TO THE THEORY OF SPATIALLY NONUNIFORM BOSE - SYSTEMS WITH BROKEN SYMMETRY

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The model of the self-consistent field for spatially nonuniform many-particle Bose-systems with broken symmetry is constructed. The self-consistent coupled equations for wave functions of the quasiparticles and of the particles of a condensate, and also system of the equations for normal and anomalous one-particle of density matrixes are deduced. The many-particle wave function is found. The thermodynamic of many-particle Bose-system on the basis of microscopic consideration in the self-consistent field model is constructed. We emphasize on the essential distinction of states with a Bose-condensate in model of ideal gas and in system of interating Bose-particles caused by obligatory presence alongside with one-particle, also of pair condensate, even at anyhow weak interaction.

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1. INTRODUCTION

The self-consistent field model frequently are used for account of atomic shells [1], structures of atomic nucleus [2], of properties of molecules and solid [3]. By the important feature of self-consistent field model is an opportunity of the description in his frameworks of stateses with lower symmetry, than symmetry initial Hamiltonian. In particular, Bogolybov generalize Hartee-Fock model on states with broken symmetry concerning phase transformation [4], that has allowed describing supercunducting state Fermi-systems with s-pairing. The self-consistent field model was used basically for theoretical research of properties Fermi systems. Only later works have appeared in wich self-consistent field model apply for study of Bose-systems [5]. The generalization semifenomenological of the Fermi-liquid approach on a case of a superfluid Bose-liquid is carried out in work [6]. The self-consistent field model bring to a conclusion about existence the one-particle excitations with energy of activation at a zero momentum, on necessity of that for many-particle Bose-systems is inverted attention in the book N.N. Bogolybov and N.N. Bogolybov (Jr.) [7]. The excitations with the sound law dispersion, predicted Landau [8], can be found from the non-stationary self-consistent equations. As the many-particle Bose-systems at low temperatures always, irrespective from the nature interparticle of interaction, passes in states with the broken phase symmetry, it is natural to use self-consistent field model, that with success used for the description of states with spontaneously broken symmetries, as initial at construction of the microscopic theory many-particle Bose-systems. In this work in the general form the self-consistent field model for Bose-systems by finite temperature is constructed. That model allow theoretically to investigate and the spatially nonuniform states. The method of consideration is analogous to that what was advanced for Fermi-systems in [9].

Is shown, that the states of system with the broken phase invariancy, even is anyhow weak interacting of Bose-particles, considerable different from ideal Bose-gas with condensate. Is constructed termodynemic of Bose-system with a condensate in self-consistent field approach. The many-particle wave function is found. To the advantages of the offered approach it is necessary to attribute that in it all the particles of system are considered on equally basis. The condensate of particles with a zero momentum arises in spatially uniform states as a consequence of the general theory.

2. THE SELF-CONSISTENT FIELD EQUATIONS

Let's consider system of Bose-particles with spin a zero interacting by means of pair potential $U(\mathbf{r},\mathbf{r}')$ Hamiltonian, which is

$$H = \int dx dx' \Psi^+(x) H(x,x') \Psi(x') + \frac{1}{2}\int dx dx' \Psi^+(x)\Psi^+(x') U(x,x') \Psi(x')\Psi(x), \quad (1)$$

where

$$H(x,x') = -\frac{\hbar^2}{2m}\Delta\,\delta(x-x') + U_0(x)\delta(x-x') - \mu\,\delta(x-x'), \quad (2)$$

$x=\{\mathbf{r}\}$, $U_0(x)$ – potential of an external field, μ - cemical potential. The field operators we define by the formula $\Psi(x) = \sum_j \varphi_j(x)\, a_j$. The Bose-operators a_j^+, a_j are creation and destruction operators of particles in a state j. The wave functions $\varphi_j(x)$ satisfy the one-particle Schroedinger equation. For transition to self-consistent field approximation initial Hamiltonian (1) represent in form of the sum two composed

$$H = H_0 + H_C, \quad (3)$$

where first composed - self-consistent Hamiltonian, including the terms not above square-law on the field operators:

$$H_0 = \int dx dx' \Big\{ \Psi^+(x)\left[H(x,x') + W(x,x')\right]\Psi(x') + \frac{1}{2}\Psi^+(x)\Delta(x,x')\Psi^+(x') + \frac{1}{2}\Psi(x')\Delta^*(x,x')\Psi(x) \Big\} + \int dx \left[F(x)\Psi^+(x) + F^*(x)\Psi(x)\right] + E_0', \quad (4)$$

and second – the correlation Hamiltonian that take into account of the correlations of the particles which have been not included in self-consistent field approximation. Hamiltonian (4), as against a case Fermi-system [9], contains also linear the members Ψ, Ψ^+. The self-consistent fields $F(x), W(x,x'), \Delta(x,x')$ and a nonoperator part E_0' in H_0 found from a condition best approximation of Hamiltonian H_0 to initial Hamiltonian H. So, in self-consistent field approach many-particle systems are characterized H_0 and the influence the correlation Hamiltonian can be taken into account under the pertrubation theory. In this work we shall be limited to the consideration of Bose-system in the framework of self-consistent field model, neglecting the effects, caused the correlation Hamiltonian.

Hamiltonian (4) is resulted in the diagonal form, if it to write down in the terms of the "displace" Bose-operators $\Phi(x), \Phi^+(x)$, so

$$\Psi(x) = \chi(x) + \Phi(x). \qquad (5)$$

The function $\chi(x)$ is selected so that in H_0 have dropped out linear on the field operators the terms. In result we receive a condition:

$$\int dx' \left[\Omega(x,x')\chi(x') + \Delta(x,x')\chi^*(x')\right] + F(x) = 0, \qquad (6)$$

where $\Omega(x,x') = H(x,x') + W(x,x')$. . Take account to the last condition self-consistent Hamiltonian by means of Bogolybov transformation

$$\Phi(x) = \sum_i \left[u_i(x)\gamma_i + v_i^*(x)\gamma_i^+\right], \qquad (7)$$

we transformated in a diagonal form:

$$H_0 = E_0 + \sum_i \varepsilon_i \gamma_i^+ \gamma_i, \qquad (8)$$

where i - complete set of quantum numbers describing of a quasiparticle state. As we see, the self-consistent field approximation naturally leads to appear of the quasiparticles in Bose-systems. The conditions of transformation from Hamiltonian (4) to (8) are the equations on coefficients of the Bogolybov transformation, which have sense of the components of the quasiparticle wave function:

$$\int dx' \left[\Omega(x,x') u_i(x') + \Delta(x,x') v_i(x')\right] = \varepsilon_i u_i(x), \qquad (9)$$

$$\int dx' \left[\Omega^*(x,x') v_i(x') + \Delta^*(x,x') u_i(x')\right] = -\varepsilon_i v_i(x). \qquad (10)$$

The self-consistent fields can be found from a condition minimum of difference self-consistent Hamiltonian H_0 from H, that gives

$$W(x,x') = U(x,x')\tilde{\rho}(x,x') + \delta(x-x')\int dx'' U(x,x'')\tilde{\rho}(x'',x''), \qquad (11)$$

$$\Delta(x,x') = U(x,x')\tau(x,x'), \qquad (12)$$

$$F(x) = -2\chi(x)\int dx' U(x,x')|\chi(x')|^2, \qquad (13)$$

where complete the one-particle density-matrixes are defined by ratio

$$\tilde{\rho}(x,x') = \langle \Psi^+(x')\Psi(x)\rangle_0 = \rho(x,x') + \chi^*(x')\chi(x), \qquad (14)$$

$$\tilde{\tau}(x,x') = \langle \Psi(x')\Psi(x)\rangle_0 = \tau(x,x') + \chi(x')\chi(x). \qquad (15)$$

In (14), (15) averaging are made with the statistical operator

$$\rho_0 = \exp\beta(\Omega_0 - H_0), \qquad \beta = 1/T, \qquad (16)$$

T – temperature. The normalization constant

$$\Omega_0 = -T\ln\left[Sp\exp(-\beta H_0)\right], \qquad (17)$$

is determined by a condition $Sp\,\rho_0 = 1$, it is meaningful the thermodynamic potential of system in a self-consistent field approximation. The overcondensate density matrixes is:

$$\rho(x,x') = \sum_i \left[u_i(x) u_i^*(x') f_i + v_i^*(x) v_i(x')(1+f_i)\right], \qquad (18)$$

$$\tau(x,x') = \sum_i \left[u_i(x) v_i^*(x') f_i + v_i^*(x) u_i(x')(1+f_i)\right], \qquad (19)$$

where

$$f_i = \langle \gamma_i^+ \gamma_i \rangle_0 = f(\varepsilon_i) = \left[\exp(\beta\varepsilon_i) - 1\right]^{-1} \qquad (20)$$

is the Bose-quasiparticle distribution function. From eq. (8) lead, that $\langle \Phi(x)\rangle_0 = \langle \Phi^+(x)\rangle_0 = 0$ and consequently

$$\chi(x) = \langle \Psi(x)\rangle_0, \qquad \chi^*(x) = \langle \Psi^+(x)\rangle_0. \qquad (21)$$

Thus, it is possible to treat $\chi(x)$ as the function which determining the density of number particles in an one-partial Bose-condensate in the self-consistent field model.

By the account of eq. (11), (12), the equations of the self-coordination (9), (10) accept a form:

$$\left[-\frac{\hbar^2}{2m}\Delta + U_0(x) - \mu + \int dx' U(x,x')\tilde{\rho}(x',x')\right] u_i(x) + \int dx' U(x,x')\left[\tilde{\rho}(x,x') u_i(x') + \tilde{\tau}(x,x') v_i(x')\right] = \varepsilon_i u_i(x), \qquad (22)$$

$$\left[-\frac{\hbar^2}{2m}\Delta + U_0(x) - \mu + \int dx' U(x,x')\tilde{\rho}(x',x')\right] v_i(x) + \int dx' U(x,x')\left[\tilde{\rho}^*(x,x') v_i(x') + \tilde{\tau}^*(x,x') u_i(x')\right] = -\varepsilon_i v_i(x) \qquad (23)$$

Besides the equations (22), (23) is necessary to receive some one equation, as uncertain Bose-condensate function. From (6) and (13) we find

$$\left\{-\frac{\hbar^2}{2m}\Delta + U_0(x) - \mu + \int dx' U(x,x')\left[\tilde{\rho}(x',x') - 2|\chi(x')|^2\right]\right\}\chi(x) + \int dx' U(x,x')\left[\tilde{\rho}(x,x')\chi(x') + \tilde{\tau}(x,x')\chi^*(x')\right] = 0. \qquad (24)$$

The equation (24), together with (22), (23) and (20) completely describes system of many Bose-particles in the self-consistent approximation. This system of the equations has three type of the solutions:

I) $\chi(x) = v_i(x) = 0, \quad u_i(x) \neq 0;$

II) $\chi(x) = 0, \quad v_i(x) \neq 0, \quad u_i(x) \neq 0;$

III) $\chi(x) \neq 0, \quad v_i(x) \neq 0, \quad u_i(x) \neq 0.$

The first type of the solutions (I) is described stateses with not broken symmetry to phase transformation

$$\Psi(x) \to \Psi(x)e^{i\xi} \quad (\xi \text{ - arbitrary phase}). \qquad (25)$$

The system does not contain in this "normal" state neither one-particle or pair condensates and has no property of superfluidity. The second type of the solutions (II) describes stateses with the broken concerning transformation (25) symmetry in the consequence appearance of the pair condencate, which is analogous the pair condensate in superfluid Fermi-systems [9]. In this case Bose-system has property of the superfluidity. The superfluidity of Bose-system, cause by pair correlations was investigated in works [10,11]. The solutions such as III describe the superfluid stateses with the broken phase symmetry containing as one-partial and pair Bose-condensates. Let's pay attention that there are absent the solutions, in which

$$\chi(x) \neq 0, \quad v_i(x) = 0, \quad u_i(x) \neq 0. \tag{26}$$

Such solution would respond a case of the ideal Bose-gas below than point of the Bose-transition, in which there is a Bose-condensate and of the overcondensate particles. Thus, the system of noninteracting particles with a Bose-condensate and system of interacting (even ahyhow is weak) Bose-particles with the broken phase symmetry are two essential various systems. The application of the ideal gas model with a condensate as base, lead to the difficulties at construction of the consecutive theory many-particle of Bose-systems with broken symmetries. It is connected with that wtat it is impossible to describe in the ideal gas model the pair correlations that always existing in the superfluid systems of the interacting particles and playing a not less essential role, than one-particle Bose-condensate.

3. THE TERMODYNAMIC PROPERTIES

The complete energy of a system of the particles in self-consistent field approximation can be submitted as the sum of three contributions $E = E_1 + E_2 + E_3$, where E_1 –energy is determined by overcondensating particles, E_2 –energy of the condensating particles, E_3 – energy of the"interaction" condensating and overcondensating of the particles. The first contribution can be written down as the sum

$$E_1 = T^{(1)} + U_E^{(1)} + U_D^{(1)} + U_{EX}^{(1)} + U_C^{(1)},$$

where: $T^{(1)} = -\frac{\hbar^2}{2m}\int dx dx' \delta(x - x')\Delta\rho(x,x')$ – kinetic energy, $U_E^{(1)} = \int dx U_0(x) n_Q(x)$ – energy in external a field, $U_D^{(1)} = \frac{1}{2}\int dx dx' U(x,x') n_Q(x) n_Q(x')$ – energy direct of the interaction, $U_{EX}^{(1)} = \frac{1}{2}\int dx dx' U(x,x')|\rho(x,x')|^2$ – energy of the exchange interaction, $U_C = \frac{1}{2}\int dx dx' U(x,x')|\tau(x,x')|^2$ – energy of the pair condensate . Here $n_Q(x) = \rho(x,x)$ – the density of the number overcondensate particles. A condensate part of energy can be written as the sum

$$E_2 = T^{(2)} + U_E^{(2)} + U_D^{(2)},$$

where $T^{(2)} = -\frac{\hbar^2}{4m}\int dx\left[\chi^*(x)\Delta\chi(x) + \chi(x)\Delta\chi^*(x)\right]$ – kinetic energy of a condensate, $U_E^{(2)} = \int dx U_0(x)|\chi(x)|^2$ – energy of the one-particle condensate in the external field, $U_D^{(2)} = \frac{1}{2}\int dx dx' U(x,x')|\chi(x)|^2|\chi(x')|^2$ – energy of the interaction of the condensate particles. The third contribution to the complete energy is determined by interaction overcondensate of the particles and the condensate:

$$E_3 = \int dx dx' U(x,x')\Big[\rho(x,x')\chi^*(x)\chi(x') + n_Q(x)|\chi(x')|^2 + \frac{1}{2}\tau(x,x')\chi^*(x)\chi^*(x') + \frac{1}{2}\tau^*(x,x')\chi(x)\chi(x')\Big] \tag{27}$$

The thermodynamic potential of the system of Bose-particles in self-consistent field approximation shall present as:

$$\Omega_0 = -\left(U_D^{(1)} + U_{EX}^{(1)} + U_C^{(1)} + U_D^{(2)}\right) - \sum_i \varepsilon_i \int dx|v_i(x)|^2 - \int dx dx' U(x,x')\Big[\rho(x,x')\chi(x)\chi^*(x') + n_Q(x)|\chi(x')|^2 + \frac{1}{2}\tau^*(x,x')\chi(x)\chi(x') + \frac{1}{2}\tau(x,x')\chi^*(x)\chi^*(x')\Big] + T\sum_i \ln\left(1 - e^{-\beta\varepsilon_i}\right). \tag{28}$$

It is possible to show, that the variation of the thermodynamic potential which determined by the formula (17), is equal average on the self-consistent state from a variation of the Hamiltonian:

$$\delta\Omega_0 = \langle\delta H_0\rangle_0. \tag{29}$$

Having expressed self-consistent Hamiltonian through $\chi(x), \rho(x,x'), \tau(x,x')$ and varying it with the account (29), we receive:

$$\frac{\delta\Omega_0}{\delta\chi^*(x)} = \left\langle\frac{\delta H_0}{\delta\chi^*(x)}\right\rangle_0 = \frac{\delta\Omega_0}{\delta\rho(x,x')} = \left\langle\frac{\delta H_0}{\delta\rho(x,x')}\right\rangle_0 = = \frac{\delta\Omega_0}{\delta\tau^*(x,x')} = \left\langle\frac{\delta H_0}{\delta\tau^*(x,x')}\right\rangle_0 = 0. \tag{30}$$

Thus, the connections of fields $F(x), W(x,x'), \Delta(x,x')$ with the condensate wave function $\chi(x)$ and the one-particle density matrixes $\rho(x,x'), \tau(x,x')$ (11), (12), (13), established with help the variational principe leads to extremeness of thermodynamic potential concerning its variation on $\delta\chi, \delta\rho, \delta\tau$.

From the equations (18), (19), (22), (23) follows system of the equations for a finding one-particle of the density matrixes:

$$-\frac{\hbar^2}{2m}(\Delta - \Delta')\tilde{\rho}(x,x') + [U_0(x) - U_0(x')]\tilde{\rho}(x,x') + \int dx''[U(x,x'') - U(x',x'')]\left[\tilde{\rho}(x,x'')\tilde{\rho}(x'',x') + \tilde{\rho}(x,x')\tilde{\rho}(x'',x'') + \tilde{\tau}(x,x'')\tilde{\tau}^*(x'',x') - 2\chi(x)\chi^*(x')|\chi(x'')|^2\right] = 0, \tag{31}$$

$$-\frac{\hbar^2}{2m}(\Delta + \Delta')\tilde{\tau}(x,x') + [U_0(x) + U_0(x') + U(x,x') - 2\mu]\tilde{\tau}(x,x') + \int dx''[U(x,x'') + U(x',x'')]\left[\tilde{\rho}(x,x'')\tilde{\tau}(x'',x') + \tilde{\rho}(x'',x'')\tilde{\tau}(x,x') + \tilde{\rho}(x',x'')\tilde{\tau}(x'',x) - 2\chi(x)\chi(x')|\chi(x'')|^2\right] = 0. \tag{32}$$

To Eq. (31), (32) it is necessary to attach and the equation (24). The correlation Hamiltonian $H_C = H - H_0$ is expressed through of the overcondensate density matrixes and can be written compactly through normal products of the operators. The perturbation theory can be advanced for Bose-systems, is similar, how it is realized for Fermi-systems [9].

4. SPATIALLY UNIFORM STATE

Let's take advantage of the received equations for the analysis of spatially uniform system. It is consider this important special case. In the spatially uniform systems of the states of the particles is characterized by their momentum $i = k \equiv \{\mathbf{k}\}$ and the wave functions look like plane waves. Let's consider the short-range interparticle potential: $U(x,x') = U_0 \delta(x - x')$. In normal Bose-system (the values with a stroke) the quasiparticle wave functions, the excitation spectra and the distribution function look like:

$$u'_k(x) = \frac{1}{\sqrt{V}} e^{-kx}, \qquad \varepsilon'_k = \frac{\hbar^2 k^2}{2m} - \tilde{\mu}',$$

$$f'_k = \left(e^{\beta \varepsilon'_k} - 1 \right)^{-1}, \tag{33}$$

where $\tilde{\mu}' = \mu' - 2U_0 n'$ – the effective chemical potential, V – volume occupied by a system. The connection chemical potential with the density of number particles is determined by the formula conterminous to the formula for the ideal Bose-gas higher of a point condensation, if in last to replace chemical potential μ' on effective chemical potential $\tilde{\mu}'$. The condition of Bose-condensation is $\mu' = \mu'_0 = 2U_0 n$. Temperature of the Bose-condensation is defined the same formula, as well as in a case of ideal Bose-gas. Below point of the Bose - condensation the self-consistent equations (26), (27), (29) suppose the solutions of a kind of the plane waves:

$$u_k(x) = \frac{u_k}{\sqrt{V}} e^{-ikx}, \quad v_k(x) = \frac{v_k}{\sqrt{V}} e^{-ikx}, \quad \chi = const. \tag{34}$$

Factors u_k, v_k, agrees (9), (10), satisfy of the algebraic equations system

$$\Delta^* u_k + \left(\frac{\hbar^2 k^2}{2m} - \mu + 2U_0 n + \varepsilon_k \right) v_k = 0, \tag{35}$$

$$\left(\frac{\hbar^2 k^2}{2m} - \mu + 2U_0 n - \varepsilon_k \right) u_k + \Delta v_k = 0, \tag{36}$$

where $\Delta = U_0 \tilde{\tau}(x,x)$, and $n = \tilde{\rho}(x,x)$ – complete density of number particles. In view of a condition normalization $|u_k|^2 - |v_k|^2 = 1$ is received:

$$|u_k|^2 = \frac{1}{2}\left(\frac{\xi_k}{\varepsilon_k} + 1 \right), \; |v_k|^2 = \frac{1}{2}\left(\frac{\xi_k}{\varepsilon_k} - 1 \right), \; u_k v_k^* = -\frac{\Delta}{2\varepsilon_k}, \tag{37}$$

where $\xi_k = \left(\hbar^2 k^2 / 2m \right) - \mu + 2U_0 n$. From (35), (36) the energy of quasiparticles follows

$$\varepsilon_k = \sqrt{\xi_k^2 - |\Delta|^2}, \tag{38}$$

and from (24) establishing connection of the chemical potential with the condensate wave function :

$$\left[-\mu + 2U_0 n_Q \right] \chi + \Delta \chi^* = 0. \tag{39}$$

The complete density of number particles can be presentd as $n = |\chi|^2 + n_q + n_p$, where first composed is the density of number particles in a Bose-condensate, second $n_q = V^{-1} \sum_k f_k$ – density of number particles that forms the quasiparticle excitations, and third $n_p = (2V)^{-1} \sum_k (\xi_k / \varepsilon_k - 1)(1 + 2f_k)$ – the density of number particles, that Cooper-pair is correlated. Thus, density of number of particles which are not included in an one-particle condensate is $n_Q = n_q + n_p$. The equation, determining Δ, looks like:

$$\Delta = \frac{U_0 \chi^2}{1 + U_0 J} = \Theta \chi^2, \tag{40}$$

where

$$J = \frac{1}{2V} \sum_k \frac{1 + 2f_k}{\varepsilon_k}, \qquad \Theta = \frac{U_0}{1 + U_0 J}. \tag{41}$$

In the states with a Bose-condensate the chemical potential, agrees (39), is determined by the formula

$$\mu = 2U_0 n_Q + \Theta |\chi|^2. \tag{42}$$

Substituting (40) and (42) in (38), we come to a final ratio determining the quasiparticle energy

$$\varepsilon_k = \sqrt{\left(\frac{\hbar^2 k^2}{2m} + 2(U_0 - \Theta)|\chi|^2 \right)\left(\frac{\hbar^2 k^2}{2m} + 2U_0 |\chi|^2 \right)}. \tag{43}$$

As see the quasiparticle energy at $\mathbf{k} = 0$ does not turn into zero, and accepts finite value

$$\varepsilon_0 = \frac{2U_0 |\chi|}{\sqrt{V}} \left| \sum_{k \neq 0} \langle a_k a_{-k} \rangle \right|^{1/2}. \tag{44}$$

i.e. the spectrum has a energy gap at $\mathbf{k} = 0$. It is obvious, that the energy spectrum is stable at pushing away between particles $U_0 > 0$ (positive scatterind length). The energy (44) has clear physical sense, namely, is that minimal energy, which is necessary for spending to pull out a particle from a condensate and by that to create new quasiparticle. It is quite natural, that in case of a Bose-condensate of the interacting particles this energy has final value. It is possible to establish a ratio

$$\frac{1}{V} \left| \sum_{k \neq 0} \langle a_k a_{-k} \rangle \right| = |\chi|^2 \frac{JU_0}{1 + JU_0}, \tag{45}$$

which determines the connection between the densities of an one-particle condensate and pairing condensate. So, the gap in a quasiparticle spectrum is defined the constant of the interparticle interaction, the density of number particles in condensate and the anomalous pairing averages, that describing pair correlations in the interacting system of Bose-particles with the broken phase symmetry. The discussion of the solutions having a gap in a quasipartial spectrum, also contains in work [6]. The distribution function of quasiparticles, both are higher and lower the temperatures of Bose-transition, has in a point $\mathbf{k} = 0$ finite magnitude, except for temperature of condensation T_0, where the distribution function diverge at $\mathbf{k} \to 0$ as $f_k \to k^{-2}$.

For square-law self-consistent Hamiltonian (8), the own vectors of stateses can be found. For this purpose it is convenient pass to new Hamiltonian, connected with initial one unitary transformation. New Hamiltonian $\tilde{H}_0 = U^+ H_0 U$, has same own of the value as H_0, but these values correspond new own vectors $|\tilde{I}\rangle = U^+ |I\rangle$. By means of two successive unitary transformations U_1 and U_2 we shall pass from H_0 to

$$\tilde{H}_0 = U_2^+ U_1^+ H_0 U_1 U_2 = E_0 + \sum_k \varepsilon_k a_k^+ a_k. \tag{46}$$

The unitary transformation

$$U_1 = e^{\left(\chi a_0^+ - \chi^* a_0 \right)} \tag{47}$$

eliminates the linear on the particle operators a_0^+, a_0 terms and the unitary transformation

$$U_2 = e^{\frac{1}{2}\sum_k \left(\psi_k a_k^+ a_{-k}^+ - \psi_{\cdot k} a_k a_{-k} \right)}, \quad \psi_k = \psi_{-k} \tag{48}$$

eliminates the square-law terms, which are not invariant concerning the phase transformations. The parameters χ, ψ_k in (47), (48) are determined by equations

$$F = -2\sqrt{V} U_0 |\chi|^2 \chi,$$
$$u_k = ch|\psi_k|, \quad v_k^* = \frac{\psi_k}{|\psi_k|} sh|\psi_k|. \tag{49}$$

To each own vector of the Schreodinger equation with Hamiltonian (46) corresponds an own vector of the Schreodinger equation with Hamiltonian H_0. The vacuum vector of the self-consistent field Hamiltonian is received in result the action of the operator $U_1 U_2$ on a vacuum vector of particles:

$$|0_q\rangle = e^{V\Lambda_0 \chi^{*2} - \frac{1}{2}\left(\Gamma_0 + V|\chi|^2\right)} e^{\sqrt{V}\left(\chi - 2\Lambda_0 \chi^*\right) a_0^+} \times$$
$$\times e^{\Lambda_0 a_0^{+2}} \prod_{k \neq -k \neq 0} e^{-\Gamma_k} e^{\Lambda_k a_k^+ a_{-k}^+} |0\rangle. \tag{50}$$

In eq. (50) the designations are used:

$$\Lambda_0 = \frac{\psi_0}{2|\psi_0|} th|\psi_0|, \quad \Gamma_0 = \ln ch|\psi_0|,$$

$$\Lambda_k = \frac{\psi_k}{|\psi_k|} th|\psi_k|, \quad \Gamma_k = \ln ch|\psi_k| .$$

5. CONCLUSION

The statical self-consistent field model describes the contribution in the thermodynamic of the multiparticle system of the one-particle overcondensate excitftions and of the condensate stateses. For Fermi-systems at low temperature this contribution is determining. Therefore theory of Fermi-systems based on the one-particle description, is suitable in this case for research of the real systems. For Bose-systems it not so, as here with downturn of temperature the contribution of one-particle exitations in the thermodynamic falls, and the contribution collective exitations are grows. The realistic theory of many-partile Bose-systems should take into account alongside with one-particie exitations, as well collective excitations. Though the static self-consistent field model not satisfy to this requirement, his theoretical study is important for several reasons. First, one allows better to understand structure of the states of Bose-system with broken by the phase invariancy, in particular demonstrate essential difference of such states from a state of ideal Bose-gas with a condensate. Secondly, allows finding the contribution of one-particle degrees of freedom to the observable characteristics of a system. Thirdly, the offered model serves natural initial approximation for construction the quantum field perturbation theory and the diagram techniques for Bose-systems with spontaneously broken simmetries, similar by that is advanced for Fermi-systems in work [9].

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REFERENCES

1. D. Hartree. *The calculation of atomic structures.* M.: “Izdatel’stvo inostrannoy literatury”, 1960, 272 p.
2. B.I. Barts, Yu.L. Bolotin, E.V. Inopin, V.Yu. Gonchar. *A Hartree-Fock method in the theory of a nucleus.* Kiev: “Naukova dumka”, 1982, 208 p.
3. J.C. Slater. *The self-consistent field for molecules and solids.* M.: “Mir’, 1978, 664 p.
4. N.N. Bogolybov. *Methods of the self-consistent field for molecules and solids* About one vary principle in many-body problem // *DAN USSR*. 1958, v. 119, p. 244-246.
5. A. Griffin. Conserving and gapless approximations for an inhomogeneous Bose gas at finite temperatures // *Phys. Rev. B.* 1996-II, v. 53. p. 9341-9347.
6. V.V. Krasilnikov, S.V. Peletminskii. On theory superfluid Bose-liquid // *Fiz. Elem. Chastits At. Yadra*. 1993, v.24, p. 463-511.
7. N.N. Bogolybov, N.N. Bogolybov (Jr.). *Introduction in the quantum statistical mechanics*. M., 1984, 384 p.
8. L.D. Landau. On theory of superfluidity helium II. // *J. Phys. USSR.* 1947, v. 11, p. 91-93.
9. Yu.M. Poluektov. Quantum field description of the many-particle Fermi-systems with broken symmetries // *The Journal of Kharkiv National University. Phys. series " Nuclei, particles, fields".* 2001. №522, v. 3 (14), p. 3-16.
10. M. Girardeau, R. Arnowitt. Theory of many-boson system: pair theory // *Phys. Rev. B*. 1959, v.113, p. 755-761.
11. P.S. Kondratenko.To the question about “even” mechanism of the superfluidity in bose-system // *Theor. Math. Phys.* 1975, v. 22, p. 278.
12. N.N. Bogolybov. On the theory of superfluidity. // *Izv. AN USSR, Phys. series*. 1947, v. 11. p. 77-90.
13. Yu.M. Poluektov, L.V. Karnatsevich. *Energy of excitation in liquid He^4 and their display in a spectrum of scattering of neutrons*. Kharkov: Prepr. KPTI, 2001-1, 2001, 56 p.
14. Yu.M. Poluektov. On the self-consistent description of collective states of the conduction electrons in crystals // *Fiz. Nizk. Temp.* 1996, v. 22, p. 402-419.
15. L.D. Landau. Theory of Fermi-liquid // *JETF*. 1956, v. 30, p. 1056-1064.
16. Yu.M. Poluektov. On the Hartree-Fock approximation taking into account the space symmetry of crystals // *Fiz. Nizk. Temp*. 1996, v. 21, p. 300-309.

17. V.V. Krasilnikov, S.V. Peletminskii, A.A. Rozhkov, A.A. Yatsenko. On theory superfluid Fermi-liquid // *Fiz. Elem. Chastits At. Yadra.* 1988, v. 19, p. 1440-1466.