THE THEORY OF KINETIC PROCESSES IN ANISOTROPIC PHONON SYSTEMS

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We present a theoretical investigation of the kinetic properties of strongly anisotropic phonon systems. Such systems can be created in superfluid helium by heat pulses. The general expression for the rates of four-phonon processes are obtained. This expression shows that there is an asymmetry between the creation and decay of the high-energy phonons in the anisotropic phonon systems. Solutions of this expression are then considered. The results presented in this work explain the phenomena which are observed in the anisotropic phonon systems and they will stimulate the conception of new experiments.

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1. INTRODUCTION

Systems with anisotropic distribution of phonons in momentum space were created in experiments [1-3] with the help of a heater immersed in superfluid helium ^{4}He at such a low temperature that the contribution of thermal excitations can be neglected. The heater is a metal film evaporated onto glass. When current flows through the metal film the surface of the film injects phonons into the superfluid helium within a narrow cone with a solid angle $\Omega_p << 1$ and with an axis perpendicular to the surface of the heater. For a gold film the phonons injected to helium occupied a solid angle $\Omega_p = 0.125$ sr in momentum space. The dimensions in coordinate space of such strongly anisotropic phonon system are defined by the area of the heater and the duration of the thermal pulse.

Isotropic phonon systems, for which in momentum space there is no special direction, have been intensively explored theoretically and experimentally during several decades. The first theoretical work, which began systematic theoretical examination of the strongly anisotropic phonon systems in superfluid helium, was published only in 1999 [4]. This work explained the unique characteristics observed in such systems [1-3] and stimulated the design of new experiments. It was shown [4,5,6], that in the strongly anisotropic phonon systems of superfluid helium the kinetic processes differ greatly from those in the isotropic case. The main aim of the present work is to continue the theoretical analysis of kinetic processes in the anisotropic phonon systems of superfluid helium begun in 1999.

2. RATES OF PHONON INTERACTIONS IN SUPERFLUID HELIUM

The interaction rates in the phonon system of superfluid ^{4}He are determined by the unusual form of the phonon energy ε_i and momentum p_i relationship, which we write as

$$\varepsilon_i = c(p_i + f_i) \tag{1}$$

where $f_i = f(p_i)$. The deviation in (1) from a linear dependence is small $(f_i << p_i)$ but nevertheless it determines the rates of the kinetic processes amongst the phonons in superfluid helium.

At the saturated vapor pressure, for phonons with $\varepsilon_i < \varepsilon_c = 10K$ the function $f(p_i < p_c) > 0$. This corresponds to anomalous dispersion. For such phonons the conservation laws of energy and momentum allow processes which do not conserve the number of phonons. The fastest of these is the three-phonon process (3pp) where one phonon decays into two or two phonons merge into one. The rate of such process ν_{3pp} in the general case was calculated in [7].

For phonons with $p_i > p_c$ function $f(p_i > p_c) < 0$ (normal dispersion). For phonons with normal dispersion the conservation laws of energy and momentum prohibit the three-phonon processes. Then the fastest process is the four-phonon process (4pp). The rate of this process is ν_{4pp} [2], [8] is much smaller that the rate ν_{3pp}. The strong inequality

$$\nu_{3pp} >> \nu_{4pp} \tag{2}$$

shows us that phonons of superfluid helium break-up into two subsystems: one subsystem of high-energy phonons (h-phonons) with $p_i > p_c$, in which the equilibrium is attained relatively slowly and second subsystem of low-energy phonons (l-phonons) with $p_i < p_c$, in which the equilibrium occurs relatively quickly.

On the time scale of the problem under consideration the equilibrium in the subsystem of l-phonons occurs instantly and their energy distribution is given by the Bose-Einstein distribution function.

The slow establishment of equilibrium in the subsystem of h-phonons may be described by a kinetic equation for the distribution function $n(\vec{p}) \equiv n_i$, which we write as:

$$\frac{dn_1}{dt} = N_b - N_d \tag{3}$$

where N_b and N_d are the rates of increasing (b, born) and decreasing (d, decay) number of h$_1$-phonons (the momentum of h$_1$-phonon is $\vec{p}_1$). For four-phonon processes with the conservation laws of energy

$$\varepsilon_1 + \varepsilon_2 = \varepsilon_3 + \varepsilon_4 \tag{4}$$

and momentum

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_3 + \vec{p}_4 \tag{5}$$

these rates can be written as

$$N_{b,d} = \int_{\Omega_{b,d}} W n_{b,d} \delta\left(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4\right) \times \delta\left(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4\right) d^3 p_2 d^3 p_3 d^3 p_4 \tag{6}$$

where $W = W(\vec{p}_1, \vec{p}_2 \mid \vec{p}_3, \vec{p}_4)$ is defined by the transition probability density;

$$n_b = n_3 n_4 (1 + n_1)(1 + n_2)$$

$$n_d = n_1 n_2 (1 + n_3)(1 + n_4)\ ; \tag{7}$$

Ω_b and Ω_d are the sets of maximum values of solid angles of phonons Ω_{bi} (i=3,4) and Ω_{d2}, taking part in processes of h$_1$-phonon creation and decay, respectively. In the isotropic case $\Omega_{bi} = \Omega_{d2} = 4\pi$ and in the anisotropic phonon system $\Omega_{bi} = \Omega_{d2} = \Omega_p$. In relations (4) - (7) and below it is supposed, that the phonon "1" has the momentum $p_1 \geq p_c$ while other three phonons may have momentum less than p_c or greater than p_c.

According to (3) the stationary state of the h-phonon subsystem is defined by the equality

$$N_b = N_d\ . \tag{8}$$

In an isotropic case, when $\Omega_b = \Omega_d$, we obtain from the relations (6) - (8) the equation defining a stationary distribution function

$$n_3 n_4 (1 + n_1)(1 + n_2) = n_1 n_2 (1 + n_3)(1 + n_4)\ . \tag{9}$$

The solution of the equation (9), taking into account (4) and (5), is Bose-Einstein energy distribution function

$$n_i^{(0)} = \left(e^{\frac{\varepsilon_i}{k_B T}} - 1 \right)^{-1}\ . \tag{10}$$

We define the rates of creation $\nu_b^{(n)}$ and decay $\nu_d^{(n)}$ for phonons with momentum $\vec{p}_1$ and an arbitrary distribution function $n(\vec{p}_i)$ with a help of the relations

$$N_b = n_1^{(0)} \nu_b^{(n)}\ ;\ N_d = n_1 \nu_d^{(n)} \tag{11}$$

The rates of creation and decay, calculated using the distribution (10) with a help of equations (6) and (11) we designate accordingly $\nu_b^{(0)}$ and $\nu_d^{(0)}$. In the isotropic phonon system according to (8)-(11) these rates are equal. However in the anisotropic phonon system, when $\Omega_b \neq \Omega_d$, these rates are not equal and according to (3) and (11) their difference

$$\nu_b^{(0)} - \nu_d^{(0)} = \frac{1}{n_1^{(0)}} \frac{dn_1}{dt} \tag{12}$$

defines the initial rate of change of the Bose-Einstein distribution function in the phonon system.

In Ref. [5], the creation $\nu_b^{(0)}$ and decay $\nu_d^{(0)}$ rates of phonons with momentum $\vec{p}_1$ directed along the symmetry axis of the anisotropic phonon system with $\Omega_p << 1$ were obtained. The results presented in [5] show us the unusual character of the kinetic processes in anisotropic phonon systems and allow us to understand the difference between the stationary h-phonon distribution function and Bose-Einstein distribution (10). However even the complete examination of kinetic processes in the anisotropic phonon systems is possible only after getting the rates $\nu_b^{(0)}$ and $\nu_d^{(0)}$ for phonons with arbitrary directions of momenta $\vec{p}_1$ relative to the symmetry axis of the anisotropic phonon system. Here we obtain a solution of this problem in integral form and we discuss the consequences which follows from this general solution.

We write the integrals (6) in a spherical coordinate system with the polar axis directed along the symmetry axis z of the phonon system, so that $\vec{p}_i(p_i, \theta_i, \varphi_i)$. In equation (6) we integrate with respect to the variable p_4 using δ-function expressing the energy conservation law; also we take into account the relation (11). As a result we obtain

$$\nu_{b,d}^{(0)} = \frac{1}{c} \int_{\Omega_{b,d}} W n_0 \delta\left(p_{1x} + p_{2x} - p_{3x} - p_{4x}\right) \times \delta\left(p_{1y} + p_{2y} - p_{3y} - p_{4y}\right) \delta\left(p_{1z} + p_{2z} - p_{3z} - p_{4z}\right) \times p_2^2 p_3^2 p_4^2 dp_2 dp_3 d\varphi_2 d\varphi_3 d\varphi_4 d\zeta_2 d\zeta_3 d\zeta_4 \tag{13}$$

where $n_0 = n_2^{(0)}\left(1 + n_3^{(0)}\right)\left(1 + n_4^{(0)}\right)$ contains the distribution functions (10), $p_4 = p_1 + p_2 - p_3 - \phi$ is a function of independent variables p_2 and p_3;

$$\phi = f_3 + f_4 - f_1 - f_2 \tag{14}$$

$p_{ix} = p_{i\perp} \cos\varphi_i$, $\quad p_{i\perp} = p_i \sin\theta_i$; $\quad p_{iy} = p_{i\perp} \sin\varphi_i$; $p_{iz} = p_i \cos\theta_i$, $\zeta_i = 1 - \cos\theta_i$.

Let us integrate the expression (13) with respect to the variables φ_3 and φ_4 with the help of the first and the second δ-functions contained in the integrand (13).

As a result we have

$$\nu_{b,d}^{(0)} = \frac{1}{c} \int_{\Omega_{b,d}} \delta\left(p_1 \zeta_1 + p_2 \zeta_2 - p_3 \zeta_3 - p_4 \zeta_4 - \phi\right) \times I_\varphi n_0 p_2^2 p_3^2 p_4^2 dp_2 dp_3 d\zeta_2 d\zeta_3 d\zeta_4 \tag{15}$$

Here

$$I_\varphi = 4 \int W \frac{\eta\left(\tilde{R}_\varphi\right)}{\sqrt{\tilde{R}_\varphi}} d\varphi_2 \tag{16}$$

where $\eta\left(\tilde{R}_\varphi\right)$ - is a step-function equal to unity at $\tilde{R}_\varphi > 0$ and to zero at $\tilde{R}_\varphi < 0$, and $\tilde{R}_\varphi = 4 p_{3\perp}^2 p_{4\perp}^2 - \left(p_{\Sigma\perp}^2 - p_{3\perp}^2 - p_{4\perp}^2\right)^2$; $\vec{p}_{\Sigma\perp} = \vec{p}_{1\perp} + \vec{p}_{2\perp}$.

The function W can be considered as an axially symmetric function if it does not depend on angles, or as in our case of small phonon dispersion, when the momentum of all reacting phonons can be considered as being parallel. Supposing a weak dependence of W on

φ_2 we consider $W(\varphi_2)$ as a constant $\overline{W}$, where φ_2 is substituted with its effective value.

So we can rewrite the integral I_φ in the following way:

$$I_\varphi = \frac{16}{\sqrt{R_\varphi}} \overline{W} K(\alpha) \eta(1-\alpha) \eta(R_\varphi),$$

where

$$K(\alpha) = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1-\alpha^2 \sin^2 \varphi}}$$

is the full elliptic integral of the first kind,

$$\alpha = 4\sqrt{\frac{p_{1\perp} p_{2\perp} p_{3\perp} p_{4\perp}}{R_\varphi}}$$

is the parameter of $K(\alpha)$ and

$$R_\varphi = 2\sum_{i>k=1}^{4} p_{i\perp}^2 p_{k\perp}^2 + 8 p_{1\perp} p_{2\perp} p_{3\perp} p_{4\perp} - \sum_{i=1}^{4} p_{i\perp}^4$$

is a function of the components of transverse momentum.

Upper bounds of the integration of $\zeta_{b,d}^{(i)}$ with the variables $\zeta_i (i = 2,3,4)$ in (15) are defined by the parameter of anisotropy

$$\zeta_p = 1 - \cos\theta_p = \Omega_p / 2\pi \qquad (17)$$

and are different for creation processes

$$\zeta_b^{(2)} = 2; \quad \zeta_b^{(3)} = \zeta_p; \quad \zeta_b^{(4)} = \zeta_p \qquad (18)$$

and decay processes

$$\zeta_d^{(2)} = \zeta_p; \quad \zeta_d^{(3)} = 2; \quad \zeta_d^{(4)} = 2 \qquad (19)$$

The integration limits in (15) for variables p_2 and p_3 are determined by the conversation laws (4), (5) and values of the moduli of momentum of the phonons participating in the four-phonon processes.

3. THE ASYMMETRY BETWEEN THE CREATION AND THE DECAY OF HIGH-ENERGY PHONONS IN ANISOTROPIC PHONON SYSTEMS

Asymmetry between creation and decay of h_1-phonons follows from the different limits of integration (18) and (19) the expression (15) for the rates $\nu_b^{(0)}$ and $\nu_d^{(0)}$. Thus, from conservation laws (4), (5) it could easily be shown that in the anisotropic phonon systems, the restrictions imposed on creation and decay processes of h_1-phonons are different.

Taking the second power of the left and right parts of equality (5) taking into account the relations (4) and (1) we get

$$\zeta_{21} = \frac{p_1 + p_2}{p_1 p_2}\phi + \zeta_{34}\frac{p_3 p_4}{p_1 p_2} \qquad (20)$$

where $\zeta_{ik} = 1 - \frac{\vec{p}_i \vec{p}_k}{p_i p_k}$.

In the anisotropic phonon system, where $\theta_i < \theta_p$, the following situation is possible: there is no $\vec{p}_2$ phonon, which can annihilate the given $\vec{p}_1$-phonon within the angle satisfying (20). So, according to (20), when $\phi > 0$ the decay of a phonon moving along z-axis is possible only under the following condition

$$\zeta_p > \zeta_{2\min} = \frac{p_1 + p_2}{p_1 p_2}\phi \qquad (21)$$

For the creation of the given $\vec{p}_1$ phonon with $\theta_1 = 0$, according to (1), (4), (5), we have another restriction

$$\zeta_p > \zeta_{4\min} = \frac{p_2 - p_3}{p_1 p_4}\phi \qquad (22)$$

Inequalities (21) and (22) can be extended to the case of the arbitrary $\theta_1 < \theta_p$.

In the extreme case of an isotropic phonon system, when $\zeta_p = 2$, inequalities (21) and (22) are always true and the processes of creation and decay are symmetric. In strongly anisotropic phonon systems at $\zeta_p << 1$, we can have the situation when one of inequalities (21), (22) is satisfied and other is not. This depends on values $p_i (i = 1,2,3,4)$, which define the signs and magnitudes of the factors contained in the right hand parts of equalities (21) and (22). In view of this, it is convenient to examine separately the different types of processes, which describe the interactions of h_1-phonon with both l-phonons and with other h-phonons.

There are only five possible types of four-phonon interactions.

$$\begin{aligned}&1.\ h_1 + l_2 \leftrightarrow l_3 + l_4;\ 2.\ h_1 + l_2 \leftrightarrow h_3 + l_4;\\ &3.\ h_1 + l_2 \leftrightarrow h_3 + h_4;\ 4.\ h_1 + h_2 \leftrightarrow h_3 + l_4;\\ &5.\ h_1 + h_2 \leftrightarrow h_3 + h_4\end{aligned} \qquad (23)$$

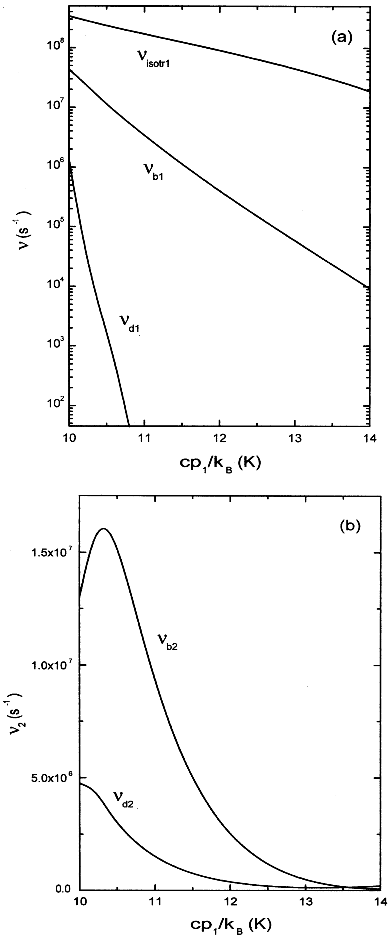
The arrow to the right indicates the decay of phonon "1" and to the left – creation.

The division of processes into five types leads to the division of the integration area, with the variables p_2 and p_3 (15) into five areas. Accordingly, each integral (15) can be written as the sum of five integrals. As the result, for each of five processes this gives the rates of creation $\nu_{b_j}^{(0)}$ and decay $\nu_{d_j}^{(0)} (j = 1,2,3,4,5)$. The limits of integration in $\nu_{b,d_j}^{(0)}$ with the variable p_2 and p_3 are determined by the following: 1. The type of creation or decay process. 2. Conservation laws (4) and (5); and 3. Inequalities (21) and (22) which lead to the appearance of the η-functions in the integrands for $\nu_{d_j}^{(0)}$, when $\zeta_{2\min} > 0$ and for $\nu_{b_j}^{(0)}$, when $\zeta_{4\min} > 0$.

According to (15) and (16) the integrals for $\nu_{b,d_j}^{(0)}$ become simpler for the case of creation or decay of $\vec{p}_1$ phonons moving along the z axis. Rates $\nu_{b,d_j}^{(0)}$ calculated for the case $\theta_1 = 0$ we denote ν_{b,d_j} where the superscript (0) is understood.

When $p_{1\perp} = 0$ the integral $K(\alpha)$ is equal to $\pi/2$. In (15) it is possible to make the integration numerically and obtain graphs of the dependencies ν_{b,d_j} on one of three general parameters of the problem p_1, T, θ_p when the other two parameters are fixed.

To explain the physical reasons for these dependencies for the rates of creation and decay, it is necessary to carry out analytical evaluations alongside the computer calculations. Unfortunately, even at $\theta_1 = 0$ integrals (15) cannot be expressed in terms of elementary functions. So the analytical approximation of the integrals was carried out for each ν_{b,d_j} by the replacement of the variables of integration, which only cause slow changes in the intergrands, by their typical values. After this, the integration of the remaining expressions can be done. The analytical expressions obtained in this way give numerical values which are close to the computer calculations.



Creation and decay rates vs the momentum of the h_1-phonon for the case of h-phonons interacting with l-phonons. The values of temperature T=1K and the parameter of anisotropy $\zeta_p = 2\cdot 10^{-2}$ corresponds with those in experiments. The first process: a; the second process: b

In the figure the dependencies of creation and decay rates for cases of interactions of h-phonons with l-phonons from p_1 are shown. The values T=1K and ζ_p=2 $\cdot 10^{-2}$ are typical for experiments [3].

The first process is exceptionally important in creating the h-phonon distribution function (Fig. a) where the phonons interchange between h-and l-subsystems. In Fig. a, for comparison, the value of the rate for the isotropic case is shown:

$$\nu_{isotr1} = \nu_{b1}(\zeta_p = 2, T = 1K) = \nu_{d1}(\zeta_p = 2, T = 1K). \quad (24)$$

Note in Fig. a the inequality $\nu_{b1} << \nu_{isotr1}$, and its growth with increasing p_1 is determined by the strong anisotropy of the system $(\zeta_p = 2\cdot 10^{-2} << 1)$ and restriction (22).

Unlike the isotropic case (24) at $\zeta_p = 2\cdot 10^{-2}$ the rate of creation is much greater than the rate of decay. So the different values and the momentum dependence of the rates ν_{b1} and ν_{d1}, shown in Fig. a are caused by the strong anisotropy of the system and the fact that in the first process $p_1 > p_2$ and $\phi > 0$. As a result at $\zeta_p << 1$ inequality (21) leads to tighter restrictions than the inequality (22). So according to equality (22), at any p_1 there exist p_2 and p_3, which satisfy the inequality (22) and rate ν_{b1} differs from zero at any p_1. The situation is different for decay process: according to equality (21), there can be an initial momentum $p_1 = p_0$ where the inequality (21) is not satisfied. As a result $\nu_{d1}(p_1 > p_0) = 0$ and the lifetime of such phonons, due to the first process, becomes infinite.

It is possible to obtain the analytical expression for p_0 from (21) in the following way. We factorize the functions f_i contained in ϕ (14) near their typical values of momentum: functions $f_{1,3}$ - near p_c, and f_4 - near p_2. We find a minimum value of $\zeta_{2\min}$. To do this we must place the maximum value of momentum p_2, which according to (4) is equal $p_{2up} = 2p_c - p_1$ for the first process, into the right hand part of equality (21). Then we replace the inequality (21) by equality. The resulting equation

$$\zeta_p = \zeta_{2\min}(p_2 = p_{2up}; p_1 = p_0)$$

allows us to obtain the analytical expression for p_0;

$$p_0 = p_c\left(1+\sqrt{\frac{\zeta_p c}{2p_c}\left|\frac{\partial^2\varepsilon}{\partial p^2}\right|_{p=p_c}}^{\,-\frac{1}{2}}\right) \quad (25)$$

For $\zeta_p = 2\cdot 10^{-2}$ the relation (25) gives a value $\frac{cp_o}{k_B} = 11K$, which coincides with the results of the computer calculation shown on Fig. a.

The infinite lifetime and the finite rate of phonons created for $p_1 > p_0$ is the cause of the fact that in the anisotropic phonon system the first process cannot create a dynamic equilibrium between the h-and l-phonon subsystems. However, this equilibrium is ensured by other processes.

The second process according to Fig. b operates essentially over the whole momentum space $\nu_{b2} > \nu_{d2}$ because the inequality (21) in this case is more rigid, than (22). Unlike the first process ν_{d2} differs from zero at all values of p_1 because in the second process, the function $\phi < 0$ at $p_1 < p_3$ and there is no restriction (21), because in this case $\zeta_{2\min} < 0$. However the second process cannot compensate for the effect of the first process at $p_1 > p_0$ because the second process maintains the total number of h-phonons.

Importantly, the dynamic equilibrium between h-and l-phonon subsystems is ensured by the fourth process, for which $\nu_{d4} > \nu_{b4}$ in those area of momentum, where $\nu_{d1} = 0$. This result follows from the fact that for all the momenta of phonons taking part in the fourth process, the function $\phi < 0$. As a result for the decay processes $\zeta_{2\min} < 0$ and there is no restriction (21) while for creation processes restriction (22) is implemented at $p_2 < p_3$, when $\zeta_{4\min} > 0$.

The fifth process is similar to second. The function φ (14) may have different signs because of the relative sizes between the momenta of the phonons participating in the fifth process. In the fifth process, as well as in second, the total number of h-phonons is maintained and it also leads to the concentration of h-phonons in momentum space, along z-axis.

In the third process $\nu_{b3} > \nu_{d3}$ because at all values of momentum function $\phi > 0$ and restriction (21) is always implemented. Rates ν_{b3} and ν_{d3} come to zero at $p_1 \to p_c$, thus the volume of momentum space of a variable p_3, which in the third process satisfies an inequality, $p_c \le p_3 \le p_1$ tends to zero.

The results of prior computer calculations obtained now with a help of the formula (15) at the arbitrary value $\theta_1 < \theta_p$ testify that the rates $\nu_{b1}^{(0)}$ and $\nu_{d1}^{(0)}$ with the increasing of an angle θ_1 differ only numerically from values shown in Fig. a. As for the second process there are not only the quantitative, but also qualitative modifications. Beginning with some value $\theta_1 = \theta_{eqv}$ in a wide range of p_1 values numerical values $\nu_{b2}^{(0)}$ become close to the numerical values $\nu_{d2}^{(0)}$. At $\theta_1 > \theta_{eqv}$ the rate ν_{b2} is less than the rate ν_{d2}. And this not only quantitatively, but also qualitatively differs from the graphs presented in Fig. b. As a result the second process will lead to the concentration of h-phonons in momentum space along the z-axis. The effect of h-phonons concentration near z-axis was found in experiments [3] where the h-phonons were confined in a cone with an angle 4°, while l-phonons moved in a cone with an angle 11,4°.

CONCLUSION

A pulse of phonons moving in superfluid helium in one direction is an unusual physical system with unique properties determined by its strong anisotropy. Such system has been studied experimentally for more than ten years [1]-[3]. This paper is the continuation of the theoretical analysis of the anisotropic phonon systems being done by physicists of University of Exeter (UK) and the Kharkov National University (UA) since 1998. As a result of these collaborations it was shown in [5] and [8], that in anisotropic phonon systems, the kinetic processes differ from the usual ones in isotropic phonon systems where the momentum distribution has no unique direction. We have obtained a common expression (15) for the creation and decay rates of high-energy phonons and have examined the consequences following from this expression.

The asymmetry between creation and decay of h-phonons in the anisotropic phonon systems is a consequence of different limits of integration (18) and (19) in expression (15). Its physical reasons are explained by the different requirements for decay (21) and creation (22) processes in anisotropic phonon systems. Such asymmetry lead to the interesting predictions that in the anisotropic phonon systems, the energy distribution function of h-phonons should be numerically much greater than that in the Bose-Einstein distribution, and it should have an unusual momentum dependence.

However a complete kinetic theory of the anisotropic phonon system in superfluid helium is possible only after an evaluation of all creation and decay rates for h-phonons moving at arbitrary angles $\theta_1 < \theta_p$ relative to the symmetry axis z of the anisotropic phonon system. Such a problem demands finding all the solutions contained in (15). At the moment there are only solutions for all the rates in extreme case $\theta_1 = 0$, and the previous results of numerical calculations obtained from (15) for the first and the second processes at arbitrary values $\theta_1 < \theta_p$. This indicates that it is necessity to carry out further theoretical analysis in this field. This should be accompanied by further experiments, because, although at first, the experiments [1]-[3] stimulated creating the theory [4] and [6], now in some respects the roles are reversed.

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