COUPLED HF AZIMUTHAL WAVES IN MAGNETOACTIVE WAVEGUIDE PARTIALLY FILLED BY CURRENT-CARRYING PLASMA

O.I. Girka, I.O. Girka, V.O. Girka, I.V. Pavlenko
Kharkiv National University, Svobody sq., 4, Kharkiv 61077, Ukraine
E-mail: girkai@univer.kharkov.ua

Coupled surface extraordinarily polarized waves and bulk ordinarily polarized waves are proved to propagate along azimuthal direction in the cylindrical metal waveguide that is partially filled by cold magneto-active plasma in the frequency range above upper hybrid resonance. Their interaction and linear conversion is investigated under the condition of an external magnetic field with both axial and azimuthal components application.

PACS: 52.35.-g, 52.40. Fd

1. INTRODUCTION

Capabilities of using the ring-type charged particle flows in sources of coherent radiation are intensively studied [1-4] with the goal to increase an efficiency of electronic devices utilized in the field of high frequency engineering alongside with elaborating the devices working on longitudinal charged particle beams (see, for example, [5] and references therein). In paper [6], it is proposed to use the ring-type flows for excitation of extraordinarily polarized azimuthal surface waves (ASW). For azimuthal electromagnetic oscillations, the dependence of the wave fields on coordinates and time t is chosen in the following way: $f(r)exp[i(m\vartheta - i\omega t)]$, where ϑ is azimuthal angle. Thus usage of plasma filling of waveguides as contrasted to vacuum devices allows continuous frequency controlling of electromagnetic signal in definite frequency range. The dispersion properties of the ASW propagating across an axial steady magnetic field in the cylindrical metal waveguide, partially-filled by plasma, are studied in [7]. As far as the eigen modes are excited in waveguide structures with a maximum increment, then research into the properties of eigen electromagnetic oscillations of magneto-active plasma-filled waveguides is interesting from practical point of view for plasma electronics.

Influence of steady azimuthal magnetic field $B_{0\vartheta}$, caused, for example, by electric current, which is carrying by the plasma column, for the case of the waveguide that is partially-filled by plasma, on low-frequency (LF) ASW dispersion properties is studied in [8]. Availability of an external steady magnetic field with two components: axial and azimuthal (in cylindrical coordinate system $B_0 = B_{0z} e_z + B_{0\vartheta} e_\vartheta$) does not allow presenting the set of Maxwell equations for azimuthal electromagnetic oscilaltions in the form of two independent subsystems, which describe respectively, ASW with field components E_r, E_ϑ, B_z and ordinarily polarized bulk azimuthal electromagnetic wave with field components E_z, B_r, B_ϑ. LF ASW can resonantly interact with ordinarily polarized surface wave, if the dielectric layer, which separates the plasma column from a metal wall of the chamber, is broad enough and if the external axial magnetic field is nonzero one [8]. Then the correction to the eigen frequency of the ASW, caused by the azimuthal magnetic field, becomes proportional to the first power of the $B_{0\vartheta}$.

In the present paper, the propagation of azimuthal electromagnetic waves in magneto-active metal waveguide, which is partially filled by plasma at presence of an external magnetic field with both axial and azimuthal components, is considered. The interaction of surface azimuthal waves with ordinarily polarized azimuthal bulk waves (ABW) is the object of the research in the present paper. Here the dispersion relation for azimuthal waves is derived. This relation allows taking into the account weak (as compared with the axial component) external azimuthal magnetic field and describing linear interaction between ASW and ABW. In the paper, the correction to eigen frequency of azimuthal waves caused by an external azimuthal magnetic field is determined, and its influence on spatial distribution of electromagnetic fields of the waves is studied.

2. FORMULATION OF THE PROBLEM

Let's consider cylindrical metal waveguide with ideally conductive wall and circular cross-section of radius b. It is partially filled by cold magneto-active plasma column of radius a with a radially inhomogeneous density profile. The plasma column is separated from the chambers' wall by dielectric layer with a permeability $\varepsilon_d=1$. Plasma is considered to be homogeneous along axis z and azimuthal angle ϑ (in cylindrical coordinates). Electrodynamical properties of plasma are described by permeability tensor ε_{ij}, obtained in the approach of cold magneto-active plasma [9,10]. It is assumed, that an external constant magnetic field has two components: axial B_{0z} and azimuthal $B_{0\vartheta}(r)$, respectively. We restrict our consideration by the case of small external azimuthal magnetic fields, $\beta \equiv B_{0\vartheta}/B_{0z} << 1$, which one corresponds to flowing of enough weak axial currents. The choice of such limiting case is explained, on one hand, by a simplicity of its experimental realization, and on the other hand, by the fact that it can be investigated both numerically and analytically.

Neglecting the small addenda of the order above the first one with respect to the small parameter β, from Maxwell equations we derive the following set of differential equations for axial components of the ABW electric field E_z and ASW magnetic field B_z:

$$\frac{1}{r}\frac{d}{dr} r \frac{dE_z}{dr} + \left(k_o^2 - \frac{m^2}{r^2} \right) E_z = \hat{M} \cdot B_z, \quad (1)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\frac{r}{k_н^2}\frac{\partial B_z}{\partial r} - \left[1 + \frac{m^2}{r^2 k_x^2} - \frac{m}{r}\frac{\partial}{\partial r}\left(\frac{\mu}{k_x^2} \right) \right] B_z = \frac{\dot{K} E_z}{k_x^2}. \quad (2)$$

The following nomenclature is applied here:

$$\hat{M} = -ik\beta\frac{d}{dr} - ik\beta\frac{k_o^2}{k_x^2}\left(\frac{d}{dr}+\frac{m\mu}{r}\right), \quad (3)$$

$$\hat{K} = \frac{ik}{\varepsilon_1}\left[\frac{-\beta}{r}\left(\varepsilon_1^2-\varepsilon_1\varepsilon_3-\varepsilon_2^2\right)-\beta\left(\varepsilon_1^2-\varepsilon_1\varepsilon_3-\varepsilon_2^2\right)\frac{d}{dr} - \beta\varepsilon_2\varepsilon_3\frac{m}{r} - \frac{d\left(\beta\left(\varepsilon_1^2-\varepsilon_1\varepsilon_3-\varepsilon_2^2\right)\right)}{dr} + \beta\left(\varepsilon_1^2-\varepsilon_1\varepsilon_3-\varepsilon_2^2\right)\frac{d\ln\left(\varepsilon_1^2-\varepsilon_2^2\right)}{dr}\right]. \quad (4)$$

The radial wave number of *H*-wave (that is ordinarily polarized) k_o and depth of penetration of *E*-wave (that is extraordinarily polarized) k_x^{-1} in plasma are determined through the components of permeability tensor ε_{ij} of cold plasma [9,10] as follows:

$$k_o^2=k^2\varepsilon_3, \qquad k_x^2=k^2\varepsilon_1(\mu^2-1), \quad (5)$$

where $ck=\omega$, $\mu=\varepsilon_2/\varepsilon_1$. The fields of investigated waves meet the following boundary conditions: the amplitudes of the waves' fields are finite values inside the waveguide; the tangential components of electrical and magnetic fields of the waves are continuous at the plasma-dielectric interface; the tangential components of the waves' electrical field are equal to zero on the internal surface of the metal chamber.

3. SPATIAL DISTRIBUTION OF THE WAVES' FIELDS

We derive their solutions by the method of variation of constants in the following form:

$$E_z = \left(A_1 - A_2\int_a^r r\tilde{\varphi}_o\hat{M}\psi_x dr\right)\varphi_o + \tilde{\varphi}_o A_2\int_0^r r\varphi_o\hat{M}\psi_x dr, \quad (6)$$

$$B_z = \left(A_2 - A_1\int_a^r\frac{\tilde{\psi}_x\hat{K}\varphi_o dr}{W(\psi_x,\tilde{\psi}_x)}\right)\psi_x + \tilde{\psi}_x A_1\int_0^r\frac{\psi_x\hat{K}\varphi_o dr}{W(\psi_x,\tilde{\psi}_x)}, \quad (7)$$

where $\psi_x(r)$ is the solution of the equation (2) with the zero right hand side, which is finite one at $r = 0$, $\tilde{\psi}_x(r)$ is the solution of the equation (2) with the zero right hand side, which is linearly independent from the solution $\psi_x(r)$. The Wronskian of the functions $\psi_x(r)$ and $\tilde{\psi}_x(r)$ is equal:

$$W(\psi_x,\tilde{\psi}_x) = \psi_x\, d\tilde{\psi}_x/dr - \tilde{\psi}_x\, d\psi_x/dr \propto -k_x^2/r, \quad (8)$$

where φ_o and $\tilde{\varphi}_o$ are the linearly independent from each other solutions of the equation (1) with the zero right hand side. The solution $\varphi_o(r)$ is finite one at the axis ($r=0$), and $\tilde{\varphi}_o(r)$ is a singular solution at the axis. Wronskian of the functions $\varphi_o(r)$ and $\tilde{\varphi}_o(r)$ is inversely proportional to r. Solutions (6) and (7) for the fields of azimuthal waves contain only two constants of integrating A_1 and A_2, because two other constants are determined from the boundary conditions, that fields E_z and B_z are finite values at the waveguide axis ($r=0$).

Other components of the wave fields are expressed through E_z and B_z as follows:

$$E_r = \frac{1}{k_x^2}\left(\frac{km}{r}B_z + k\beta\frac{dB_z}{dr} - i\beta k_o^2\mu E_z\right), \quad (9)$$

$$B_\vartheta = \frac{i}{k}\frac{dE_z}{dr}, \qquad B_r = \frac{mE_z}{kr},$$

$$E_\vartheta = \frac{1}{k_x^2}\left[i\frac{km\mu}{r}B_z + ik\frac{dB_z}{dr} + \beta\left(k_o^2+k_x^2\right)E_z\right]. \quad (10)$$

Note, that, as it should be for ordinarily polarized waves [10], the properties of ABW don't depend on the value of an external axial magnetic field B_{oz} and, as it is shown in [8], ordinarily polarized ASW can't propagate in the metal waveguide, which is entirely filled by cold plasma.

The radial dependence of the waves' fields in dielectric ($a < r < b$) region can be expressed through Bessel cylindrical function of the first kind $J_n(\xi)$ and Neumann function $N_n(\xi)$:

$$E_z^{(d)}(r,\vartheta) = C_1\left[J_m(\kappa r)N_m(\kappa b) - J_m(\kappa b)N_m(\kappa r)\right]\exp(im\vartheta - i\omega t), \quad (11)$$

$$B_z^{(d)}(r,\vartheta) = C_2 P_m(\kappa r)\exp(im\vartheta - i\omega t), \quad (12)$$

$$P_m(\kappa r) = J_m(\kappa r)N'_m(\kappa b) - J'_m(\kappa b)N_m(\kappa r). \quad (13)$$

Here $\kappa^2=\varepsilon_d k^2$, $C_{1,2}$ – are normalization factors, the prime denotes the derivative with respect to the argument. The expressions (11) and (13) are constructed so that tangential components of the waves' electric field are equal to zero on the metal wall interface.

4. DISPERSION RELATION

Application of boundary conditions which consist in continuity of tangential components of electrical and magnetic fields of the considered waves E_ϑ, E_z, B_ϑ and B_z on plasma - dielectric interface ($r=a$) allows to deduce the following dispersion relation:

$$D_oD_x = D(\beta). \quad (14)$$

Here $D_o=0$ is dispersion relation of ABW in the absence of an external steady azimuthal magnetic field,

$$D_o = \varphi_o\left(\frac{d\varphi_o}{dr}\right)^{-1} - \frac{1}{\kappa}\frac{J_m(\kappa a)N_m(\kappa b) - J_m(\kappa b)N_m(\kappa a)}{J'_m(\kappa a)N_m(\kappa b) - J_m(\kappa b)N'_m(\kappa a)}, \quad (15)$$

$D_x=0$ is dispersion relation of ASW in the absence of an external constant azimuthal magnetic field,

$$D_x = -\frac{k^2}{k_x^2}\left(\frac{1}{\psi_x}\frac{d\psi_x}{dr} + \frac{m\mu}{a}\right) - \kappa\frac{J'_m(\kappa a)N'_m(\kappa b) - J'_m(\kappa b)N'_m(\kappa a)}{J_m(\kappa a)N'_m(\kappa b) - J'_m(\kappa b)N_m(\kappa a)}, \quad (16)$$

$D(\beta)$ is coupling coefficient which describes interaction between ABW and ASW, its value is of the second order of smallness on β,

$$D(\beta) = \left(\frac{d\varphi_o}{dr}\right)^{-2}\frac{1}{a}\int_0^a r\varphi_o\hat{M}\psi_x dr\left[\frac{k^2W(\psi_x,\tilde{\psi}_x)}{k_x^2\psi_x^2}\int_0^a\frac{\psi_x\hat{K}\varphi_o dr}{W(\psi_x,\tilde{\psi}_x)} - i\kappa\beta\left(1+\frac{k_o^2}{k_x^2}\right)\frac{\varphi_o}{\psi_x}\right]. \quad (17)$$

Let's note, that all values included in expressions (15) - (17), which are the functions of radius r and are not written under the sign of integral, have been computed in the point $r=a$.

We derive the solution of the equation (14) in the following form: $\omega=\omega_x+\Delta\omega_x$ ($|\Delta\omega_x| << \omega_x$) and $\omega=\omega_o+\Delta\omega_o$ ($|\Delta\omega_o| <<\omega_o$), where ω_x and ω_o are eigen frequencies of the ASW and ABW, respectively, calculated in the absence of an external azimuthal magnetic field. One can found from equation (14), that the correction $\Delta\omega_x$ to the eigen frequency of ASW, which is caused by external steady azimuthal magnetic field, is as follows:

$$\Delta\omega_x = D(\beta)\left(D_o\,\partial D_x/\partial\omega\right)^{-1}_{|\omega=\omega_x} \propto B_{0\vartheta}^2. \quad (18)$$

The correction $\Delta\omega_o$ to eigen frequency of ABW can be written in the following form:

$$\Delta\omega_o = D(\beta)\left(D_x\,\partial D_o/\partial\omega\right)^{-1}_{|\omega=\omega_o} \propto B_{0\vartheta}^2. \quad (19)$$

Under the definite conditions dispersion curves of the ABW and ASW intersect: $\omega_x=\omega_o$. Under these conditions, the influence of the field $B_{0\vartheta}$ on the interaction between the ABW and ASW increases, and the correction to their eigen frequency becomes linear with respect to an external steady azimuthal magnetic field (namely, proportional to the parameter β):

$$(\Delta\omega_x)^2=(\Delta\omega_o)^2=D(\beta)\left(\partial D_x/\partial\omega\right)^{-1}_{|\omega=\omega_x}\left(\partial D_o/\partial\omega\right)^{-1}_{|\omega=\omega_o}. \quad (20)$$

5. ANALYTICAL SOLUTION OF THE DISPERSION EQUATION

We undertake the further consideration for the case of homogeneous radial density profile. Such limitation of the consideration can be justified by the following reasons. First, this model describes the case of semiconductor plasma. Second, the radial non-uniformity of the plasma density can be neglected, if plasma density varies weakly in radial layer which thickness is of the order of the depth of the waves' field penetration into the plasma. Thus, for example, in the case of gas discharge maintenance due to propagation of surface waves, the uniformity of produced plasma density is provided just at distances about penetration-depth of the operating wave into the plasma.

Let's assume also, that the plasma density value is high enough ($\Omega_e^2>>\omega_e^2$, where Ω_e and ω_e are Langmuir and electron cyclotron frequencies, respectively). Such inequality is always true for *n*-semiconductor plasma, but it can be realized also in laboratory gas plasma in the case of utilization of not too strong steady axial magnetic field. Such limiting case introduces the greatest interest for plasma technologies [11], since utilization of a strong magnetic field results in essential rising in price of a unit volume of produced plasma. Let's mark also, that the dispersion properties of ASW in magnetized, $\Omega_e^2<\omega_e^2$, cylindrical plasma waveguides are studied in [12].

In the homogeneous plasma chamber ABW exist in frequency range $\omega > \Omega_e$. Since the inequality $k_o^2 > 0$ is satisfied under this condition, then one can consider the value k_o (5) as a radial wave number of the ABW. In the case of radial homogeneous plasma the equations (1) and (2) for E_z and B_z have the form of inhomogeneous Bessel equations. The solutions of the respective homogeneous Bessel equations, which are restricted on the waveguide axis, can be expressed through cylindrical functions of imaginary and real arguments, accordingly:

$$\psi_x=I_m(k_x r),\ \varphi_o=J_m(k_o r). \quad (21)$$

The solutions, which are linearly independent from the indicated above solutions of equations (1) and (2) with the zero right hand sides, are functions of McDonald and Neumann functions:

$$\tilde{\psi}_x = K_m(k_x r),\ \tilde{\varphi}_o = N_m(k_o r). \quad (22)$$

The solutions (21) and (22) are applicable, if $k_o^2 > 0$ and $k_x^2 > 0$. In its turn, just the condition $k_x^2 > 0$ determines the frequency ranges of ASW existence [7]. Let's write them down:

$$|\omega_e|\sqrt{\Omega_i^2(\Omega_e^2+\omega_e^2)^{-1}} < \omega < |\omega_e|,$$

$$|\omega_e| < \omega < \sqrt{0{,}25\omega_e^2+\Omega_e^2} - 0.5|\omega_e|, \quad (23)$$

here Ω_i is ion plasma frequency. Analyzing the ranges (23), one can see that they are low enough, so it is natural to refer to them as to the low frequency (LF) ranges. The minimum frequency is the lower hybrid frequency and the maximum frequency is the cut-off frequency. Besides that, the condition $k_x^2 > 0$ is valid also in the narrow frequency range that is little bit higher than the upper hybrid frequency:

$$\sqrt{\omega_e^2+\Omega_e^2} < \omega < 0{,}5|\omega_e| + \sqrt{0{,}25\omega_e^2+\Omega_e^2}. \quad (24)$$

It is natural to refer to this frequency range as to high frequency (HF) range.

The analysis of value k_o^2 testifies that it has opposite signs in the ranges (23) and (24): namely, $k_o^2 < 0$ in the LF ranges (23), and it becomes positive, $k_o^2 > 0$, in the HF range (24). Thus, the resonant linear interplay between ordinarily polarized bulk waves and ASW can take place only in the HF range (24).

Let's assume, that the steady azimuthal magnetic field $B_{o\vartheta}$ depends on radius r linearly: $B_{o\vartheta}(r)= B_{o\vartheta}(a)\, r/a$. Such choice of radial dependence for the magnetic field is explained by the following reasons. If the azimuthal magnetic field is produced by an axial current of completely ionized plasma, which conductivity is determined by pair Coulomb collisions, then it means that the temperature profile of electrons is homogeneous one. (Let's note also, that the participation of electrons in an axial motion, which is connected with the current, does not result in Doppler shift of frequency due to zero value of the axial wave number.) It is necessary to point out, that in this case it is possible to calculate the integrals in the expression (17) for $D(\beta)$ in the explicit form.

The conditions of resonant linear interaction between ordinarily polarized ASW and extraordinarily polarized ASW can be provided only in the case of a wide dielectric layer between plasma and metal wall of the waveguide (when a lot of half-wavelengths can be located along the radial coordinate in this layer) for the electromagnetic oscillations propagating along the direction of electrons' rotation on Larmor orbits [8].

In the case of a narrow dielectric layer ($\Delta=(b-a)/a<1$) and homogeneous plasma, the dispersion relations for ABW (15) and ASW (16) in an axial magnetic field are essentially simplified:

$$D_o=\frac{J_m(k_o a)}{k_o J'_m(k_o a)} + (b-a)(1-0.5\Delta), \quad (25)$$

$$D_x=-\frac{k^2}{k_x^2}\left(\frac{k_x I'_m(k_x a)}{I_m(k_x a)}+\frac{m\mu}{a}\right)-$$

$$-\Delta\left[\kappa^2a^2-m^2+0.5\Delta(3m^2-\kappa^2a^2)\right]/a. \quad (26)$$

Analysis of the dispersion relation (26) $D_x=0$ for ASW in an axial magnetic field ($B_{0\vartheta}=0$) demonstrates, that in narrow waveguides ($a << |m|\delta$, where $\delta=c/\Omega_e$ is the skin-depth) HF ASW do not propagate [9]. An analytical estimation of the waveguide radius, in which one HF ASW can exist in absence of the dielectric layer, can be determined by the following inequality: $a > |m/\omega_e|c$.

The availability of a narrow dielectric gap leads to little decreasing of the HF ASW eigen frequency. In wide waveguides ($a >> |m|\delta$) its value can be approximately determined as follows:

$$\omega_x \approx \sqrt{\Omega_e^2\left(1+k_{ef}^2\right)+\omega_e^2} - m\omega_e\varepsilon_d\Delta\,(1-k_{ef}^2/\varepsilon_d). \quad (27)$$

Effective wave number $k_{ef} = |m|\delta/a$ is utilized here.

If the applied magnetic field and the sizes of the waveguide are not too large, so that the following inequality takes place:

$$a^2\omega_e^2/c^2 < j_{|m|,s}^2 - m^2, \quad (28)$$

then the eigen bulk ordinarily polarized azimuthal waves do not propagate in the HF range (24). Here $j_{|m|,s}$ is the magnitude of the s-th root of the cylindrical Bessel function of the first kind of the $|m|$-th order, i.e. $J_{|m|}(j_{|m|,s})=0$.

With increasing of the waveguide radius and/or the magnetic field value, the condition of coincidence of eigen frequencies ASW and ABW can be realized. This condition follows from the inequality (28), in which the sign of inequality is replaced by the sign of equality. In this case eigen frequency of ASW takes the form $\omega=\omega_x+\Delta\omega_x$, where ω_x is given by the equation (27), and the correction $\Delta\omega_x$ (18) to ASW frequency is equal to:

$$\Delta\omega_x \approx \beta|m|1.5Z^{-2.5}j_{|m|,s}^{-2}\Omega_e, \quad (29)$$

where $Z=\Omega_e/|\omega_e| > 1$. One can draw a conclusion from the analysis of expression (29), that the resonant influence of $B_{0\vartheta}$ on the ASW spectra strengthens at increasing an axial constant magnetic field, and also at decreasing the plasma density, azimuthal wave number and radius of the waveguide chamber. The correction (29) to the wave frequency appears to be small as compared with the main addend (27) even if $B_{0\vartheta}$ is not a small, because the smallness of this correction is provided with small multiplicands $Z^{-2.5}j_{|m|,s}^{-2}$. At the same time, it is necessary to note, that for definite values of plasma parameters, namely, if $j_{|m|,s}^2 - m^2 < a^2\omega_e^2/c^2 < j_{|m|,s+1}^2 - m^2$, i.e. in the intervals between two neighboring points of intersection for dispersion curves of ordinarily and extraordinarily polarized modes, the derivative $d\varphi_o/da$ located in the denominators in equations (15) and (17) becomes equal to zero. It means, that for such values of parameters, the influence of the steady azimuthal magnetic field on the dispersion properties of azimuthal waves becomes even more than that nearby the points of intersections for dispersion curves of ordinarily and extraordinarily polarized modes.

6. NUMERICAL RESEARCH OF DISPERSION PROPERTIES

In the Fig.1 the fine structure of spectral interaction between ASW and the first, the second and the third radial modes of ABW, which propagate with azimuthal number $m=-1$, is shown. We have introduced the dimensionless variable y: $y^4=\omega^2/\omega_e^2-1-Z^2$. The level $y=0$ corresponds to lower boundary of HF frequency range (24). The higher boundary (dashed line) of HF frequency range (24) corresponds to the level $y \approx 1.261$. The abscissa axis is represented in a logarithmic scale. At construction of dispersion curves in the Fig.1, the following values of the waveguide parameters are chosen: $\beta=0.1$, $Z=3$, $\Delta=0$.

The behavior of dispersion curves indicated by numbers 6 and 7 differs weakly from that of the curves indicated by numbers 4 and 5. That is why we do not demonstrate in Fig.1 fine structure of spectral interaction between ASW and higher radial modes of ABW. Let's note only the following common features of these curves. In process of increasing of radial number of the mode, first, splitting of the frequencies in the transformation point of the waves becomes less and less pronounced. Second, the range of values of k_{ef} between two adjacent points of the waves' transformation becomes more and more narrow. Third, at decreasing of k_{ef} the ASW frequency value becomes more closed to the lower boundary of the frequency range (24), and value of dimensionless parameter y turns to zero point, respectively.

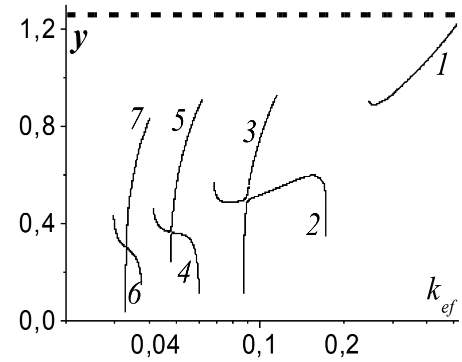

Fig.1. Fine structure of spectral interaction between ASW and first three radial modes of ABW in absence of the dielectric layer. $m=-1$, $Z=3$, $\beta=0,1$

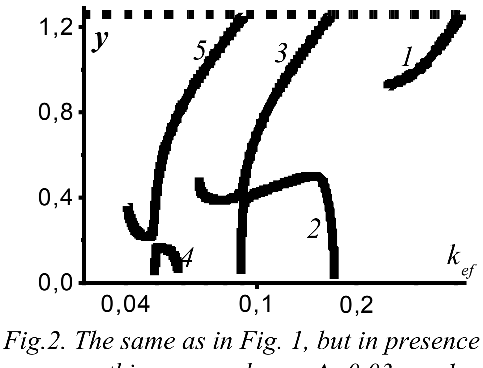

Fig.2. The same as in Fig. 1, but in presence of very thin vacuum layer, $\Delta=0,03$, $\varepsilon_d=1$

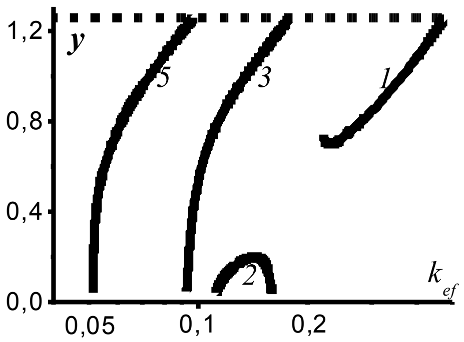

Fig.3. The same, as in a Fig.2, but in the case of more wide vacuum layer: Δ=0,1

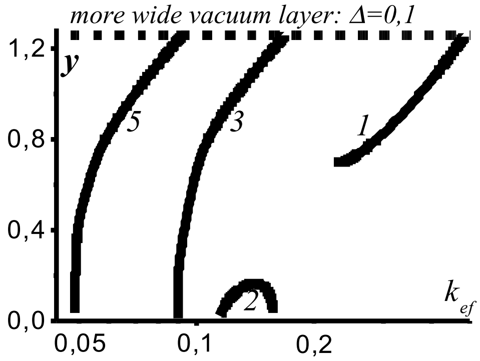

Fig.4. The same, as in the Fig.2, but in presence of the dielectric layer: Δ=0,0333, ε_d=3

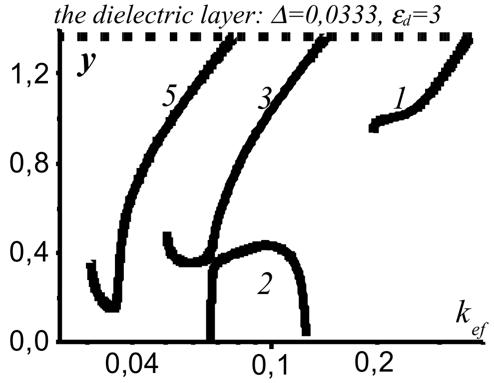

Fig.5. The same, as in Fig.2, but in the case of more weak constant axial magnetic field: Z=4

Let's point out, that the interaction between ASW and ABW results in appearance of forbidden bands in the frequency spectrum (non-passing bands of a signal), that is representative for problems on wave propagation in mediums with a periodic spatial non-uniformity (see, for example, [13]). The appearance of forbidden bands in the frequency spectrum on the graph of these waves frequency dependence on k_{ef} is accompanied by appearance of ranges of forbidden values of the effective wave number, which in fact is dimensionless inverse radius of the plasma cylinder. It is explained by the fact that for these ranges of plasma radius values ABW are not eigen modes anymore. That is why at approaching to these forbidden ranges of k_{ef} the correction to frequency of ASW (these waves are coupled with ABW) sharply increases (being calculated in the first approximation from the equation (18)). The width of these forbidden ranges of k_{ef} is larger for smaller radii of plasma a/δ.

The fine structure of spectral interaction between ASW and two radial modes of ABW in presence of very thin vacuum ($\varepsilon_d=1$) layer: Δ=0.03, and for the same values of other parameters of the waveguide is shown in Fig.2. As contrasted to Fig.1 value of ASW frequency has notably decreased.

The changes which take place at further increase of the vacuum layer width up to Δ=0.1 under the same values of other parameters of the waveguide are shown in Fig.3. As contrasted to the curves, which were introduced in Fig.2, the branch of the dispersion curve indicated by number 4 has vanished completely; further decreasing of ASW frequency takes place; and the range of k_{ef}, within which the waves can exist, moves to the right.

In Fig.4 as contrasted to Fig.3 width of the dielectric layer is chosen three times less: Δ=0.1/3, and the dielectric permeability is chosen three times larger: ε_d=3. In accordance with conceptions of General Physics, three layers with the same value of permeability exert the same influence as one layer with the tripled permeability value.

In Fig.5 as contrasted to Fig.2 steady axial magnetic field is reduced a little: Z=4. This reduction is accompanied by the narrowing of frequency range (24) and its approaching to Langmuir frequency. As the result the ABW dispersion curves move in the direction of smaller k_{ef} as contrasted to Fig.2.

Let's estimate an opportunity of experimental observation of the studied phenomenon. For example, for the concentration $n=10^{11}$ cm^{-3} the Langmuir frequency is equal to $\Omega_e \approx 1.8\times10^{10}$ sec^{-1}. To get the chosen ratio between Langmuir and electron cyclotron frequencies (Z=3) the axial magnetic field of moderate value $B_{0z}\approx$ 483 G is necessary. In accordance with the condition (28), minimum radius of plasma is necessary to meet the resonant conditions for the wave with azimuthal mode number $m=-1$. In this case it appears, that $a^2\omega_{ce}^2/c^2 = j_{1,1}^2 - 1 \approx 13.68$, whence it is possible to evaluate the radius of plasma: $a \approx 12$ cm. As it was noted above, ASW propagate only in a wide waveguides (radius of plasma $a > Z|m|\delta$). For the waveguide with radius $a \approx 12$ cm this inequality holds true, since skin-depth is $\delta \approx 1.7$ cm. To get the steady azimuthal magnetic field that is ten times weaker than the axial one, it is necessary to supply the axial current $I \approx 3$ KA in plasma. Then the splitting of the wave frequency nearby the intersection point for the dispersion curves of ASW and first radial mode of ABW is equal to 1.8×10^{7} sec^{-1}, though minimum spacing interval between the curves indicated by numbers 2 and 3 in the Fig.2 along ordinate makes only $(y_2 - y_1) \approx 0.087$.

CONCLUSIONS

In the present paper, the linear interaction of extraordinarily polarized azimuthal surface waves and ordinarily polarized azimuthal bulk waves is studied for the case of their propagation in metal waveguides which are partially-filled by plasma with an axial current. This phenomenon can be experimentally observed in wide plasma waveguides for the waves propagating with negative values of azimuthal mode number if utilized steady axial magnetic field is not small, so that $|\omega_e| \sim j_{|m|,s}c/a$. The property of surface waves to propagate only in one direction across the utilized steady magnetic field along the plasma – metal interface is well-known [9,15]. They are called as unidirectional waves and are rather useful in such plasma - filled radio devices, in which it is necessary to provide for the absence of reflected signal.

Influence of the following plasma waveguide parameters: width of the dielectric layer, permeability of the layer, value of the axial steady magnetic field, - on the fine structure of spectral interaction between ASW and the radial modes of ABW is studied numerically (see Fig.2-5). The effect of the steady azimuthal magnetic field $B_{0\vartheta}$ on the spatial distribution of the waves' fields is determined (see equations (6) and (7)) by the aid of perturbation theory with taking into account the addenda of the first order of smallness, which are proportional to $B_{0\vartheta}$.

Far from the conditions of the waves' linear resonant interaction, the correction to eigen frequency of ASW, caused by the external azimuthal magnetic field, is proportional to the square of $B_{0\vartheta}$: $\Delta\omega_x \propto (B_{0\vartheta})^2$ (whereas the corresponding frequency correction, introduced by steady axial magnetic field, is proportional to the first power of B_{0z}). In other words, transversal component of the external magnetic field (with respect to the direction of the wave propagation) affects on the dispersion properties of ASW stronger, than the longitudinal component. Such dependence of eigen frequency on the direction of an external magnetic field corresponds to the conclusions, which were obtained in [14] for the case of surface MHD waves propagation along flat plasma – metal interface.

It is shown that nearby the points, in which the dispersion curves of ASW and ABW cross (see the equation (28)), the resonant linear interaction between extraordinarily polarized surface and ordinarily polarized bulk azimuthal waves takes place. Convenient analytical expression (29) for the resonant correction to the eigen frequency caused by a steady azimuthal magnetic field is derived. In this case the correction to the eigen frequency appears to be the value of the first order of smallness: $\Delta\omega_x \propto B_{0\vartheta}$.

However, intersection points for the dispersion curves of ASW and ABW do not indicate those conditions under which a steady azimuthal magnetic field $B_{0\vartheta}$ exerts the strongest influence on the ASW dispersion properties. Let's remind that under the conditions which are determined by these points, the integer number of ABW half-wavelengths compose the plasma radius. Therefore, just as it happens in the case of linear interaction between ASW and ABW, the boundary conditions can be satisfied due to a small frequency shift because of its simultaneous strong influencing on the spatial distribution of the both interacting modes.

As the result of bulk nature of ABW fields' spatial distribution, for the definite sets of values of the waveguide parameters the conditions are realized, under which an odd number of quarters of forced ABW wavelength compose the plasma radius. These conditions are observed approximately in the middle between intersection points for dispersion curves of ABW radial modes and ASW. In this case to meet the boundary conditions the frequency correction would be so large, that the corresponding increasing of the wavelength allows withdrawing a superfluous quarter of the wavelength. This circumstance distinguishes the linear interaction between ASW and ABW qualitatively from the interaction between extraordinarily polarized and ordinarily polarized surface azimuthal waves.

REFERENCES

1. H. Saito, J.S. Wurtele // *Phys. Fluids*. 1987, v.30, №7, p.2209.
2. I.H. Kho, A.I. Lin, Liu Chen // *Phys. Fluids*. 1988, v.31, №10, p.3120.
3. R.C. Davidson, K.T. Tsang // *Lasers and Particle Beams.* 1988, v.6, pt.4, p.661.
4. Y.Y. Law // *Proc. Symp. Non - neutral plasma Physics*. Washington, 1988, p.210.
5. M. Kuzelev, A. Rukhadze. *Electrodynamics of dense electron beams in plasma.* M.: Nauka, 1990.
6. V. Girka, A. Kondratenko, S. Puzyrkov // *Plasma Physics Reports*. 2003, v.29, №2, p.112.
7. V.O. Girka, I.O. Girka, A.N. Kondratenko, V.I. Tkachenko // *Soviet Journal of Communications Technology and Electronics.* 1988, v.33, №8, p.37.
8. V.O. Girka, I.O. Girka // *Soviet Journal of Communications Technology and Electronics.* 1992, v.37, №4, p.23.
9. A. Kondratenko. *Surface and bulk waves in restricted plasma.* M.: "Energoatomizdat", 1985.
10. A.F. Alexandrov, L.S. Bogdankevich, A.A. Ruhadze. *Oscillations and waves in plasma mediums.* M.: "Moscow State University", 1990.
11. *Microwave discharges: Fundamentals and Applications,* ed. by Ferreira C.M. and Moisan M. 1993, v.302 of NATO Advanced Study Institute, Series B: Physics. (New York: Plenum)
12. I. Girka, P. Kovtun // *Technical Physics*. 1998, v.43, №12, p.1424.
13. M.B. Vinogradova, O.V. Rudenko, A.P. Suhorukov // *Wave theory*. M.: Nauka, 1979.
14. N.A. Azarenkov, K.N. Ostrikov // *Physics Reports.* 1999, v.308, p.333.

СВЯЗАННЫЕ ВЧ-АЗИМУТАЛЬНЫЕ ВОЛНЫ В МАГНИТОАКТИВНОМ ВОЛНОВОДЕ, ЧАСТИЧНО ЗАПОЛНЕННОМ ТОКОВЕДУЩЕЙ ПЛАЗМОЙ

А.И. Гирка, И.А. Гирка, В.А. Гирка, И.В. Павленко

Показано, что связанные поверхностные необыкновенно поляризованные волны и объемные обыкновенно поляризованные волны могут распространяться в азимутальном направлении в цилиндрическом металлическом волноводе, который частично заполнен холодной магнитоактивной плазмой в частотном диапазоне выше верхнего гибридного резонанса. Их взаимодействие и линейная конверсия исследованы в условиях приложения внешнего магнитного поля с аксиальной, а также азимутальной составляющими.

ЗВ'ЯЗАНІ ВЧ-АЗИМУТАЛЬНІ ХВИЛІ В МАГНІТОАКТИВНОМУ ХВИЛЕВОДІ, ЯКИЙ ЧАСТКОВО

ЗАПОВНЕНО СТРУМОВЕДУЧОЮ ПЛАЗМОЮ

О.І. Гірка, І.О. Гірка, В.О. Гірка, І.В. Павленко

Показано, що зв’язані поверхневі незвичайно поляризовані хвилі та об’ємні звичайно поляризовані хвилі можуть поширюватись в азимутальному напрямку в циліндричному металевому хвилеводі, який частково заповнено холодною магнітоактивною плазмою в частотному діапазоні вище верхнього гібридного резонансу. Їхні взаємодія та лінійна конверсія досліджені за умов прикладення зовнішнього магнітного поля з аксіальною, а також азимутальною складовими.