CALCULATION OF STRETCHING FACTOR FOR OPTICAL PULSE STRETCHER

V.P. Leshchenko, A.I. Povrozin, S.Y. Karelin
National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

A task of temporal stretching of pulses in an optical pulse stretcher of CPA-laser systems is considered. The output pulse profiles are obtained and the stretching factors are calculated on the foundation of the solution of the wave equation for a passage through an optical stretcher of the ultrashort pulses with the different profiles. The calculations for the optical stretcher of Offner triplet type had made.

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An optical stretcher of pulses that stretches pulses of a driving oscillator for the tens of femtoseconds to as rule the hundreds of picoseconds [1] is an important structural part of the CPA-laser systems. The stretching of the femtosecond pulses performs in CPA-laser systems due to the phase modulation. A long-wavelength component of the pulse passes a path shorter than short-wavelength one due to this modulation and thus an output pulse duration increases. This effect was achieved in the first stretchers, consisting of two antiparallel diffraction gratings with a telescopic system between them, introducing an additional dispersion in the entire optical system [2 - 4]. Therefore the CPA-laser systems with the stretchers, using only reflective optics, were created [5 - 9]. These stretchers include the Offner triplet [10], which in creating of the CPA-laser systems was often preferable. The calculation method presented below is applicable not only to the Offner triplet, but also for other stretcher schemes. The consideration at an example of the above type stretcher made to allow comparison of the obtaining results with published data [9].

An optical scheme of the Offner triplet includes the minimum elements (Fig. 1): a diffraction grating 1, a concave mirror M_1, a convex mirror M_2, a reflector M_3 and a return mirror П.

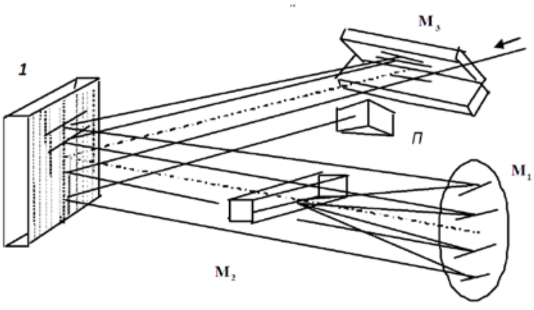

Fig. 1.

An important task in a determining of the maximum value for the stretching factor of the driving oscillator pulse is to establish the connection between the stretcher parameters and the temporal profile of this pulse. However, this problem has not considered in a literature.

The aim of this work is to create of a mathematical model for the passage of the pulse through the stretcher that is suitable for the engineering calculations of the pulse stretching factors with the different time profiles. An influence of the third and fourth orders dispersion in the stretching of the pulse not previously considered in a literature takes into account in this case. The wave equation for complex electric field amplitude *A(z,t)* of the pulse passing through the stretcher with taking into account of this influence is in the form [11]:

$$\frac{\partial A}{\partial z}-\frac{i}{2}K_2\frac{\partial^2 A}{\partial t^2}-\frac{1}{6}K_3\frac{\partial^3 A}{\partial t^3}+\frac{i}{24}K_4\frac{\partial^4 A}{\partial t^4}=0, \quad (1)$$

where $K = 2\pi/\lambda$ is a wave vector; λ is a wave length; $K_2=\frac{\partial^2 K}{\partial\omega^2}$ is a dispersion of a group velocity of the wave packet;

$$K_3=\frac{\partial^3 K}{\partial\omega^3}+\frac{3}{K}\frac{\partial^2 K}{\partial\omega^2}\frac{\partial K}{\partial\omega};$$

$$K_4=\frac{\partial^4 K}{\partial\omega^4}+\frac{4}{K}\frac{\partial^2 K}{\partial\omega^2}\frac{\partial K}{\partial\omega}+\frac{3}{K}\left(\frac{\partial^2 K}{\partial\omega^2}\right)^2;$$

ω is a frequency of a radiation.

Here and hereafter the digital codes "2" and higher ones corresponds to the order dispersion. The values of the wave vector and its derivatives had taken at the average frequency of the wave packet.

The solution of the equation (1) gives the opportunity to compare the influence on the pulse width and shape of the second, third and fourth orders dispersion. First, we consider the influence on the stretching factor of only second order of the dispersion. The solution of the equation (1) in such a representation founded by the Fourier integral transforms method [12] is on the form:

$$A(t,z)=\int_{-\infty}^{\infty}A_0(t_1)G(t-t_1,z)dt_1, \quad (2)$$

where $A_0(t_1) = A(0,t_1)$ – the electric field amplitude at the input to the stretcher, $G(t\text{-}t_1,z)$ – influence function:

$$G(t-t_1,z)=\frac{\exp\left[\frac{i(t-t_1)^2}{2K_2 z}\right]}{\sqrt{2\pi i K_2 z}}. \quad (3)$$

We assume, that at the input of the stretcher the pulse electric field amplitude has a Gaussian temporal profile:

$$A_G(t_1)=A_{0G}\exp\left[-2\ln 2\left(\frac{t_1}{\tau_{0G}}\right)^2\right], \quad (4)$$

where A_{0G} – the maximum amplitude of the electric field, the index "G" means "Gaussian", τ_{0G} – a pulse duration determined at a level $J=\frac{1}{2}|A_{0G}|^2$, that determines the specific form of the expression (4).

When the pulse passes through the stretcher, a spatial coordinate z equals its effective length L. The second order dispersion is on the form:

$$\Phi_2 = \frac{\partial^2 \Phi}{\partial \omega^2} = K_2 L, \quad (5)$$

where K_2 is given by expression [13]:

$$K_2 = \frac{\lambda_0^3}{2\pi c^2 d^2 \cos^2 \theta_0}, \quad (6)$$

where λ_0 is the average wavelength of the wave packet; θ_0 is the diffraction angle at a wavelength λ_0; d is a constant of the grating; c is a velocity of a light in a vacuo.

The expression for the effective length of the Offner triplet obtained on the basis of the theory of the optical systems [14] and is on the form:

$$L = \frac{3R - 8r + 2R\frac{r}{S} + S\left(\frac{R}{r} + 8\frac{r}{R} - 6\right)}{6 + 4r\left(\frac{1}{S} - \frac{2}{R}\right) - \frac{R}{r} - 1{,}5\frac{R}{S}}, \quad (7)$$

where R – a radius of the concave mirror, r – a radius of the convex mirror, S – distance between the center of the concave mirror and the middle of the diffraction grating.

Substituting (3), (4) and (5) in (2) with followed squaring a number gives an expression for the intensity profile of the output stretcher pulse:

$$J_{2G}(t, \Phi_2) = \frac{A_{0G}^2}{V_{0G}} \exp\left[-\frac{4\ln 2}{V_{0G}^2}\left(\frac{t}{\tau_{0G}}\right)^2\right], \quad (8)$$

where V_{0G} – stretching factor of the pulse with a Gaussian intensity distribution:

$$V_{0G} = \sqrt{1 + \left(\frac{4\ln 2\Phi_2}{\tau_{0G}^2}\right)^2}. \quad (9)$$

As it is clear from a literature the driving oscillator of the pulses with Gaussian temporal intensity profile and the profile proportional to the square of the hyperbolic secant are in practice most often in the CPA-laser systems. The electric field amplitude of this pulse is on the form:

$$A_S(t) = A_{0S} \operatorname{sech}\frac{1.763t}{\tau_{0S}}, \quad (10)$$

where A_{0S} – a peak amplitude of the electric field , τ_{0S} – the pulse duration on the level $\frac{1}{2}|A_{0S}|^2$. Index “S” means a belonging to the function "sech".

The solution of the equation (1) taking into account only the second order dispersion for the pulses with an amplitude of electric field (10) at the stretcher input is on the form:

$$A_{2S}(t, \Phi_2) = \int_{-\infty}^{\infty} \frac{A_{0S} \exp\left[\frac{i(t - t_1)^2}{2\Phi_2}\right]}{\sqrt{2\pi i \Phi_2}} \operatorname{sech}\frac{1{,}763 t_1}{\tau_{0S}} dt_1. \quad (11)$$

The squaring a number (11) gives the expression for the intensity profile of the stretcher output:

$$J_{2S}(t, \Phi_2) = \left| \int_{-\infty}^{\infty} \frac{A_{0S} \exp\left[\frac{i(t - t_1)^2}{2\Phi_2}\right]}{\sqrt{2\pi i \Phi_2}} \operatorname{sech}\frac{1{,}763 t_1}{\tau_{0S}} dt_1 \right|^2. \quad (12)$$

The stretching factor in this case is defined:

$$V_{OS} = \tau_{2S}/\tau_{0S},$$

where τ_{2S} – is determined by measuring at the level of half $J_{2S}(t,\Phi_2)$.

A consistent taking into account of the influence of the second and third orders dispersion and then the second, third and fourth orders dispersion for output pulse with the input Gaussian profile (4) gives a solution of the wave equation (1) on the form:

$$A_{3G}(t, \Phi_0, \Phi_1, \Phi_2, \Phi_3) = A_{0G} \int_{-\infty}^{\infty} \exp\{-i[\omega t + \frac{\Phi_2 \omega^2}{2} - \frac{1}{6}(\Phi_3 + \frac{3}{\Phi_0}\Phi_1\Phi_2)\omega^3] - \frac{\tau_{0G}^2 \omega^2}{8\ln 2}\} d\omega, \quad (13)$$

$$A_{4G}(t, \Phi_0, \Phi_1, \Phi_2, \Phi_3, \Phi_4) = A_{0G} \int_{-\infty}^{\infty} \exp\{-i[\omega t + \frac{\Phi_2 \omega^2}{2} - \frac{1}{6}(\Phi_3 + \frac{3}{\Phi_0}\Phi_1\Phi_2)\omega^3 + \frac{1}{24}(\Phi_4 + \frac{4}{\Phi_0}\Phi_1\Phi_3 + \frac{3}{\Phi_0}\Phi_2^2)\omega^4] - \frac{\tau_{0G}^2 \omega^2}{8\ln 2}\} d\omega, \quad (14)$$

here Φ_0 is the phase shift acquired in the stretcher. It is on the form [15]:

$$\Phi_0 = -\frac{\omega_0 L}{c} \cos\theta_0 \left(\cos\theta_0 + \cos\gamma\right), \quad (15)$$

where ω_0 – average frequency of the wave packet, θ_0 – an angle of a diffraction at the frequency ω_0, γ – an angle of incidence of the pulse to the diffraction grating.

$\Phi_1 = \frac{\partial \Phi}{\partial \omega}$ – a first derivative with respect to frequency of the phase. It is on the form [15]:

$$\Phi_1 = -\frac{L}{c}\left[1 + \cos(\gamma - \theta_0)\right], \quad (16)$$

$\Phi_3 = \frac{\partial^3 \Phi}{\partial \omega^3}$ and $\Phi_4 = \frac{\partial^4 \Phi}{\partial \omega^4}$ – the third and fourth orders dispersion, respectively. According to [13] the expressions for them are on the forms:

$$\Phi_3 = -\frac{3\lambda_0^4 L}{4\pi^2 c^3 d^2 \cos^2\theta_0}\left[1 + \frac{\lambda_0}{d}\frac{\sin\theta_0}{\cos^2\theta_0}\right], \quad (17)$$

$$\Phi_4 = \frac{3\lambda_0^5 L}{8\pi^3 c^4 d^2 \cos^2\theta_0}\{4 + 8\frac{\lambda_0}{d}\frac{\sin\theta_0}{\cos^2\theta_0} + \frac{\lambda_0^2}{d^2}\left[1 + tg^2\theta_0\left(6 + 5tg^2\theta_0\right)\right]\}. \quad (18)$$

The calculations Φ_0, Φ_1, Φ_2, Φ_3, Φ_4 performed for the Offner triplet with specific operating parameters:

R = 1024 mm, r = 512 mm, S = 774 mm, λ_0 = 800 nm, θ_0 = 20.14°, d = 1/1200 mm (for the formulas (5) - (7), taken from [8]). The calculations of the integrals (13) and (14) performed numerically.

The solutions of the equation (1) for the input pulse having a temporal profile of the input field (10) prepared similarly. The field amplitude of this pulse with taking into account the second and third orders is on the form:

$$A_{3S}(t,\Phi_0,\Phi_1,\Phi_2,\Phi_3) = A_{0S}\int_{-\infty}^{\infty}\operatorname{sech}\frac{\pi\tau_{0S}\omega}{3,526}\times \\ \times\exp\{-i[\omega t+\frac{\Phi_2\omega^2}{2}-\frac{1}{6}(\Phi_3+\frac{3}{\Phi_0}\Phi_1\Phi_2)\omega^3]\}d\omega. \quad (19)$$

And the field amplitude of the same pulse with taking into account the second, third and fourth orders is on the form:

$$A_{4CS}(t,\Phi_0,\Phi_1,\Phi_2,\Phi_3,\Phi_4) = A_{0S}\int_{-\infty}^{\infty}\operatorname{sech}\frac{\pi\tau_{0S}\omega}{3,526}\times \\ \times\exp\{-i[\omega t+\frac{\Phi_2\omega^2}{2}-\frac{1}{6}(\Phi_3+\frac{3}{\Phi_0}\Phi_1\Phi_2)\omega^3+ \quad (20) \\ +\frac{1}{24}(\Phi_4+\frac{4}{\Phi_0}\Phi_1\Phi_3+\frac{3}{\Phi_0}\Phi_2^2)\omega^4]\}d\omega.$$

Fig. 2 shows the profiles of the pulses at the stretcher output with taking into account the second order dispersion for the input Gaussian pulse (8) and the input pulses with a hyperbolic secant profile (12). Expression (12) has been calculated numerically.

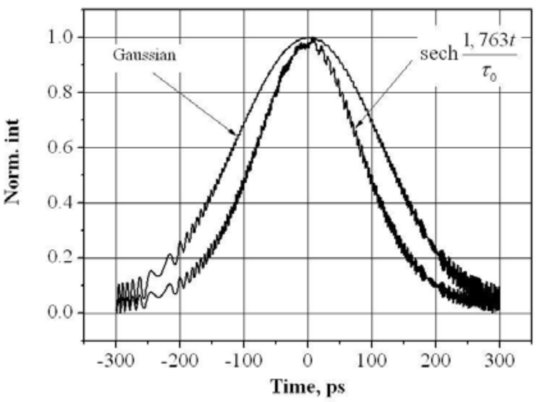

Fig. 2.

The results of calculations of Gaussian pulses profiles at the stretcher output with taking into account the third (13) and fourth (14) orders dispersion have coincided with the output Gaussian pulse profiles (8) calculated with taking into account only the second order dispersion with the accuracy 0.6%. A similar coincidence of the pulse profiles calculations with taking into account the third (19) and fourth (20) orders dispersion received in the comparison theirs with the pulse profile (12) with taking into account only the second order dispersion for the case of the hyperbolic secant. Duration of the input pulses in the calculations taken $\tau_{0G}=\tau_{0S}$ =30 fs.

As can be seen from Fig. 2 the duration of the stretched input Gaussian pulse on the half intensity level is 273 ps, and of the stretched input pulse with the profile of the hyperbolic secant is 195 ps.

According to the formula (9) and Fig. 2 the stretching factor was V_0 = 9118. This value is 9.7% lower than the experimental one, obtained under the same parameters of stretcher [9]. This discrepancy may be cause of the fact that the calculations do not take into account the phase shift caused by the spatial structure of the beam [16].

The value of the stretching factor for the input pulse with the profile of the hyperbolic secant is equal to 6500.

The following conclusions from the obtained solutions of the wave equation for the passage of the pulses with the input Gaussian profile and the input profile of the hyperbolic secant through the stretcher taking into account the second , third and fourth orders dispersion have been made:

1. The negligible influence on the results of the calculations of the profiles pulses passing through the stretcher of the third and fourth orders dispersion allows to simplify the calculation of the stretching factor. The calculation of the stretching factor may be performed with taking into account only the second order dispersion.

2. The discrepancy between the value calculation of the stretching factor for the Gaussian shape pulse and the value of the stretching factor experimentally determined in the range of 10% is acceptable and demonstrates the possible applicability of the presented methods of the calculations for the practical purposes.

3. To achieve the largest stretching factor for the same duration of the input pulses with the different profiles preferable to use a pulse with the Gaussian profile, as in this case, the stretching factor in 1.4 times larger than that of the pulse with the profile of hyperbolic secant .

4. The maximum stretching factor of the output pulse with the Gaussian profile at the input stretcher is determined analytically and may be obtained by reducing its input duration and the increasing of the second order dispersion of the stretcher. The maximum stretching factor of the pulse with temporal profile of the hyperbolic secant can be defined numerically by selection of the specific initial data.

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РАСЧЕТ КОЭФФИЦИЕНТА РАСШИРЕНИЯ ДЛЯ ОПТИЧЕСКОГО РАСШИРИТЕЛЯ ИМПУЛЬСОВ

В.П. Лещенко, А.И. Поврозин, С.Ю. Карелин

Рассмотрена задача о временном расширении импульсов в оптическом расширителе импульсов CPA-лазерной системы. На основе решения волнового уравнения для прохождения сверхкоротких импульсов различного профиля через оптический расширитель импульсов получены их выходные профили и рассчитаны коэффициенты расширения. Расчеты выполнены для оптического расширителя типа триплета Оффнера.

РОЗРАХУНОК КОЕФІЦІЄНТА РОЗШИРЕННЯ ДЛЯ ОПТИЧНОГО РОЗШИРЮВАЧА ІМПУЛЬСІВ

В.П. Лещенко, А.І. Поврозін, С.Ю. Карелін

Розглянуто задачу про часове розширення імпульсів в оптичному розширювачі імпульсів CPA-лазерної системи. На основі рішення хвильового рівняння для проходження надкоротких імпульсів різного профілю через оптичний розширювач отримано їх вихідні профілі та розраховано коефіцієнти розширення. Розрахунки виконано для оптичного розширювача типу триплету Оффнера.