. MAIN REMARKS ON TRAVELLING WAVE ELECTRON LINAC THEORY

Considering electron bunch acceleration in travelling wave acceleration structure few remarks on classical theory describing bunch movement along acceleration structure should be given [1].

In the first, the equation stated below is of great importance in order to obtain electric field radiated by moving along DLWG charge:

$$dW = -qEdz\,, \qquad (1)$$

$$\frac{dP}{dz} = -\frac{\omega P}{v_g Q} = -2\alpha P\,, \qquad (2)$$

$$\frac{dE}{dz} = -\alpha E\,. \qquad (3)$$

Here dW is accelerated particle energy gain due travelling dz distance along waveguide axis, q is charge quantity, E is on-axis electric field amplitude, α is attenuation factor, v_g is group velocity, P is instantaneous power and Q is disk loaded waveguide (DLWG) Q-factor. It should be stated also that equations (2)-(3) described by Perry B. Wilson are correct for DLWG feeding by RF power source in steady mode. It is wrong to believe that current energy speed equals group velocity. But actually group velocity is responsible for transient mode and power source current energy speed equals bunch velocity.

A relation mentioned by Wilson is series resistance R_p which plays important part too (further R_{sh} is shunt impedance):

$$R_p = \alpha R_{sh} = \frac{\omega R_{sh}}{2 v_g Q}\,, \qquad (4)$$

$$R_p = \frac{E^2}{2P}\,. \qquad (5)$$

In the next equation minus sign should be put before multiplication of direct current and beam field:

$$\frac{dP}{dz} = I_0 E_b - 2\alpha P\,. \qquad (6)$$

It is considered that there is no power feed. Energy balance in the whole accelerating structure gives (6). E_b is beam field which resists short bunch with I_0 current. Energy gain generated by beam is power loss.

The next equation means beam generates 2 waves. The second component is in antiphase, i.e. in power source phase and attenuating as well as power source field:

$$E_b(z) = I_0 R_{sh}(1 - e^{-\alpha z})\,. \qquad (7)$$

It contradicts physics of the process. Moreover boundary conditions are failed when $z = 0$ for bridged waveguide.

It is cleared that fundamental theory of beam loading which states that charge energy loss due radiation equals half of induced voltage gives simple way of beam field calculation.

2. BEAM FIELD CALCULATION BASED ON WILSON AND RAMO THEOREMS

We consider a new approach to define relativistic electron beam field in DLWG. Relativistic electron bunch with q charge is travelling along DLWG axis with constant $\beta = v/c = 1$ velocity. It is assumed that charge velocity is equaled to the speed of light. According to Perry B. Wilson theorem, W, energy being lost by travelling bunch with q charge, equals half of charge and induced voltage U multiplication [1]:

$$W = \frac{1}{2}qU\,. \qquad (8)$$

Assume that U voltage induced on x length equals current induced by q charge and multiplied by equivalent impedance R on accelerating gap x length.

$$U = IR\,. \qquad (9)$$

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According to Simon Ramo theorem induced current equals multiplication of charge and velocity divided by gap length [2]:

$$I = \frac{qc}{x}\,. \tag{10}$$

Define R impedance on x gap length by means of series resistance R_p:

$$R = R_p x^2\,. \tag{11}$$

According to definition:

$$R_p = \frac{E^2}{2P}\,. \tag{12}$$

In the last equation E is electric field strength on DLWG axis, P is RF power in current DLWG cross-section.

Using (8)-(12) for W, U, I, R and R_s, obtain beam energy loss gain W on DLWG x length:

$$W = \frac{1}{2}qU = \frac{1}{2}q^2 cR_p x\,. \tag{13}$$

Dissipated power equals:

$$P = \frac{W}{t} = \frac{1}{2}q^2 cR_p \frac{x}{t}\,. \tag{14}$$

Within considered case beam velocity equals the speed of light, i.e. $x/t = c$

$$P = \frac{1}{2}q^2 c^2 R_p = \frac{E^2}{2R_p}\,. \tag{15}$$

Which means that:

$$.E = qcR_p \tag{16}$$

Therefore, field E radiated by beam with q charge is defined by charge, velocity and DLWG series resistance. Current statement is also correct for velocities under the speed of light $v < c$, i.e. it is correct for any velocity of accelerating beam.

$$E = qvR_p\,. \tag{17}$$

3. ELECTRON BEAM FIELD CALCULATION IN DLWG. CLASSICAL THEORY

As it is expected simple calculation could raise doubts. That is why consider classical approach by means of electrodynamics fundamentals. Calculate q charge beam field on DLWG axis. Beam is travelling with v velocity. Point charge is considered. When travelling beam interacts with decelerating system like DLWG F force will act on charge. Force equals multiplication of q charge on E beam field.

$$F = qE\,. \tag{18}$$

Instantaneous power equals differential relation of work dA divided by time interval dt, i.e.:

$$P = \frac{dA}{dt}\,. \tag{19}$$

It should be noted that force equals relation dA to dl, where dl is elementary distance on DLWG axis.

$$F = \frac{dA}{dl}\,. \tag{20}$$

(19)-(20) conclude that instantaneous power equals multiplication of force to velocity:

$$P = F\frac{dl}{dt} = Fv\,. \tag{21}$$

Substitute F from (18) to (21) and get:

$$P = qEv\,. \tag{22}$$

Define average power in DLWG $\bar{P}$ by means of E beam field and R_s series resistance:

$$\bar{P} = \frac{E^2}{2R_p}\,. \tag{23}$$

While reminding that electromagnetic field instantaneous power P and average $\bar{P}$ differ by factor 2, get the next relation:

$$P = 2\bar{P}\,, \tag{24}$$

$$P = \frac{E^2}{p}\,. \tag{25}$$

(22) and (25) infer

$$E = qR_p v\,. \tag{26}$$

It is assumed indirectly that we have point charge bunch with q charge, v velocity on TM_{010} mode in DLWG.

When $v \neq c$ (26) could be rewritten as:

$$E = qR_p \beta c\,. \tag{27}$$

In the current equation $\beta = v/c$ and when $\beta = 1$ get:

$$E = qR_p c\,. \tag{28}$$

(28) could be rewritten also as follows. Since the speed of light equals wave length λ divided by oscillation period it is:

$$c = \frac{\lambda}{T}\,, \tag{29}$$

$$\frac{q}{T} = I\,. \tag{30}$$

I is average q charge bunch current. Time between bunches is equaled to oscillation period of RF power. Substitute (29), (30) to (28). As a result beam field in relativistic case ($\beta = 1$) equals:

$$E = IR_p \lambda\,. \tag{31}$$

For $v \neq c$:

$$E = IR_p \beta\lambda\,. \tag{32}$$

(32) might be used for calculation of electron linac bunchers. (31) is usually used for calculation of electron linac acceleration sections.

Therefore, obviously equation (28) for obtaining single bunch field in travelling wave accelerating structure calculated by means of classical theory identical with (17) calculated by means of Wilson and Ramo theorems.

4. MULTIPLE BUNCHES BEAM FIELD TRANSIENT PROCESS IN DLWG

Consider radiation process for a chain of electrons with q charge, interval λ which is equaled to wave length in DLWG (Fig.1).

The first bunch radiates field:

$$E = IR_p\lambda\,. \tag{33}$$

It is taken into account that beam is relativistic ($v = c$) and interval between point charge bunches is equaled wave length λ. Using results of [3] and considering DLWG as chain of feedthrough quarter-wave cavities ($\theta = \pi/2$ mode) or third-wave cavities ($\theta = 2\pi/3$ mode) beam field E transient process is defined as:

$$E_1 = lR_p\lambda e^{-\frac{\pi t}{Q_1 T}}\,. \tag{34}$$

In the statement above t is current time, T is oscillations period and Q_1 is DLWG loaded Q-factor. While $t = 0$: $E_1 = IR_p\lambda$.

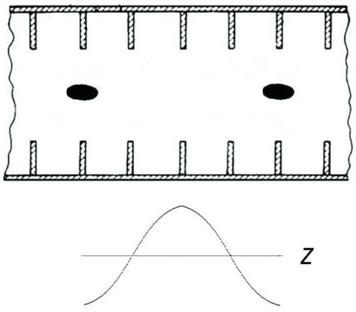

Fig.1. *The chain of bunches travelling in DLWG*

When the second bunch enters considered DLWG cross-section beam field is:

$$E_2 = IR_p\lambda + IR_p\lambda e^{-\frac{\pi}{Q_l}}\,. \tag{35}$$

And after the third bunch:

$$E_2 = IR_p\lambda + IR_p\lambda e^{-\frac{\pi}{Q_l}} + IR_p\lambda e^{-\frac{2\pi}{Q_l}}\,. \tag{36}$$

Net series is a geometric sequence. Summary field in considered cross-section is equaled:

$$E_n = \frac{IR_p\lambda\left(1 - e^{-\frac{n\pi}{Q_l}}\right)}{1 - e^{-\frac{\pi}{Q_l}}}\,. \tag{37}$$

When infinite quantity of bunches is taken into calculation following beam field is observed in DLWG with finite Q_l:

$$E_0 = \frac{IR_p\lambda}{1 - e^{-\frac{\pi}{Q_l}}}\,. \tag{38}$$

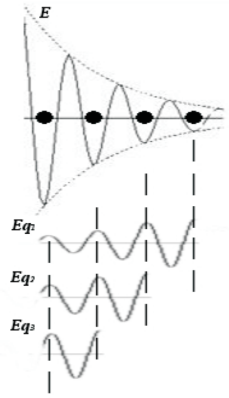

Fig.2. *Transient process for the chain of bunches beam field*

Field envelope amplitude growth is obviously characterized by Q_l size (Fig. 2). We take into account power losses in walls under T oscillation period. As it is seen from (37) maximum field is generated after travelling of n bunches in DLWG cross-section. After the bunch number n beam field decreases according to law:

$$E = \frac{IR_p\lambda\left(1 - e^{-\frac{n\pi}{Q_l}}\right)}{1 - e^{-\frac{\pi}{Q_l}}}\,. \tag{39}$$

It should be mentioned that time is counted here before the bunch number n enters considered cross-section.

5. CONCLUSIONS

As a result few remarks on classical theory for calculation of electron linac electrodynamic parameters are given within current research. Method of beam field calculation based on electrodynamics fundamentals and different from classical one is proposed. Derived way of beam field calculation is used in consideration of transient process for the chain of bunches beam field.

Moreover, this technique could be used not for DLWG only but for other electron linac accelerating structures which have their own benefits and drawbacks.

Calculation of electron beam field presented within current work allows to make more accurate calculations of some electrodynamic characteristics used in design and optimization of electron linac accelerating systems including improvement of accelerating structures design software.

Presented research has been conducted in realization of two State Contracts in Compact Accelerators Laboratory of National Research Nuclear University MEPhI which has been financed within mentioned contracts. It includes development of electron linac with $4\,MeV$ output energy (with usage of modern klystrons and magnetrons of different wave lengths) for generation of powerful bremmstrahlung in THz wavelength range and also development of linear electron accelerator with $15\,MeV$ output energy, maximum of $1\,kA$ beam current and pulse duration about $1\,ns$.

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ЭЛЕКТРОДИНАМИКА В ЛИНЕЙНЫХ УСКОРИТЕЛЯХ ЭЛЕКТРОНОВ

С. С. Проскин, В. А. Дворников, И. А. Кузьмин, И. С. Щедрин

Рассмотрен расчет ускорения сгустков в линейных ускорителях электронов, основанный на постулатах электродинамики. Перечислены основные постулаты: увеличение работы равно произведению силы на элемент пройденного расстояния; мгновенная мощность равна отношению прироста работы на отрезок времени; сила, действующая на заряд, равна величине заряда, умноженного на напряженность электрического поля. Используя электродинамическую характеристику, последовательное сопротивление круглого диафрагмированного волновода, равное отношению квадрата напряженности электрического поля к мгновенной мощности, и выше названные постулаты электродинамики, получено выражение для электрического поля излучения. Рассмотрен переходный процесс поля излучения цуга сгустков.

ЕЛЕКТРОДИНАМІКА В ЛІНІЙНИХ ПРИСКОРЮВАЧАХ ЕЛЕКТРОНІВ

С. С. Проскін, В. А. Дворніков, І. А. Кузьмін, І. С. Щедрін

Розглянуто розрахунок прискорення згустків у лінійних прискорювачах електронів, оснований на постулатах електродинаміки. Перелічені основні постулати: збільшення роботи дорівнює здобутку сили на елемент пройденої відстані; миттєва потужність дорівнює відношенню приросту роботи на відрізок часу; сила, діюча на заряд, рівна величині заряду, помноженого на напруженість электричного поля. Використовуючи електродинамічну характеристику, послідовний опір круглого діафрагмованого хвильопровода, який дорівнює відношенню квадрата напруженості електричного поля на миттєву потужність, та перелічені вище постулати електродинаміки, отриманоно вираз для електричного поля випромінювання. Розглянуто перехідний процес поля випромінювання цуга сгустків.