SLOW NUCLEAR BURNING

A.I. Akhiezer, N.A. Khizhnyak, N.F. Shulga, V.V. Pilipenko, L.N. Davydov

National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

Concepts of linear nuclear power systems of slow burning with breeding are briefly discussed. The propagation of the nuclear fission wave in a long thermal fission reactor (without breeding) is considered. The evolution of the neutron density profile with time is determined. The velocity of the burning front is estimated.

PACS: 28.41.Ak

1. INTRODUCTION

Last two decades, and especially after the Chernobyl accident, a great interest was shown in new concepts of fission nuclear reactors, as they were named safe reactors. One of such well known proposals was the concept by C. Rubbia et al.[1]. The matter concerns a subcritical nuclear reactor driven to criticality by a powerful accelerator system.

Another one was the conceptual design by E. Teller and al.[2] of a slow burning reactor with a completely automated system of operation and power output. The relative safety of slow burning concept lies in the necessity to produce, or breed the fissile fuel (e.g., ^{239}Pu) from fertile (^{238}U) in front of the moving wave of nuclear burning. Thus, the wave velocity, and the power output, depends on the sluggishness of the breeding process.

But chronologically earlier the first ideas of the slowly burning safe reactor were advanced by L. Feoktistov in 1989 [3].

2. REACTOR OF FEOKTISTOV

To describe the propagation of the nuclear burning wave Feoktistov [3] maximally simplified the problem and considered only four components (neutrons, ^{238}U, ^{239}U, ^{240}Pu) of the open nuclear reaction sequences of U-Pu fuel cycle

$$^{238}\mathrm{U}\ (n,\gamma)\ ^{239}\mathrm{U}$$
$$^{239}\mathrm{U} \rightarrow (\beta) \rightarrow {}^{239}\mathrm{Pu}$$
$$^{239}\mathrm{Pu}\ (n,\text{fission}) \ldots$$

These resulted in a set of diffusion-reaction equations

$$\frac{dn}{dt} = D\Delta n + vn\left(\sigma_{a8} N_8 - \left(\sigma_a + \sigma_f\right)_{\mathrm{Pu}} N_{\mathrm{Pu}}\right),$$

$$\frac{dN_8}{dt} = -vn\sigma_{a8} N_8 ,$$

$$\frac{dN_9}{dt} = vn\sigma_{a8} N_8 - \frac{1}{\tau_\beta} N_9 ,$$

$$\frac{dN_{\mathrm{Pu}}}{dt} = \frac{1}{\tau_\beta} N_9 - vn\left(\sigma_a + \sigma_f\right)_{\mathrm{Pu}} N_{\mathrm{Pu}} .$$

for the corresponding densities of neutrons, 238U, ^{239}U, and ^{239}Pu. Here v is the mean neutron velocity and σ_a, σ_f are the absorption and fission cross-sections. It should be emphasized that the slowness or the sluggishness of the burning process is ensured here by the very large time (τ_β =2.4 days for U-Pu cycle) and 24.5 days for Th-U cycle). Similar equations can be written for Th-U cycle with τ_β =24.5 days.

In the self-similar approach, when the densities of all reaction components are moving with the same velocity V and depend on the unique variable $z = x + Vt$, Feoktistov has proved the existence of a slowly moving front of nuclear burning. To illustrate his results we obtained the exact numeric solution of above equations (in the self-similar approach) and presented them in Fig. 1 (a,b) and Fig. 2 (a,b) as functions of $\zeta = z / l$ (l is the neutron migration length).

One can see the soliton-like moving front (from left to right) of the nuclear reaction where the density of neutrons is enhanced (Fig. 1 a). The raw 238-uranium material (Fig. 1 b) existing in front of the reaction is burned out behind the moving front. And just in front or the moving burning wave a layer of the newly born Plutonium-239 (Fig. 2 a) is formed due to breeding, which is burned out to some extent after the wave has passed. The reaction intermediate product uranium-239 also can be seen as moving with the front (Fig. 2 b). The velocity V of the burning front depends on the parameters of the system through the ratio of two Plutonium concentrations, equilibrium and critical. The described process was named in [3] as neutron fissioning wave.

An expansion of this work was done later by V. Goldin et al. [4,5]. They have used the idea by Feoktistov of self-adjusting neutron-nuclide regime, described it, however, in a much more comprehensive model. They applied a multi-group description of neutrons and an accompanying system of 16 equations

PROBLEMS OF ATOMIC SCIENCE AND TECHNOLOGY. 2001, № 6, p. 272-275.

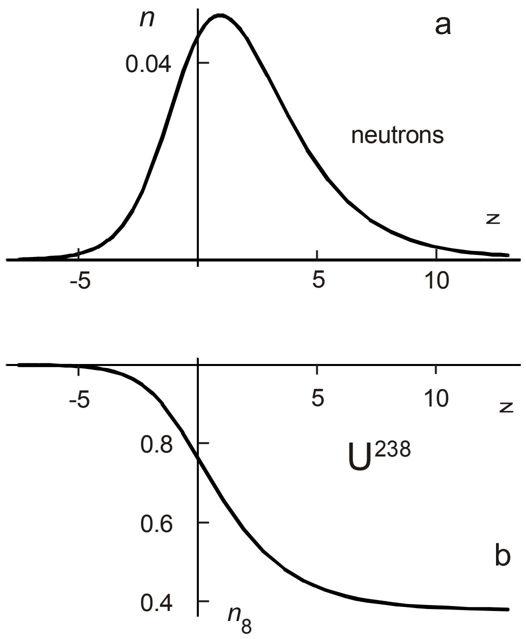

Fig. 1. *The dependence of neutron (a) and U^{238} (b) relative densities in Feoktistov reactor on the self-similar coordinate* $\zeta = z / l$

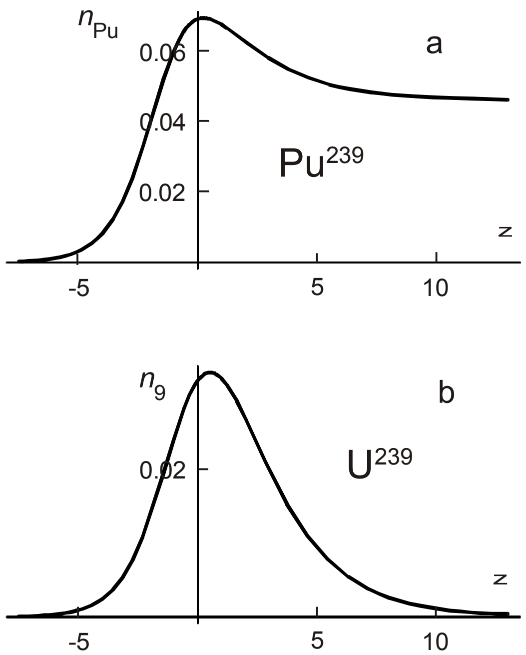

Fig. 2. *The dependence of Pu^{239} (a) and U^{239} (b) relative densities in Feoktistov reactor on the self-similar coordinate* $\zeta = z / l$

of isotope burn-out. But they considered [5], in fact, a heterogeneous reactor with alternating layers of differently enriched fissile isotope as a nuclear fuel. Actually, in their case, several burning fronts are slowly moving during the reactor campaign through a distance not exceeding the wavelength.

3. SLOW NUCLEAR BURNING IN A FULLY AUTOMATED REACTOR OF TELLER

Teller and al.[2] advocated a slow burning reactor with a fully automated system of power output. The process of nuclear burning was calculated numerically within a cylinder geometry of the reactor. They have employed 175 neutron energy-group model. Reactor design was resolved into several hundred spatial zones and a few hundred different materials. Sixteen isotopes are usually carried in each zone, representing both fertile and fissile isotopic components of nuclear fuel, in addition to reflector and coolant elements, structural materials, and various neutronic poisons, including fission products. The detailed results were not published, but the time-running of the reactor can be seen in Fig. 3 (z is the distance along the cylinder reactor axis). Teller et al. considered, unlike Feoktistov, Th-U fuel cycle: (^{232}Th (n,γ) ^{233}Pa; $^{233}Pa \rightarrow (\beta) \rightarrow$ ^{233}U; ^{233}U (n,fission)...).

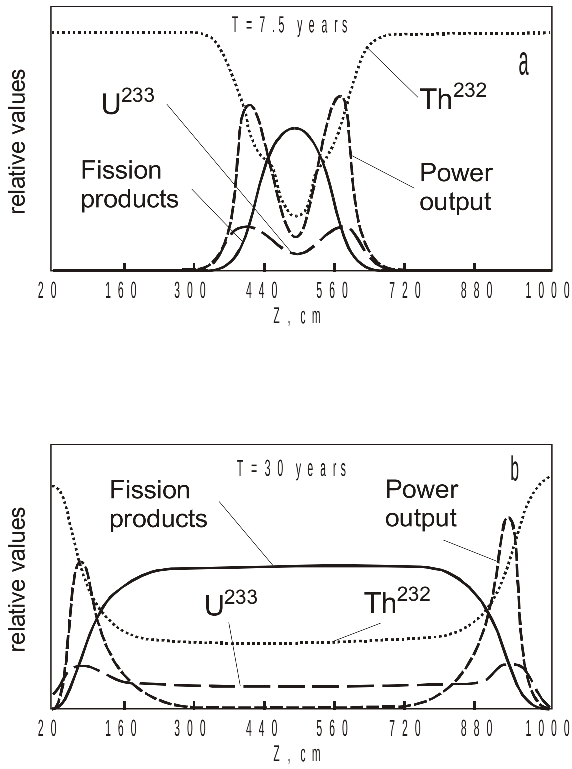

Fig. 3. *Operation of Teller fully automated reactor in time (a) after 7.5 years; (b) at the end of campaign after 30 years*

One can see the same processes to evolve as described by the simplified equations of Feoktistov: slow movement of the wave where the density of the fissile 233-Uranium and a power release are enhanced and a burned-out zone (depleted of Thorium) remained behind the front.

4. "SLOW" BURNING IN A THERMAL REACTOR

As a first step to tackle the problem of the slow burning in a breeder, we started with a hypothetical problem of the moving front of nuclear burning in a thermal reactor [6]. We show that a moving front of nuclear burning exists in this case also. However, its propagation velocity will be much higher determined mainly by the neutron diffusion. Contrary to the previous cases, the breeding will be absent, and with it the time consuming stage with β -decay also.

4.1. FORMULATION OF THE PROBLEM

Let's consider fissile medium in a form of a prolate infinite quadratic ($a \times a$) prism. The equation for neutron density $n(\mathbf{r},t)$ has a form

$$\frac{\partial n}{\partial t} = D\Delta n - \frac{1}{\tau_c} + \frac{\nu\theta_f\varphi}{\pi^{3/2} r_o^3 \tau_c} \int_V n(\mathbf{r}',t) \exp\left(- \frac{|\mathbf{r} - \mathbf{r}'|^2}{r_0^2} \right) dV' \tag{1}$$

Here D is the diffusion coefficient of thermal neutrons and τ_c is the neutron lifetime in relation to the capture. The quantities φ, θ_f in Eq. (1) describe, correspondingly, the probability that a fast neutron will be slowed down to a thermal energy, not being resonantly captured by the nuclei ^{238}U, and the probability of absorption of a thermal neutron by a nucleus of uranium. The first term in Eq. (1) is due to neutron diffusion, the second to the absorption and fission losses, an the last one is a nonlocal source of newly created fission neutrons. It describes the density of slow neutrons, produced as a result of the slowing-down of fast neutrons. The characteristic length parameter r_0 is an average length of neutron slowing-down.

The initial and edge conditions are

$$n(\mathbf{r},t)\Big|_{\substack{x=0.a \\ y=0,a}} = 0 \tag{2}$$

Searching for solution in the form

$$n(\mathbf{r},t) = n(z,t) \sin\frac{\pi}{a}x \sin\frac{\pi}{a}y \tag{3}$$

with initial condition

$$\begin{aligned} n(z,t)\big|_{t=0} &= n_0(z) \\ n_0(z) &= n_0(-z) \end{aligned} \tag{4}$$

one finds

$$n(z,t) = \frac{\exp\left(A^* t/\tau_c\right)}{2\sqrt{\pi D^* t}} \int_{-\infty}^{\infty} dt' n_0(t') \exp\left(- \frac{(z - z')^2}{4D^* t} \right) ,(5)$$

where D^* is an effective coefficient of diffusion, $D^* \equiv D + \frac{1}{4}\nu\theta_f\varphi\frac{r_0^2}{\tau_c}$ and $A^* \equiv \nu\theta_f\varphi - 1 - \frac{2\pi^2}{a^2}D^*\tau_c$ is the neutron multiplication (reproduction) factor for a reactor of finite transversal size a. If $A^* < 0$, the number of neutrons, born in nuclear fission reactions, is deficient to sustain a self-maintained chain nuclear reaction, i.e. we have a subcritical regime. With the increase of the characteristic size a the parameter A^* becomes positive (we suppose $\nu\theta_f\varphi > 1$), and we pass into the region of overcritical regime. Notice, that the condition $A^* = 0$ determines the critical transverse size of a system, where a chain reaction can be realized,

$$a_{cr} = \frac{\pi\sqrt{2\left(D\tau_c + \nu\theta_f\varphi r_o^2/4\right)}}{\sqrt{\nu\theta_f\varphi - 1}} \tag{6}$$

In the overcritical region $A^* > 0$ the self-sustaining chain reaction is possible, that is manifested in the exponential growth with time of neutron density at every point of space.

4.2. EVOLUTION OF NEUTRON DENSITY FLUCTUATION

Let us examine the evolution of fluctuation of the density of thermal neutrons that has some constant value n_0 in the range $-l \le z \le l$ at the initial moment of time. In this case Eq. (5) can be simplified and takes the form (at large distance from the initial fluctuation, $z >> l$)

$$n(z,t) \cong \frac{n_0\sqrt{D^* t}}{\sqrt{\pi}} \exp\left(A^* t/\tau_c\right) \left\{ \frac{\exp\left(- \frac{(z-l)^2}{4D^* t} \right)}{z - l} - \frac{\exp\left(- \frac{(z+l)^2}{4D^* t} \right)}{z + l} \right\} \tag{7}$$

Eq. (7) describes diffusion of thermal neutrons taking into account their multiplication. It can be simplified in some important cases. First, asymptotic expression for increasing z and fixed t : $z \to +\infty$, $t = \text{const}$,

$$n(z,t) \cong \frac{n_0}{\sqrt{\pi}} \frac{\sqrt{D^* t}}{z} \exp\left[\frac{A^* t}{\tau_c} - \frac{(z-l)^2}{4D^* t} \right]. \tag{8}$$

Second, asymptotic expression for increasing t and fixed z : $t \to +\infty$, $z = \text{const}$,

$$n(z,t) \cong \frac{n_0}{\sqrt{\pi}} \frac{l}{\sqrt{D^* t}} \exp\left[\frac{A^* t}{\tau_c} \right], \quad z \pm l << 2\sqrt{D^* t} \tag{9}$$

The first of the above two cases corresponds to an "instant" picture of neutron density distributed along the cylinder axis, and the second case describes the time evolution of neutron density at each given point z. According to Eqs. (8), (9) one can see, that at large distance from the fluctuation the density $n(z,t)$ exponentially decreases with distance, and at large time it grows exponentially with time due to multiplication.

A characteristic feature of multiplicating medium, according to Eq. (8), is the existence of such velocity $v = (z - l)/t = v_0$ of the moving point of observation, at which the observed density of thermal neutrons $n(z.t)$ very slowly decreases with distance and is given by

$$n(z,t) \cong \frac{n_0}{2\sqrt{\pi z}}\sqrt{L_0}, \quad L_0 \equiv 2\sqrt{\frac{D^*\tau_c}{A^*}} = \frac{4D^*}{n_0}. \quad (10)$$

Such sharp change in the asymptotic behavior is the characteristic feature of multiplicating medium and may be physically explained by the compensation (along the ray $z = v_0 t + l$) of the exponential growth, caused by neutron multiplication, by the exponential decrease of neutron density with distance. Namely, the decrease of neutron density with displacement at the distance δz will be compensated by the increase of neutron number in the multiplication process during the time δt, needed to perform the above displacement.

The velocity v_0 may be named the velocity of slow nuclear burning in a system. Indeed, in the case of conventional (chemical) slow burning [7], which results in the achievement of a certain reaction temperature, the velocity of slow burning is proportional to $\sqrt{\chi/\tau}$, where χ is the temperature conductivity, and τ is the characteristic reaction time. In our case the problem is in the achievement of some fixed neutron density at a given point, caused both by multiplication and diffusion. The lifetime of neutron τ_c acts as a characteristic time of reaction, and the geometric mean of diffusivity and multiplication parameter A^* plays the role of the transfer coefficient. The essential role of multiplication process is supported by the fact that the velocity of slow nuclear burning goes to zero at $A^* = 0$. This is consistent with the fact, that the velocity of purely diffusion process tends to zero at great distance from a source ($dz/dt \approx D/z, \quad z \to \infty$).

The neutron density can not grow unrestrictedly as above in linear problem, first of all due to the finite quantity of fuel. One can account for this introducing a nonlinear factor

$$1 - N(0,t)/\mathbf{N}_c \quad (11)$$

in the equation for neutron density

$$\frac{\partial n(z,t)}{\partial t} = D\frac{\partial^2 n}{\partial z^2} - 2\frac{\pi^2 D}{a^2}n - \frac{1}{\tau_c} + \\ + \frac{\nu\theta_f\varphi}{\sqrt{\pi}\, r_0 \tau_c}\left(1 - N(0,t)/\mathbf{N}_c\right)\int_{-\infty}^{\infty} n(z',t)\exp\left(-\frac{|z - z'|^2}{r_0^2}\right)dz' \quad (12)$$

where $N(0,t)$ is proportional to the total number of neutrons

$$N(0,t) = \sqrt{\frac{2}{\pi}}\int_0^\infty n(z,t)\,dz$$

and $\mathbf{N}_c$ is a sufficiently large given constant. This procedure is described in detail in [8] and we do not dwell on it any more.

Similar, though mostly local, nonlinear equations are familiar in diffusion-reaction problem in physical chemistry and other fields. The most known is the differential equation [9] describing the processes of genes dessimination and struggle for survival. With a nonlinear term written down in a more general form it was thoroughly investigated by A.N. Kolmogorov, I.G. Petrovsky and N.S. Piskunov [10]. It was, in particular, shown that the asymptotic velocity of propagation of the travelling wave solution could be also introduced. This velocity coincides with the velocity v_0 obtained in this paper.

Authors gratefully acknowledge the financial support of STCU (project №1480).

REFERENCES

1.C. Rubbia, J.A. Rubio, S. Buono, F, Carminati, N. Fietier, J. Galvez, C. Geles, Y. Kadi, R. Klapisch, P. Mandrillon, J.P. Revol, Ch. Roshe. *Conceptual design of a fast neutron operated high power energy amplifier.* Preprint CERN/AT/95-44 (ET), 1995, 240 p.

2.E. Teller, M. Ishikawa, L. Wood, R. Hyde and J. Nuckolls. Completely automated nuclear reactors for long-term operation. Int. Conf. on Emerging Nuclear Energy Systems, Obninsk, Russia, 24-28 June 1996.

3.L.P. Feoktistov. Neutron-fissioning wave // *Doklady AS USSR,* 1989, v. 309, p. 864-867 (in Russian).

4.V.Ya. Goldin, D.Yu. Anistratov. Fast neutron reactor in a self-adjusting neutron-nuclide regime // *Mathematical Modelling.* 1995, v. 7, №. 10, p. 12-32 (in Russian).

5.V.Ya. Goldin, N.V. Sosnin and Yu.V. Troshchiev. Fast neutron reactor in a self-controlled regime of 2d type // *Doklady RAS.* 1998, v. 358, № 6, p. 747-748 (in Russian).

6.A.I. Akhiezer, D.P. Belozorov, F.S. Rofe-Beketov, L.N. Davydov and Z.A. Spolnik. Propagation of a nuclear chain reaction in the diffusion approximation // *Physics of Atomic Nuclei (Yadernaya Fizika).* 1999, v. 62, № 9, p. 1474-1481.

7.L.D. Landau and E.M. Lifshits. *Fluid Mechanics*. London, 1959.

8.A.I. Akhiezer, D.P. Belozorov, F.S. Rofe-Beketov, L.N. Davydov and Z.A. Spolnik. On the theory of propagation of chain nuclear reaction // *Physica A.* 1999, v. 273, p. 272-285.

9.R.A. Fisher. *The Genetical Theory of Natural Selection*, Oxford: "Oxford University Press", 1930.

10.A.N. Kolmogorov, I.G. Petrovsky and N.S. Piskunov. Investigation of the equation of diffusion associated with the augmentation of the substance quantity, and its application to a certain biological problem // *Bulletin of Moscow University.* 1937, v. 1, Sect. A, № 6, p. 1-26 (in Russian).