STOCHASTIC RESONANCE IN NUCLEAR FISSION

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Fission decay of highly excited periodically driven compound nuclei is considered in the framework of Langevin approach. We used residual-time distribution (RTD) as a tool for studying the dynamic features in the presence of periodic perturbation. The structure of RTD essentially depends on the relation between Kramers decay rate and the frequency ω of periodic perturbation. In particular, the intensity of the first peak in RTD has a sharp maximum at certain nuclear temperature depending on ω. This maximum should be considered as fist-hand manifestation of stochastic resonance in nuclear dynamics.

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INTRODUCTION

The process of nuclear fission, connected with crucial reconstruction of a nucleus occupies an important position in nuclear dynamics and is a most perspective source of energy (together with nuclear fusion). Scientists have been studying this phenomenon for more than half of century. The book by A.I. Akhiezer and I.Ya. Pomerantchuk "Nekotorye problemy teorii yadra" [1], published in 1950, contained practically first presentation of the fission theory. The complicity of the process became move apparent as study progressed. Despite much time has passed since discovery, we cannot claim that we understand all aspects of the process. Because of that complexity of the phenomenon under consideration a wide range of models and methods (sometimes mutually contradicting) were used for its description. Each piece of progress in comprehending of nuclear structure (or, more generally, of the structure of arbitrary nonlinear system) caused immediate response in fission physics. For instance, alteration of universally recognized views on existence of shell structure of highly excited nuclei [2] had resulted in radical reconsideration of fission barrier geometry as well as made it possible to explain a series of critical experiments. An introduction of the concept of dynamical chaos into nuclear dynamics enabled scientists to review the penetration through fission barrier clue role in induced fission [3]) from new positions.

Kramers [4] was the first who considered nuclear fission as a process of overcoming the potential barrier by the brownian particle. A slow fission degree of freedom (with large collective mass) is considered as brownian particle, and fast nucleon degrees of freedom - as a heat bath. Adequacy of such description is based on the assumption that the while of equilibrium achievement in the system of nucleons degrees of freedom is much less than the characteristic time scale of collective motion. As was shown (see [5]) rate of the thermal activation is essentially varied in the presence of the weak periodic perturbation of the fixed frequency, whish depend on the bath temperature. The effect has received a title of a stochastic resonance (SR) and is increasingly becoming a concept of universal validity.

Originally SR was introduced nearly 20 years ago to explain the periodicity of the Earth's ice ages [6,7] and has found its numerous applications into such diverse fields like physics, chemistry and biology (see [5]).

The mechanism of SR can be explained in terms the motion of a particle in a symmetric double-well potential subjected to noise and time periodic forcing. The noise causes incoherent transitions between the two wells with a well-known Kramers rate [4] r_k. If we apply a weak periodic forcing noise-induced hopping between the potential wells can become synchronized with periodic signal. This statistical synchronization takes place at the condition

$$r_k^{-1} \approx \pi / \omega ,$$

(1)

where ω is a frequency of periodic forcing. Two prominent feature of SR arise from synchronization condition (1):(i) signal-to-noise ratio does not decrease with increasing noise amplitude (as it happens in linear system), but attains a maximum at a certain noise strength (optimal noise amplitude can be found from (1) as r_k is simply connected with it);(ii) the residence-time distribution (RTD) demonstrates a series of peaks, centered at odd multiples of the half driving period $T_n = 2(n - \frac{1}{2})\pi/\omega$, exponentially decreasing amplitude. Notice that if a single escape from a local potential well is the event of interest – RTD reveals the dynamics of considering system more transparently than the signal-to noise ratio. These signatures of SR are not confined to the special models, but occur in general bi- and monostable systems and for different types of noise.

The aim of the present work is to demonstrate the possibility of observation of SR in nuclear dynamics. As a specific example we consider the process of induced nuclear fission in the presence of weak periodic perturbation.

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1. DISSIPATIVE DYNAMICS IN PRESENCE PERIODIC PERTURBATION

The most general way of description of dissipative nuclear dynamics is Fokker-Planck equation [8]. However for demonstration of qualitative effects it is convenient to use Langevin equation [9] that is equivalent to Fokker-Planck equation but is more transparent. As it has been shown the description based on Langevin equation adequately represents nuclear dissipative phenomena such as heavy-ion reactions and fission decay [10-12] and possesses a number of advantages over Fokker-Planck description.

Because we intend to only qualitatively demonstrate SR in nucleus let us consider the simplest type of Langevin equation — one-dimensional problem with inertial M and friction γ parameters independent on coordinates. Fission coordinate R is considered as a coordinate of Brownian particle. The rest degrees of freedom play a role of heat bath being modeled by random force $\xi(t)$.

The particle motion in the presence of external periodic force $A\cos\omega t$ is described by Langevin equation for canonically conjugate variables $\{P, R\}$,

$$dR/dt = P/M$$
$$dP/dt = -\beta P - dV/dR + A\cos\omega t + \xi(t) \qquad (2)$$
$$\beta = \gamma/M$$

$\xi(t)$ is stochastic force possessing statistical properties of white noise

$$\langle \xi(t) \rangle = 0, \langle \xi(t)\xi(t') \rangle = 2D\delta(t - t'), D = \gamma T$$

(3)

The nuclear temperature $T(MeV) = \sqrt{E^*/a}$ where E^* is an excitation energy and the level density parameter $a = \bar{A}/10$ ($\bar{A}$ being a mass number). The deformation potential V is given as [10]

$$V(R) = \begin{cases} 37.46(R-1)^2(MeV), & 0 < R < 1.27 \\ 8.0 - 18.37(R-1.8)^2(MeV), & R > 1.27 \end{cases} \quad (4)$$

these are parameters of ^{205}At nucleus [10].

The discrete form of the Langevin equation is given by [11,12]

$$R_{n+1} = R_n + \tau P_n/M$$
$$P_{n+1} = P_n(1 - \beta\tau) -$$
$$\left[\frac{dV(R)}{dR}\bigg|_n - A\cos\omega t_n \right]\tau + \sqrt{\frac{2\beta MT\tau}{N}}\eta(t_n) \qquad (5)$$

Here $t_n = n\tau$ and $\eta(t_n)$ is a normalized Gaussian distributed random variable which satisfies

$$\langle \eta(t) \rangle = 0, \langle \eta(t_n)\eta(t_{n'}) \rangle = N\delta_{nn'} \qquad (6)$$

Efficiency of numerical algorithm (5) was checked for the following cases:

$V = 0, A = 0$, where numerical and analytical results for $\langle P^2 \rangle$ and $\langle R^2 \rangle$ can be compared [10];
$V \neq 0, A \neq 0$, where numerical and analytical values for Kramers decay rate r_k can be compared. According to [4]

$$r_k = \frac{\omega_{min}}{2\pi}[\sqrt{\bar{\beta}^2 + 1} - \bar{\beta}]\exp(-\Delta V/T),$$
$$\bar{\beta} = \frac{\beta}{\omega_{max}} \qquad (7)$$

Here ω_{min} is the angular frequency of the potential (4) in the potential minimum and ω_{max} — at the top of barrier, ΔV is the height of the potential barrier. Numerical value of the Kramers decay rates r_k^i for the time bin I is calculated by sampling the number of fission events $(N_f)_i$ in the ith time bin width Δt normalized to the number of events $N_{total} - \Sigma(N_f)_j$ which have not fissioned

$$r_k^i = \frac{1}{N_{total} - \Sigma(N_f)_j}\frac{(N_f)_i}{\Delta t}. \qquad (8)$$

Comparison of (7) with asymptotic value of (8) was used for determination of the time interval τ, which provides saturation for numerical integration (5). From the result one could see, that 20 steps per nuclear time $\hbar/MeV$ provides a sufficient saturation.

3. MANIFESTATION SR IN NUCLEAR FISSION

Now let us proceed to the description of expected effect- manifestation of SR in nuclear fission.. For usually considered case of symmetric double well in the absence of periodic forcing, RTD N(t) has the exponential form (see [5]) $N(t): \exp(-r_k t)$. In the presence of the periodic forcing, one observes a series of peaks, centered at odd multiples of the half driving period $T_\omega = 2\pi/\omega$. The height of these peaks decrease exponentially with their order number.

These peaks are simply explained [15]. The best time for the particle to escape potential well is when the potential barrier assumes a minimum. A phase of periodic perturbation ϕ may be chosen in such a special way that the potential barrier $V(r) - AR\cos(\omega t + \phi)$ assumes its first minimum at $t = 1/2\ T_\omega$. Thus $1/2\ T_\omega$ is a preferred residence time interval. Following a "good opportunity" to escape occurs after a full period, when potential barrier again achieves its minimum. The second peak in the RTD is therefore located at $3/2\ T_\omega$. The location of the other peaks is evident. The peak heights decay exponentially because the probabilities of the particle to jump over a potential barrier are statistically independent. As is shown for symmetric double-well potential [14], the strength P_1 of the first peak at $1/2\ T_\omega$ (the area under peak) is a measure of the synchronization between the periodic forcing and the switching between the wells. So, if the mean residence time (MRT) of the particle in one potential well is much larger than the period of the driving, the particle is not likely to jump the first time the relevant potential barrier assumes its minimum. The RTD exhibits in such a case a larger number

of peaks where P_1 is small. If the MRT is much shorter than the period of the driving RTD has already decayed practically to zero before the time $1/2\,T_\omega$ is reached and the weight P_1 is again small. Optimal synchronization, i.e., maximum P_1, is reached when the MRT matches half the period of the driving, i.e., condition (1). This resonance condition can be achieved by varying the noise intensity D (or ω).

We will show that the analogous correlation between periodic forcing and escape time takes place for a decay of excited states (fission) with a single potential minimum as well. For RTD constructing (and following P_1 calculation) we use the numerical solutions of Langevin equation (5). Let us study evolution of P_1 within the temperature interval

$$1MeV \nmid T \rfloor 6MeV\sqrt{a^2 \quad b^2}\;.$$

Let us fix a frequency of periodic perturbation $\omega = 0.0267 MeV/\hbar$ ($T_\omega/2 = 117\hbar/MeV$) – a resonance frequency at $T = 3MeV$ (following Eq. (1)). The result of numerical procedure for RTD under fixed parameters of periodic perturbation [$A = 3, \omega = 0.0267$] are presented in Fig. 1. Nuclear friction β in all numerical calculations is chosen to be $1MeV/\hbar$.
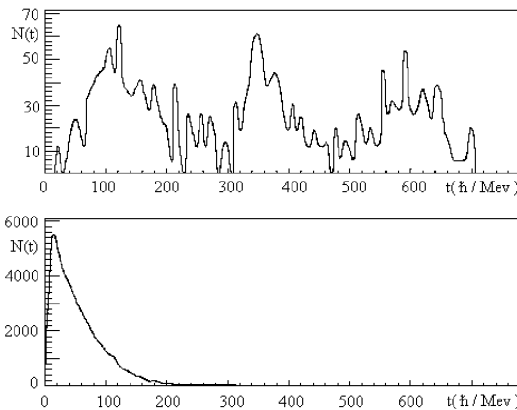

Fig. 1. RTD for T=1 and T=4

In accordance with the expected behavior at $T = 1MeV$ (for low r_k), one can distinctly see three peaks located near $t = T_\omega/2(\sim 117), 3/2T_\omega(\sim 353)$ and $5/2T_\omega(\sim 588)$, and at T=4MeV almost all RTD is concentrated near $t = 0$ (with width less than $T_\omega/2$). The corresponding variations of P_1 (which represent the measure of SR) are depicted in Fig. 2 for two frequencies of periodic perturbation, corresponding to temperatures 2 and 3 MeV. Maximum of intensities P_1 are close to the predicted values of temperature.

The above calculated P_1 can be estimated theoretically for single-well situation using a model similar to two-state model [5]. Let us evaluated RTD for a single-well case. Rate equation for a number of a fissile nuclei should be written as

$$\frac{dn}{dt} = -nr_k e^{-\varepsilon \cos \omega t}\,, \tag{9}$$

where $\varepsilon = A/T$. At low temperature Eq. (9) properly describes simulated process. The solution of Eq. (9) is

$$\ln\frac{n(t)}{n_0} = -r_k \int_0^t \exp(-\varepsilon \cos\omega t')dt'$$

$$= -r_0 t + \sum_{n=1}^{\prime} (-1)^n I_n(\varepsilon)\frac{2r_k}{n\omega}\sin\omega t\,, \tag{10}$$

where $r_0 = r_k I_0(\varepsilon) > r_k$; $I_n(\varepsilon)$ is modified Bessel functions. RTD in this model is given by $N(t) = -\dfrac{dn}{dt}$, so that

$$N(\frac{\pi}{\omega}) = r_k \exp(-\frac{r_0}{\pi\,\omega} + \varepsilon)$$
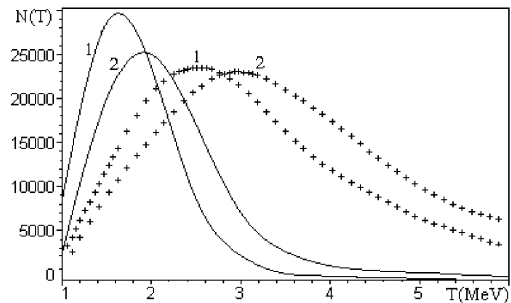
$$\tag{11}$$

Fig. 2. $P_1(t)$ at $\omega=0.0267$ (crossed) and $\omega=0/007$ (solid); 1 corresponds to Eg.(12) and 2 is for the numerical calculation

Using Eqs. (9)-(11) we obtain instead of Eq. (1) new condition for resonant temperature $T_{RES}(\omega)$, which provides maximal value for $N(\pi/\omega)/\omega$, whose dependence on $\omega/2$ and T properly represents $P_1(\omega,T)$ calculated above:

$$r_k^{-1} = \frac{\pi}{\omega}\frac{\lambda I_0(\varepsilon) - I_1(\varepsilon)}{\lambda - 1}\quad, \tag{12}$$

Here $\lambda \triangleq V/A$. Numerical solution to Eq. (12) for $T_{RES}(\omega)$ is presented in Fig. 3 [together with a solution to $r_k^{-1} = 2\pi/\omega$, which much better than (1) approximate curve (12)].

Evaluated $P_1(T)$ depicted in Fig. 2 is to be compared with numerical results. The scale of $P_1(T)$ is chosen in such away that the height in its maximum for $\omega = 0.0267$ coincides with the numerical data. Higher T_{RES} in the latter case is connected with non-equilibrium distribution within a long interval near $t = 0$ (which can be easily seen in Fig. 1).

The first maximum in RTD is shifted from π/ω, so it may seem more reasonable to evaluate the height in true maximum. The calculation shows that this height dependence on ω resembles that presented in Fig. 2, excepting the region of high T, where the curve $N(t)$ does not possess any maximum. Nevertheless $N(\pi/\omega)$

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is easily defined observable and studying its dependence on T allows one to determine the important nuclear characteristics, as for example, nuclear friction.
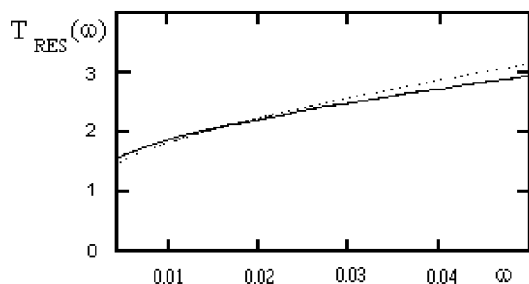

Fig. 3. *Resonant conditions (12) and (13) for $T_{RES}(\omega)$*

Let us now briefly consider the possible sources of periodic perturbation. One of possibilities to consider a fissile nucleus as a component of a double nuclear system formed, for example, in heavy-ion collisions [15]. In this case, the deformation potential will experience periodic perturbation similar to tide waves on the Earth caused by the Moon rotation. In the case of asymmetric fission, alternating electric field may be the source of periodic perturbation.

4. OPEN PROBLEMS

In conclusion, we shall stop on a number of open questions connected to prospect of research of the nuclear stochastic resonance.

In Section 2 we used model of Brownian motion, where the heat bath influences the motion of the Brownian particle (collective motion), while the bath itself is not affected by its coupling to the collective motion. In particular, temperature of the bath remains constant. At the description of the fission process we assume that the bath represent the nuclear internal degrees of freedom of nuclei. The statistical properties of this bath (including temperature) are influenced by the coupling to the collective motion. We must then permit that the temperature of the bath rises as its energy increases during the course of the fission process. Consequence it is that the coefficient D of Eq. 3, which determines the strength of the fluctuation of the Langevin force $\xi(t)$ is not constant, but is continually re-adjusted. Such self-consistent scheme usually used at the description of the fission process [12], should essentially improve considering model.

There is a natural question, whether it is possible to observe a nuclear stochastic resonance in spontaneous fission. The equivalent statement of a question consists in a possibility of observation of stochastic resonance on a quantum scale. Indeed, quantum mechanics provides an additional channel to overcome a potential barrier. This additional channel is provided by quantum tunneling. Because quantum noise persists even at absolute zero temperature, the transport on a quantum scale should naturally be aided by quantum fluctuations as well. As show estimations [16] for strongly damped systems quantum contributions can enhance the classical effect up to two orders of magnitude.

All these questions require special consideration.

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