RELATIVISTIC ELECTRON BEAM PINCHING IN PLASMA AND CUMULATION OF IONS

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A process of the high-current relativistic electron beam pinching propagating in plasma has been considered. A time of pinching of an electron - ion system has been defined. Maximum compression ratio, the ion density and temperature are found.

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1. INTRODUCTION

The effect of a radial cumulation and acceleration of ions under action of short laser pulse is known [1]. The physical sense of effect of a cumulation of ions by laser pulse consists in the following. Under an influence of a ponderomotive potential of the laser radiation acting on electrons of plasma, in last there is a separation of charges and there will arise a polarization electric field. In this electric field the ions obtain a radial momentum. Ions motion to axis lead to essential increase of their density and energy.

The analogous effect of a cumulative action of ions is possible at propagation of a high-current relativistic electron beam (REB) in plasma. In case of dense high-energy REB $n_b >> n_p$, where n_b is a density of an electron beam, n_p is a density of background plasma, in the region of its propagation there will be only ions. Electrons of plasma will be blow out for boundary of an electron beam. If on a first step we consider that ions are immobile, then REB will be in an equilibrium, i.e. focusing force of the space charge ions will be compensated by defocusing force of a electromagnetic eigen- field. In plasma the REB equilibrium occurs at realization of a condition $n_i = n_b / \gamma^2$, where n_i is a ion density and γ is a relativistic factor of electrons. It is obvious, that the equilibrium state is forced and it exist only at presence of a radial electric field. In this electric field will take place ion displacement to an axis. Their density in area of REB will increase and accordingly the REB density will grow, i.e. there arise electron-ion sys-tem contraction.

Such REB pinching process and cumulation of ions is below discussed. The values of both a time and minimum sizes of pinch have been estimated.

The center-boundary potential difference of a continuous electron beam is determined by simple expression $eU = I_b mc^2 / I_A$, where e is a charge and m is a mass of an electron, a c is a velocity of light, I_b is a be-am current and I_A =17kA. From this expression it is pos-sible to estimate a maximum energy of the accelerated ions. So, for example, for a current the REB, I_b =64kA, the energy of ions on an axis of a system will attain W_i =2 MeV.

2. STATEMENT OF PROBLEM. THE BASIC EQUATIONS

In previous homogeneous plasma an axial- symmet-ric REB is injected. The quasi-static electromagnetic field in such system is described by the equations:

$$\frac{1}{r}\frac{d}{dr} rE_r = -4\pi e(n_b - n_i), \quad (1)$$

$$\frac{1}{r}\frac{d}{dr} rH_\varphi = -4\pi \frac{v_b}{c} n_b, \quad (2)$$

where E_r is radial electrical and H_φ is azimuth magnetic fields, V_0 is a velocity of an electron beam, r is radial coordinate. In an electromagnetic field the electrons the REB and ions will be displaced in a radial direction. We describe motion of both beam electrons and ions within the framework of hydrodynamic equations:

$$\frac{\partial v_{re}}{\partial t} + v_0 \frac{\partial v_{re}}{\partial z} + v_{re}\frac{\partial v_{re}}{\partial r} = -\frac{e}{m\gamma}\left(E_r - \beta_0 H_\varphi\right),$$

$$\frac{\partial v_{re}}{\partial t} + v_0 \frac{\partial v_{re}}{\partial z} + v_{re}\frac{\partial v_{re}}{\partial r} = -\frac{e}{m\gamma}\left(E_r - \beta_0 H_\varphi\right)$$

$$\frac{\partial n_e}{\partial t} + v_0 \frac{\partial n_e}{\partial z} + \frac{1}{r}\frac{\partial r n_e v_{re}}{\partial r} = 0, \quad (3)$$

$$\frac{\partial v_{ri}}{\partial t} + v_{ri}\frac{\partial v_{ri}}{\partial r} = \frac{e}{M} E_r,$$

$$\frac{\partial n_i}{\partial t} + \frac{1}{r}\frac{\partial r n_i v_{ri}}{\partial r} = 0, \quad (4)$$

where $V_{re,ri}$ V_{ri} are radial velocities of electrons and ions correspondingly, n_e , n_i are their densities, M is an ion mass.

A solution of a self-consistent system of the nonlinear equations (1) - (4) we search in a paraxial approximation [2,3]

$$v_{re,ri} = r u_{1e,i}(z,t), \qquad n_{e,i} = n_{0e,0i}(z,t). \quad (5)$$

From Maxwell equations (1), (2) we obtain expressions for electric and magnetic fields

$$E_r = -2\pi e(n_{0e} - n_{0i}) r, \quad (6a)$$

$$H_\varphi = -2\pi e \frac{v_0}{c} n_{0e} r. \quad (6b)$$

Having substituted expressions (5), (6) in equations of motion and continuities (3), (4) and having equated coefficients at equal degrees, we shall obtain the following system of the nonlinear differential equations

$$\frac{\partial u_{1e}}{\partial t}+v_0\frac{\partial u_{1e}}{\partial z}+u^2{}_{1e}=\frac{2\pi e^2}{m\gamma_0}\left(\frac{n_{0e}}{\gamma_0^2}-n_{0i}\right), \quad (7a)$$

$$\frac{\partial n_{0e}}{\partial t}+v_0\frac{\partial n_{0e}}{\partial z}+2n_{0e}u_{1e}=0, \quad (7b)$$

$$\frac{\partial u_{1i}}{\partial t}+u_{1i}^2=-\frac{2\pi e^2}{M}\left(n_{0e}-n_{0i}\right), \quad (8a)$$

$$\frac{\partial n_{0i}}{\partial t}+2n_{oi}u_{1i}=0. \quad (8b)$$

Taking into account an relation on masses of electrons and ions, it is possible to assume, that in each instant the REB is in equilibrium with an ion system. It means, that the conditions is satisfied

$$n_{0i}=n_{0e}/\gamma_0^2. \quad (9)$$

Having substituted a relation (9) in equations of motion of ions (8), we obtain the following system of the nonlinear equations,

$$\frac{du_{1i}}{dt}+u_{1i}^2=-\frac{2\pi e^2}{M}n_{oi}\left(\gamma_0^2-1\right), \quad (10a)$$

$$\frac{dn_{0i}}{dt}+2n_{oi}u_{1i}=0. \quad (10b)$$

3. DYNAMICS OF A PINCHING OF AN ELECTRON - ION SYSTEM

Let us considered a dimensionless function $\psi=\ln(n_{0i}/n_0)$ as well as a time $\tau=\omega_i t$, where $\omega_i=\left[4\pi e^2 n_0 M^{-1}\left(\gamma_0^2-1\right)\right]^{1/2}$.

The set of equations (10) is equivalent to a nonlinear differential second-order equation for function ψ

$$\frac{d^2\psi}{d\tau^2}-\frac{1}{2}\left(\frac{d\psi}{d\tau}\right)^2-e^{\psi}=0 \quad (11)$$

with the following initial conditions

$$\psi(0)=0,\ \psi'(0)=0. \quad (12)$$

The equation (11) does not contain parameters and is the universal. The precise solution of this equation in the implicit form is given by expression, which have two arbitrary constants. The partial solution, satisfying to initial conditions (12), has following form:

$$\sqrt{2}\int_0^{\sqrt{\psi}} ds e^{-s^2/2}=\tau. \quad (13)$$

Immediately from solution (13) follows, that for finite time

$$\psi+2\ln\psi=\ln 2-2\ln(\tau^*-\tau)$$

a function and consequently, both an ion and electron densities, formally become unlimited. In vicinity of a singularity implicit representation (13) can be simplified if to use asymptotic expansion of an error function

$$\frac{n_{0i}}{n_0}=\frac{2}{\left(\tau^*-\tau\right)^2\ln\left(2/\left(\tau^*-\tau\right)^2\right)}. \quad (14)$$

Solving the equation (14) method of sequential approximations, we obtain the expressions for function and, accordingly, for an ion density in vicinity of a singularity.

A time of development of a collapse can be estimated by formula:

$$t^*=\frac{r_b}{c}\left(\frac{\pi M}{4m}\frac{V_o}{c}\frac{I_A}{I_b}\frac{\gamma_0^2}{\gamma_0^2-1}\right)^{1/2}. \quad (15)$$

For the REB with a current 32kA, an energy 1MeV, initial radius 1cm a time of development of a collapse for ions of a deuterium equally, approximately, 1.34 ns. The length on which there is a collapse of an electron - ion system for the indicated parameters is equal 40 cm.

4. AN ESTIMATION OF COMPRESSION RATIO OF AN ELECTRON - ION SYSTEM

During the REB compression in an ion system gas-kinetic pressure of particles will be increased. A force of gas-kinetic pressure is equilibrant for an electrostatic force and process of collapse will be stopped. Compression ratio and pinch radius can be determined from balance of the indicated forces. The finite temperature of ions below will be taken into account, considering that the electron beam remains cold. The equilibrium equation has a form

$$E=\frac{1}{n_i}\frac{\partial P_i}{\partial r}, \quad (16)$$

where P_i is a gas-kinetic pressure of ions. Process of collapse has isoentropic character with an equation of state for monoatomic gas

$$P_i=n_0 T_i\left(n_i/n_0\right)^{5/3},$$

where T_i is an initial ion temperature. Using a Poisson equation (1), a balance equation of forces (16) we obtain a following expression:

$$\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}N_i^{2/3}+\frac{1}{a^2}N_i=0, \quad (17)$$

where $N_i=\dfrac{n_i(r)}{n_0}$, $a^2=r_b^2\dfrac{5}{8}\dfrac{T_i}{mc^2}\dfrac{V_o}{c}\dfrac{I_A}{I_b}\dfrac{\gamma_0^2}{\gamma_0^2-1}$.

We shall introduce the dimensionless radial coordinate $\rho=rN_{\max}^{1/6}/a$ and new function $\chi=(N_i(r)/N_{\max})^{2/3}$ (where $N_{\max}=n_i(r=0)/n_0$ is a maximum ion density compression ratio). Such replacement allows to transform the equation (17) to a relation

$$\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho}\chi+\chi^{3/2}=0 \quad (18)$$

with initial conditions:

$$\chi(\rho=0)=1, \qquad \left(\frac{d\chi}{d\rho}\right)_{\rho=0}=0.$$

The equation (18) also has the universal character. In a point $\chi=\lambda_0=2.658$ a function $\chi(\rho)$ and, accordingly, a density of particles will be equal zero. For an equilibrium value of pinch radius we have the following expression

$$\frac{r_{eq}}{a}=\frac{\lambda_0}{N_{\max}^{1/6}}. \qquad (19)$$

The relation (19) determines a pinch minimum size and a particle density compression ratio. The second relation between these magnitudes follows from a conservation condition of a number of the beam electrons during its collapse

$$2\pi\int_0^{r_{eq}} n_e(r)rdr = I_e/eV_0. \qquad (20)$$

From a relation (20) we obtain an expression for compression coefficient

$$N_{\max}=\frac{1}{(2\nu_0)^{3/2}}\frac{r_b^3}{a^3},$$ where the number ν_0 =1.06.

Accordingly for a minimum value of a pinch radius from expression (19) follows:

$$\left(\frac{r_{eq}}{r_b}\right)=(2\nu_0)^{1/4}\lambda_0\left(\frac{a}{r_b}\right)^{3/2}. \qquad (21)$$

As follows from a relation (21), a minimum value of a pinch radius is proportionally $T_i^{3/2}$. The ion temperature will be increased to a value

$$T_{i\max}=T_iN_{\max}^{2/3}.$$

For initial ion temperature 50 eV and the above mentioned parameters the REB we obtain following values: a compression coefficient, $N_{\max}=$ 2.2·10^5, ion density, $n_{i\max}=5.6\cdot10^{16}\,\text{cm}^{-3}$ and their temperature, $T_{i\max}=$ 700keV. We shall mark, that this value of temperature close to a maximum energy of ions which they can obtain in a radial direction in a of a space charge field of the REB and that is equal 1 MeV.

5. SUMMARY

A compression process of a high-current REB, propagating in plasma is in-progress explored. An effect is stipulated by that in a equilibrium state of the REB in plasma on ions acts force of electrostatic compression. The ion system compression reduces the REB collapse. The estimations of a pinching time and its length for an electron - ion system are obtained. During compression the temperature of ions will increase. In a finish state a gas-kinetic pressure force counterpoises compression to an electrostatic force. Maximum compression ratio, ion density and ion maximum temperature are determined.

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ПИНЧЕВАНИЕ РЕЛЯТИВИСТСКОГО ЭЛЕКТРОННОГО ПУЧКА В ПЛАЗМЕ И КУМУЛЯЦИЯ ИОНОВ

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Рассмотрен процесс пинчевания релятивистского электронного пучка, распространяющегося в плазме. Определено время пинчевания электронно-ионной системы. Найдены максимальная степень компрессии, ионные плотность и температура.

ПИНЧУВАННЯ РЕЛЯТИВІСТСЬКОГО ЕЛЕКТРОННОГО ПУЧКА У ПЛАЗМІ ТА КУМУЛЯЦІЯ ІОНІВ

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Розглянуто процес пінчування релятивістського електронного пучка, що поширюється у плазмі. Визначено час пінчування електронно-іонної системи. Знайдені максимальна ступінь компресії, іонна густина та температура.