PARAMAGNETIC RESONANCE
IN ELECTRON-IMPURITY SYSTEMS

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The equation for the magnetization is obtained on the basis of the kinetic equation for an isotropic distribution function of electrons scattering on massive impurity centers in the presence of magnetic and electric fields. The analytical solution of the Cauchy problem for a given initial distribution of the magnetization under conditions of paramagnetic resonance is obtained. The estimated dynamic frequency shift of the forced precession has nonlocal and nonlinear dependence on the nonuniform distribution of the initial magnetization. The dynamic frequency shift of the free precession has only nonlocal character. Time and space dependence of the internal field is obtained. All results are expressed in terms of the initial distribution of the magnetization without specifying its functional form and in terms of the propagation function. These results may be used for analysis of spin diffusion in natural and manmade materials and also in magnetometry.

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1. INTRODUCTION

The system of electrons, interacting among themselves and with motionless potential impurity centres randomly distributed in uniform external fields is described by a distribution function f, obeying the kinetic equation [1,2]

$$\frac{\partial}{\partial t} f + i[w, f]_- + \mathbf{v}\frac{\partial}{\partial \mathbf{x}} f + q\mathbf{E}\frac{\partial}{\partial \mathbf{p}} f +$$
$$\frac{q}{c}[\mathbf{v}, \mathbf{B}]\frac{\partial}{\partial \mathbf{p}} f = Lf + L^{ee} f \qquad (1)$$

where $f \equiv f_{\mathbf{p}}(\mathbf{x}, t)$ is the distribution function of the electrons, which is a matrix in electron spin space; q and $\mathbf{v} = \frac{\partial e_{\mathbf{p}}}{\partial \mathbf{p}}$ are the electron charge and velocity, respectively; L and L^{ee} are the electron-impurity and electron-electron collision integrals; $\mathbf{B}$ is the magnetic field, $\mathbf{E}$ is the static electric field; $w = -\mu_0 \vec{\sigma}\mathbf{B}$ (μ_0 is the Bohr magneton, $\vec{\sigma}$ are the Pauli matrices). We assume that massive charged impurities whose kinetics is not considered here form neutralizing electrical background.

We shall define the distribution function of electrons on energy e [3]

$$n(e, \mathbf{x}, t) = \langle f \rangle = \frac{1}{\rho(e)}\int dV_{\mathbf{p}} f_{\mathbf{p}}(\mathbf{x}, t)\delta(e - e_{\mathbf{p}}),$$

$$dV_{\mathbf{p}} = \frac{d^3 p}{(2\pi)^3}, \qquad (2)$$

where $\rho(e) = \int dV_{\mathbf{p}}\delta(e - e_{\mathbf{p}})$ is the electron density of states, and the brackets mean averaging defined by the formula (2). It follows from Eq. (2), that $\langle \overline{L}f \rangle = 0$, where

$$\overline{L} = L + L'(\mathbf{B}), \quad L'(\mathbf{B}) = -\frac{q}{c}[\mathbf{v}, \mathbf{B}]\frac{\partial}{\partial \mathbf{p}}. \qquad (3)$$

Indeed, the electron-impurity collision integral has the form

$$(Lf)(\mathbf{p}) = 2\pi N\int dV_{\mathbf{p}'} w(\mathbf{p}, \mathbf{p}')\delta(e_{\mathbf{p}} - e_{\mathbf{p}'})(f_{\mathbf{p}'} - f_{\mathbf{p}}),$$

and hence $\langle Lf \rangle = 0$. Here N is the impurity density, $w(\mathbf{p}, \mathbf{p}')$ is the probability per unit time of electron scattering on the impurity centre. As $\varepsilon_{ikl}\frac{\partial v_k}{\partial p_i} = \varepsilon_{ikl}\frac{\partial^2 e_{\mathbf{p}}}{\partial p_i \partial p_k} = 0$, hence the mean

$$\left\langle [\mathbf{v}, \mathbf{B}]\frac{\partial}{\partial \mathbf{p}} f \right\rangle = \varepsilon_{ikl}\frac{1}{\rho(e)}\int dV_{\mathbf{p}} v_k B_l(\frac{\partial}{\partial p_i} f)\delta(e - e_{\mathbf{p}}),$$

and after integration by parts we have $\langle \overline{L}f \rangle = 0$.

The operator $\overline{L}$ has property

$$\overline{L}(\mathbf{B}) = \overline{L}^+(-\mathbf{B}), \qquad (4)$$

where $+$ means the conjugate operation, defined by the formula $(x, y) \equiv \langle x, y \rangle$. By virtue of the definition of the operators L and L' we have $(Lx, y) = (x, Ly)$, $(L'(\mathbf{B})x, y) = (x, L'(-\mathbf{B})y)$, i. e. Eq. (4) is valid.

As the result of averaging Eq. (1) we obtain the equation for distribution function $n(e, \mathbf{x}, t)$:

$$\frac{\partial}{\partial t} n + i[w, n]_- = \langle L^{ee} f \rangle - \frac{\partial}{\partial x_k} j_k -$$
$$qE_k \frac{1}{\rho(e)}\frac{\partial}{\partial e}(\rho(e)j_k) \qquad (5)$$

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To close this equation one has to express the current j_k in terms of n. It can be done, if the frequency of electron-impurity collisions τ_{eimp}^{-1} is much greater than the frequencies of electron-electron collisions τ_{ee}^{-1}, and if the times t large in comparison with the corresponding relaxation time τ_{eimp}, the electron distribution function becomes some functional of n, e. i., the electron distribution function becomes independent of the electron momentum direction due to the collisions of electrons with impurities. On this basis it is possible to show, that in the linear approximation with respect to the gradients and on electric field the diffusion current is [4]

$$j_k = -D_{ki}(\mathbf{B})\left(\frac{\partial}{\partial x_i}n + qE_i\frac{\partial}{\partial e}n\right), \tag{6}$$

where $D_{ki}(\mathbf{B}) = \left\langle \overline{L}^{-1}v_k, v_i \right\rangle$ is the diffusion coefficient of electrons in a magnetic field having the property, $D_{ki}(\mathbf{B}) = D_{ik}(-\mathbf{B})$, which follows from (4).

In case of an isotropic electron dispersion relation $e_\mathbf{p} = e_{|\mathbf{p}|}$ we get

$$D_{ki} = d\left(\delta_{ki} + b_k b_i \omega_c^2 + \varepsilon_{ikl}b_i\omega_c\right),$$
$$d = \frac{\tau_{eimp}\mathbf{v}^2}{3(1+\omega_c^2)}, \mathbf{b} = \frac{\mathbf{B}}{|\mathbf{B}|}, \omega_c = \tau_{eimp}\Omega_c \tag{7}$$

The cyclotron frequency Ω_c is equal to $\Omega_c = \frac{q|\mathbf{v}|B}{c|\mathbf{p}|}$,

$$\tau_{eimp}^{-1} \equiv 2\pi N\int dV_{\mathbf{p}'}w(\mathbf{p},\mathbf{p}')\delta\left(e_\mathbf{p} - e_{\mathbf{p}'}\right)\left(1 - \frac{\mathbf{pp'}}{|\mathbf{p}\|\mathbf{p'}|}\right). \tag{8}$$

Equation (5) and (6) determine a closed equation for the distribution function $n(e,\mathbf{x},t)$, which is isotropic with respect to the moments [4].

2. MACROSCOPIC EQUATION FOR MAGNETIZATION

We define the macroscopic density of the electron magnetic moment $\mathbf{M} = (M_1, M_2, M_3)$ - magnetization by the formula

$$\mathbf{M}(\mathbf{x},t) = 2\mu_0 Sp\frac{1}{2}\vec{\sigma}\int dV_\mathbf{p}n(e,\mathbf{x},t). \tag{9}$$

In view of the relation for Pauli matrixes $\sigma_j\sigma_k - \sigma_k\sigma_j = 2i\varepsilon_{jkl}\sigma_l$ the kinetic equation (5) after multiplication by $\mu_0\vec{\sigma}$, taking the trace on the spin variables, and integration over $dV_\mathbf{p}$ takes the form

$$\frac{\partial}{\partial t}M_i + 2\mu_0[\mathbf{B},\mathbf{M}]_i + \frac{\partial}{\partial x_k}I_{ik} = 0, \tag{10}$$

where the flux density of the electron magnetic moment is equal to

$$I_{ik} \equiv -2\mu_0 Sp\frac{1}{2}\sigma_i\int dV_\mathbf{p}D_{kp}\left(\frac{\partial n}{\partial x_p} + qE_p\frac{\partial n}{\partial e}\right). \tag{11}$$

In order to obtain the closed equation for the magnetization we assume that function D^{kp} is smooth over

e, therefore it is possible to take it out under integral sign. We integrate the second term in (11) by parts on e, and assume that the surface terms are small at $e=0$, and at $e=e^F$, where e^F is the Fermi energy. This approximation allows us to write down the equation for the magnetization as

$$\frac{\partial}{\partial t}\mathbf{M} + 2\mu_0[\mathbf{B},\mathbf{M}] - $$
$$D_{kp}\left(\frac{\partial^2}{\partial x_k\partial x_p} - \frac{q}{2e_F}E_p\frac{\partial}{\partial x_k}\right)\mathbf{M} = 0 \tag{12}$$

Equation (12) without allowance of spatial non-homogeneity corresponds to the Bloch equation, and forms the basis of the theory of paramagnetic resonance. The account of nonhomogeneity is carried out in Refs. [1,5] without concrete description of the character of the diffusion mechanism. The nonlinear equation describing a collision dynamics of magnetization in the absence of external fields is obtained in Ref. [6]

The purpose of the present work is to study the magnetization dynamics of electron-impurity systems on the basis of Eq. (12) under conditions of paramagnetic resonance.

3. PARAMAGNETIC RESONANCE IN ELECTRON-IMPURITY SYSTEMS

We consider the magnetization behaviour in the case, when the external magnetic field in Eq. (12) consists of two terms $\mathbf{B}=\mathbf{B}^0+\mathbf{h}(t)$, where $\mathbf{B}^0$ is the static field, and $\mathbf{h}(t)$ is the alternating field.

To find the solution of Eq. (12) we shall develop the scheme described by Bar'yakhtar and Ivanov in Ref. [7]. For this purpose, we shall present the solution for the magnetization as an expansion in powers of the amplitude of the external alternating magnetic field

$$\mathbf{M}(\mathbf{x},t) = \sum_{k=0}^{\infty}\mathbf{m}^{(k)}(\mathbf{x},t). \tag{13}$$

After substituting Eq. (13) into Eq. (12) we have an infinite system of equations for $\mathbf{m}^{(k)}$

$$\frac{\partial}{\partial t}\mathbf{m}^{(k)} + 2\mu_0[\mathbf{B},\mathbf{m}^{(k)}] - D\mathbf{m}^{(k)} = $$
$$-2\mu_0[\mathbf{h}(t),\mathbf{m}^{(k-1)}] \tag{14}$$
$$k = 0, 1, 2,...; \quad \mathbf{m}^{(-1)} \equiv 0, \quad \mathbf{B} = (0, 0, B),$$

$$D \equiv d_F \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left(1 + \omega_c^2\right)\frac{\partial^2}{\partial z^2} - \right.$$

$$\frac{q}{2e_F}\left(E_1 + \omega_c E_2\right)\frac{\partial}{\partial x} - \frac{q}{2e_F}\left(E_2 - \omega_c E_1\right)\frac{\partial}{\partial y} \qquad (15)$$

$$\left. - \frac{q}{2e_F}E_3\left(1 + \omega_c^2\right)\frac{\partial}{\partial z}\right).$$

At first we find the solution $\mathbf{m}^{(0)}$ of the Cauchy problem with the help of the change of dependent variables,

$$m_i^{(0)}(\mathbf{x},t) = (2\pi)^{-\frac{3}{2}}\int d^3k\, e^{-i\mathbf{kx}}m_i^{(0)}(\mathbf{k},t) \qquad (16)$$
$$(i = 1, 2, 3)$$

For the Fourier-components $\mathbf{m}_i^{(0)}(\mathbf{k},t)$ we get a system of differential equations of the first order in time. This system is easily solved. Carrying out return transformation,

$$m_i^{(0)}(\mathbf{k},t) = (2\pi)^{-\frac{3}{2}}\int d^3x'\, e^{i\mathbf{kx'}}m_i^{(0)}(\mathbf{x'},t) \qquad (17)$$

we find the solution for magnetization in the form of free precession in constant fields,

$$\mathbf{m}^{(0)}(\mathbf{x},t) =$$
$$\int d^3x\, g(t,\mathbf{x'},\mathbf{x})(m(\mathbf{x'})\cos(\Omega t + \varphi(\mathbf{x'})), \qquad (18)$$
$$- m(\mathbf{x'})\sin(\Omega t + \varphi(\mathbf{x'})),\ m_3(\mathbf{x'}))$$

on the set of the known initial data of the form

$$\mathbf{m}^{(0)}(\mathbf{x},t=0) =$$
$$(m(\mathbf{x})\cos\varphi(\mathbf{x}),\ - m(\mathbf{x})\sin\varphi(\mathbf{x}),\ m_3(\mathbf{x})) \qquad (19)$$

with the propagation function equal to

$$g(t,\mathbf{x'},\mathbf{x}) = \frac{1}{\left(1 + \omega_c^2\right)^{\frac{1}{2}}}\left(\frac{1}{2(\pi d_F t)^{\frac{1}{2}}}\right)^3\exp[-\left(\frac{x'-x}{2\sqrt{d_F t}} + \frac{q}{2e_F}\frac{E_1 + \omega_c E_2}{2}\sqrt{d_F t}\right)^2 -$$

$$\left(\frac{y'-y}{2\sqrt{d_F t}} + \frac{q}{2e_F}\frac{E_2 - \omega_c E_1}{2}\sqrt{d_F t}\right)^2 - \left(\frac{z'-z}{2\left(1 + \omega_c^2\right)^{\frac{1}{2}}\sqrt{d_F t}} + \frac{q}{2e_F}\frac{\left(1 + \omega_c^2\right)^{\frac{1}{2}}E_3}{2}\sqrt{d_F t}\right)^2], \qquad (20)$$

where

$$\Omega = 2\mu_0 B, \quad d_F = \frac{v_F^2\tau_{eimp}}{3\left(1 + \omega_c^2\right)}$$

and v_F is the Fermi velocity.

This function obeys the equation

$$\frac{\partial}{\partial t}g - Dg = 0 \qquad (21)$$

and also has the properties

$$\lim_{t \to 0^+} g(t,\mathbf{x'},\mathbf{x}) = \delta(\mathbf{x'}-\mathbf{x}), \qquad (22)$$

$$\lim_{t \to 0^+} \frac{\partial}{\partial t}g(t,\mathbf{x'},\mathbf{x}) = D\delta(\mathbf{x'}-\mathbf{x}). \qquad (23)$$

An important property of the propagation function is that it satisfies the Smolukhowski-Chapman-Kolmogoroff equation

$$\int d^3x'\, g(t-t',\mathbf{x'},\mathbf{x})g(t',\mathbf{x''},\mathbf{x'}) = g(t,\mathbf{x''},\mathbf{x}). \qquad (24)$$

The particular solution $\mathbf{m}^{(1)}$ of Eq. (14) with the right hand side $- 2\mu_0[\mathbf{h}(t),\mathbf{m}^{(0)}(\mathbf{x},t)]$ can be obtained, using the semigroup property of the function $g(t,\mathbf{x'},\mathbf{x})$ (24):

$$m_1^{(1)}(\mathbf{x},t) + im_2^{(1)}(\mathbf{x},t) =$$
$$2\mu_0 i\int_0^t dt'\, e^{i\Omega(t'-t)}\int d^3x'\, g(t,\mathbf{x'},\mathbf{x})\times$$
$$[(h_1(t') + ih_2(t'))m_3(\mathbf{x'})] - h_3(t')e^{-i(\Omega t'+\varphi(\mathbf{x'}))}m(\mathbf{x'})], \qquad (25)$$

$$m_3^{(1)}(\mathbf{x},t) = 2\mu_0\int_0^t dt'\int d^3x'\, g(t,\mathbf{x'},\mathbf{x})m(\mathbf{x'})\times$$
$$[h_1(t')\sin(\Omega t'+\varphi(\mathbf{x'})) + h_2(t')\cos(\Omega t'+\varphi(\mathbf{x'}))]. \qquad (26)$$

It is seen from the formulae (25), (26), that the magnetization at the time t is determined by the field $\mathbf{h}(t)$ at all previous moments of time, starting the moment of inclusion.

We choose left rotation for external alternating magnetic field, which is perpendicular to the static field $\mathbf{B}_0$, $\mathbf{h}(t) = h(\cos\omega t, - \sin\omega t, 0)$, h is the amplitude of the field, ω is the frequency of the alternating magnetic field. Since at the paramagnetic resonance $\omega = \Omega$, we find from formulae (25), (26), that the particular solution for the magnetization linear in the field approximation is the forced precession,

$$m_1^{(1)}(\mathbf{x},t) = \omega_1 t m_3^{(0)}(\mathbf{x},t)\cos\left(\Omega t - \frac{\pi}{2}\right),$$

$$m_2^{(1)}(\mathbf{x},t) = - \omega_1 t m_3^{(0)}(\mathbf{x},t)\sin\left(\Omega t - \frac{\pi}{2}\right), \qquad (27)$$

$$m_3^{(1)}(\mathbf{x},t) = \omega_1 t A(\mathbf{x},t,)$$

$$A(\mathbf{x},t) \equiv \int d^3x'\, g(t,\mathbf{x'},\mathbf{x})m(\mathbf{x'})\sin\varphi(\mathbf{x'}),$$
$$\omega_1 = 2\mu_0 h \qquad (28)$$

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lagging in phase behind in the phase of the alternating magnetic field by $\frac{\pi}{2}$.

Having continued the procedure of iteration, it is possible to sum up series on $\omega_1 t$ and find the general exact solution for the magnetization dynamics (12) at paramagnetic resonance:

$$\mathbf{M}(\mathbf{x},t) = \mathbf{m}^{(0)}(\mathbf{x},t) + \mathbf{m}(\mathbf{x},t),$$

$$\mathbf{m}(\mathbf{x},t) =$$

$$\left(\left(A(\mathbf{x},t)(1 - \cos\omega_1 t) + m_3^{(0)}(\mathbf{x},t)\sin\omega_1 t\right)\sin\Omega\, t, \quad (29)\right.$$

$$\left(A(\mathbf{x},t)(1 - \cos\omega_1 t) + m_3^{(0)}(\mathbf{x},t)\sin\omega_1 t\right)\cos\Omega\, t,$$

$$\left. A(\mathbf{x},t)\sin\omega_1 t + m_3^{(0)}(\mathbf{x},t)(\cos\omega_1 t - 1)\right),$$

At $h = 0$ this solution transforms into $\mathbf{M}(\mathbf{x},t) = \mathbf{m}^{(0)}(\mathbf{x},t)$, see Eq. (18). As is seen from the solution (29), there is no divergence in time. Finally we come to the conclusion, that the solution of the Cauchy problem of the Eq. (12) with the initial distribution (19) in the class of square integrable functions is completely determined by the propagation function and the shape of the sample, i. e., by the integration volume. For an unbounded medium under conditions of paramagnetic resonance the solution of the Cauchy problem takes the form of Eq. (29).

The magnetization projection $M_3(\mathbf{x},t)$ oscillates. This fact implies that a inverse population occurs in the system considered.

For the analysis of the forced precession we write the solution (29) for M_1, M_2 as

$$M_1(\mathbf{x},t) = a\cos(\Omega\, t + \phi),$$

$$M_2(\mathbf{x},t) = -a\sin(\Omega\, t + \phi), \quad (30)$$

where the local amplitude and phase of precession are equal to

$$a(\mathbf{x},t) =$$

$$\sqrt{\left(-A(\mathbf{x},t)\cos\omega_1 t + m_3^{(0)}(\mathbf{x},t)\sin\omega_1 t\right)^2 + A_1^2(\mathbf{x},t)},$$

$$\phi(\mathbf{x},t) = arctg\frac{A(\mathbf{x},t)\cos\omega_1 t - m_3^{(0)}(\mathbf{x},t)\sin\omega_1 t}{A_1(\mathbf{x},t)}, \quad (31)$$

$$A_1(\mathbf{x},t) \equiv \int d^3 x'\, g(t,\mathbf{x}',\mathbf{x})m(\mathbf{x}')\cos\varphi(\mathbf{x}').$$

Expanding the phase $\phi(\mathbf{x},t)$ (31) with respect to t and restricting ourselves to the term linear in t, we get, in view of property (23), the local dynamic shift of the forced frequency Ω' with respect to Larmor precession Ω,

$$\phi(\mathbf{x},t) = \varphi(\mathbf{x}) + \Omega'(\mathbf{x},0)t + ...,$$

$$\Omega'(\mathbf{x},0) = \frac{1}{m^2(\mathbf{x})}\{[-\omega_1 m_3(\mathbf{x}) +$$

$$\int d^3 x'\left(D\delta(\mathbf{x} - \mathbf{x}')\right)m(\mathbf{x}')\sin\varphi(\mathbf{x}')]m(\mathbf{x})\cos\varphi(\mathbf{x}) \quad (32)$$

$$- [\int d^3 x'\left(D\delta(\mathbf{x} - \mathbf{x}')\right)m(\mathbf{x}')\cos\varphi(\mathbf{x}')]$$

$$(m(\mathbf{x})\sin\varphi(\mathbf{x}) - \omega_1 m_3(\mathbf{x}))\},$$

the cause of which has the meaning of the internal field at the point $\mathbf{x}$ (analog of Suhl-Nakamura field [8] in a paramagnetic medium). This field depends on initial nonuniform magnetization distribution at all points, that is, it has nonlocal character. Without nonlocality being taken into account, this shift is proportional to the amplitude of the forced field and has the simple form,

$$\Omega'(\mathbf{x},0) = -2\mu_0 h\frac{m_3(\mathbf{x})\cos\varphi(\mathbf{x})}{m(\mathbf{x})} \quad (33)$$

As it is seen from the formula (33), this shift depends non-linearly on initial distribution. This result coincides with that of Ref. [8] in view of heterogeneity. It follows from formula (32) that the dynamic shift of the free precession $\Omega'_{free}(\mathbf{x},0)$ is completely nonlocal:

$$\Omega'_{free}(\mathbf{x},0) =$$

$$-\frac{1}{m(\mathbf{x})}\int d^3 x'\left(D\delta(\mathbf{x} - \mathbf{x}')\right)m(\mathbf{x}')\sin(\varphi(\mathbf{x}) - \varphi(\mathbf{x}')) \quad (34)$$

Now it is obvious, that in the general case the dependence of dynamic shift on time and co-ordinates is

$$\Omega'(\mathbf{x},t) = \frac{\phi(\mathbf{x},t) - \varphi(\mathbf{x})}{t}. \quad (35)$$

We find the maximal amplitude of the forced precession a^{max} from the condition $M_3 = 0$, i. e.,

$$A\sin\omega_1 t + m_3^{(0)}\cos\omega_1 t = 0 \quad (36)$$

After substituting (36) in (31), we get

$$a^{max} = \sqrt{\frac{m_3^{(0)2}}{\sin^2\omega_1 t} + A_1^2}, \quad (37)$$

and the $t^{(k)}$ are determined by the solution of Eq. (36), which can be written in equivalent form as

$$Sin(\omega_1 t + \delta) = 0,\ \delta = arctg\frac{m_3^{(0)}}{A} \quad (38)$$

In the simplest case we find:

$$\delta \approx -\frac{\pi}{4},\ \omega_1 t \approx k\pi + \frac{\pi}{4},\quad k = 0,1,2,...$$

$$t^{(0)} \approx \frac{\pi}{4\omega_1},\quad t^{(1)} \approx \frac{5\pi}{4\omega_1},\quad ... ,$$

$$a^{max}\big|_{t=t^{(0)}} \approx \sqrt{3}\left|m_3^{(0)}\right|\big|_{t=t^{(0)}}. \quad (39)$$

Decaying bursts of precession amplitude a^{max} are observed. The general exact solution of the equation (12) is given in the Appendix.

4. CONCLUSIONS

The evolution dynamics in the system of electrons and impurities placed in static electrical and magnetic fields is investigated under the influence of a alternating magnetic field under conditions of paramagnetic resonance. The general formulas for all three magnetization components in their evolutionary interrelation are obtained, since experimental engineering allows one to measure these components [9]. The behaviour of forced precession is theoretically investigated. The dynamic shift of the frequency of

paramagnetic resonance caused by a nonuniform distribution of initial magnetization is found. All results are expressed in terms of the initial magnetization distribution and a propagation function. The results obtained are applied to the analysis of spin diffusion in natural and manmade materials [10,11] and also in magnetometry [9].

ACKNOWLEDGMENT

APPENDIX

If the mismatch $\Delta = \Omega - \omega$, i. e. the difference between Larmor precession Ω and the frequency of the alternating magnetic field is not equal to zero, the general exact solution of the equation (12) has the form:

$$\mathbf{M}(\mathbf{x}, \Delta, t) = \mathbf{m}^{(0)}(\mathbf{x}, t) + \mathbf{m}(\mathbf{x}, \Delta, t),$$

$$\mathbf{m}(\mathbf{x}, \Delta, t) = \Big(\big(-A\cos\gamma t + \frac{\omega_1}{\gamma} m_3^{(0)} \sin\gamma t\big)\sin\omega t +$$

$$A_1\cos\omega t - \frac{\Delta}{\gamma} A_1 \sin\gamma t \sin\omega t - A_1\cos\Omega t + A\sin\Omega t -$$

$$\Big(\frac{\Delta}{\gamma} A\sin\gamma t - \big(\omega_1\Delta m_3^{(0)} - \Delta^2 A_1\big)\frac{(1-\cos\gamma t)}{\gamma^2}\Big)\cos\omega t,$$

$$\Big(-A\cos\gamma t + \frac{\omega_1}{\gamma} m_3^{(0)}\sin\gamma t\Big)\cos\omega t - A_1\sin\omega t +$$

$$\Big(\frac{\Delta}{\gamma} A\sin\gamma t - \big(\omega_1\Delta m_3^{(0)} - \Delta^2 A_1\big)\frac{(1-\cos\gamma t)}{\gamma^2}\Big)\sin\omega t -$$

$$\frac{\Delta}{\gamma} A_1\sin\gamma t \cos\omega t + A_1\sin\Omega t + A\cos\Omega t,$$

$$m_3^{(0)}\big(\cos\gamma t - 1\big) + \frac{\omega_1}{\gamma} A\sin\gamma t +$$

$$\big(\Delta^2 m_3^{(0)} + \omega_1\Delta A_1\big)\frac{(1-\cos\gamma t)}{\gamma^2}\Big),$$

where

$$\gamma = \sqrt{\omega_1^2 + \Delta^2} \ .$$

The dynamic shift is

$$\Omega'(\mathbf{x}, \Delta, t) = \frac{\phi(\mathbf{x}, \Delta, t) - \varphi(\mathbf{x})}{t},$$

where

$$\phi(\mathbf{x}, \Delta, t) =$$

$$arctg \frac{A\cos\gamma t - \omega_1 m_3^{(0)}\dfrac{\sin\gamma t}{\gamma} + \Delta A_1 \dfrac{\sin\gamma t}{\gamma}}{A_1 - \Delta A\dfrac{\sin\gamma t}{\gamma} + \big(\omega_1\Delta m_3^{(0)} - \Delta^2 A_1\big)\dfrac{(1-\cos\gamma t)}{\gamma^2}} .$$

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