# WHAT LIMITS THE PRECISION OF GROUND-BASED STELLAR PHOTOMETRY AND POSITIONAL MEASUREMENTS? 

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#### Abstract

Nobody has reached the photometric precision better than 0.001 of a magnitude with ground-based telescopes. At the same time, we should spend barely some seconds of time to detect a million photons from any bright star and to attain a Poisson noise of one millimagnitude. The instrumental accuracy of coordinate determination with the ground-based telescopes equipped with CCD cameras is also no better than $0.1-0.2$ arcsec. Many factors limiting the precision of stellar photometry and astrometry on the ground are superimposed and occur simultaneously. We show, however, that image motions in the focal plane of telescopes, recently discovered by the authors, may be in reality the barrier both to millimagnitude photometry and milliarcsecond astrometry with ground-based instruments. It is quite essential that stellar image motions (SIM) have a chance to be found out only by using synchronous observations with several telescopes. The measurements of SIM were carried out by taking registration of a star image near to diaphragm edge, which plays a role of an optical knife. The spectra of SIM show periodic variations on the scale from a few seconds to minutes typically having amplitudes of a few tenths of arc seconds. Clearly, image motions during the integration time can cause photometric errors and frustrate exact coordinate determinations. The nature of SIM remains obscure. An important point is that image displacements measured synchronously in different telescope reference frames show significant correlations. Emphasis is given to problems of the detecting of SIM and estimating their amplitude-frequency characteristics with the Synchronous Network of Telescopes.


## INTRODUCTION

As noted by Heintze et al. in [1] the signal-to-noise ratio $S / N$ in stellar photometry increases to a certain extent only with increasing of the integration time $\tau$. Further integration over $\tau \geq 40 \mathrm{~s}$ may decrease the $S / N$ value. The $S / N$ is rarely higher than 200 (the precision better than 0.005 mag ). The mean external error of differential stellar photometry of 0.0017 mag achieved by Lockwood \& Skiff (1988) in the late 1980s with robotic telescopes seems to be till now the top limit of ground-based photometry.

In a similar manner, the instrumental accuracy of coordinate determination with the ground-based telescopes equipped with CCD cameras is typically of $0.1-0.2 \operatorname{arcsec}[2]$.

Such precision is inadequate for many studies in astronomy and astrophysics, in particular, for searching for planetary companions of solar-type stars, stellar activity and other problems. Young et al. [4] give an exhaustive treatment of the factors limiting the precision of differential stellar photometry. They suggest implementing 15 recommendations to do ground-based photometry with an overall precision of 0.001 mag or better. These include, in particular: providing a temperature-controlled environment for the detector and filters; using multiple comparison stars; using short integrations; measuring extinction several times an hour; using a set of filters that satisfies the sampling theorem; using a CCD camera centering device; using a large focal-plane diaphragm; etc.

One can imagine that myriad details of the photometer device, electronics, etc. can make problems at a level near or below a millimagnitude. The main contributors in the error budget at the millimagnitude level may become such noise sources as centering nonuniformities, inaccuracies in tracking and guiding and short-term changes in extinction. Exact values of these errors are as a rule unknown. Actually, these effects may be superimposed and occur simultaneously. Their joint action results in variations of the intensity. This limits

[^0]the accuracy that can be reached. Hence, one has to invoke special techniques to reduce all of these effects to about a millimagnitude. We shall show that it appears possible to do reduction using an a posteriori analysis based on the proper data handling, which involves some after-the-fact decisions.

Charge-coupled devices (CCDs) have unsolved problems with millimagnitude photometry too. As discussed by Young et al. [4], the intrapixel nonuniformity and small "full-well" charge require the use of rather large star images for a high precision. The problems with the extended wings of the star image require a large focal-plane aperture too. But this introduces additional errors due to the sky background and faint field stars (detected or not). As shown by Gilliland et al. [3] on an ensemble of stars in the open cluster M67, all these problems do not permit to improve the CCD photometric precision better than 0.002 mag .

The instrumental accuracy of coordinate determination is directly dependent on the photometric precision. Fitting stellar profiles from a CCD image performs measuring the position of a star. Hence, the random photometric errors introduce direct contributions to the error budget of coordinate determination. Practically, precision is rarely better than 0.1 arcsec and is often poorer.

One of our recent finding [5] consists in that the actual stellar intensity observed on the ground is an unsteady stochastic process on different time scales. In this connection there does not exist the rigorous in the mathematical sense notion of what "the brightness of a star" means. It must be redefined in a more comprehensive statistical sense. But the discussing of this property and its relation to the photometric precision goes far beyond this paper.

Practically, the problem consists in how we can estimate and improve the precision experimentally. The PointSpectrum (PS) technique presented in this paper provides a useful tool kit for improving the accuracy of the differential stellar photometry in practice. The variation of the signal-to-noise ratio $S / N$ as a function of the integration time $\tau$ shows the well-known relation $S / N \sim \tau^{1 / 2}$ for strictly stationary stochastic noises. The standard deviation of observations is the reciprocal value of the signal-to-noise ratio $S / N$. Hence, the fractional error of stellar photometry decreases as may be supposed with the $1 / 2$ power of integration time. In fact, this is not the case. The PS algorithm allows us: (1) to find an optimum value of the integration time, (2) to attain a maximum of the $S / N$ ratio. Thus, we are able to determine the brightness difference between the program and comparison stars with the best possible accuracy.

## METHODS

The photometric error may be evaluated using the sampling moments of photometric readings. The expression $\varepsilon=\sigma^{2} /<n>^{2}$ specifies the relative power of noise fluctuations in the frequency range $\Delta f=(1 / 2 \Delta t-1 / N \Delta t)$, where $\langle n\rangle$ and $\sigma^{2}$ are the sample mean and the variance for photometric readings, $\Delta t$ is the sampling time, $N$ is the length of the data segment. Choosing an appropriate value of $N$, one can calculate the power spectrum of noise fluctuations by averaging of $\varepsilon$ over time. Such finite-time mode of averaging of power provides a simple method for estimating the instrumental error in the relevant frequency domain related with the integration time $\tau=N \Delta t$. We set $N=3$. This smallest value of $\tau$ determines the so-called point spectrum (PS). Merging the photometric readings and repeating the calculation of the PS we can easily examine the relative power of noise fluctuations as a function of the integration time $\tau$. The photometric error of differential photometry $\sigma_{1,2}$ is given by

$$
\sigma_{1,2}^{2}=\frac{<n_{1}>^{2}}{<n_{2}>^{2}}\left(\frac{\sigma_{1}^{2}}{<n_{1}>^{2}}+\frac{\sigma_{2}^{2}}{<n_{2}>^{2}}\right)=\frac{<n_{1}>^{2}}{<n_{2}>^{2}} \cdot\left(\varepsilon_{1}+\varepsilon_{2}\right)
$$

where the index numbers 1,2 refer to the program star and the reference one.
Figure 1 shows the fractional photometric errors in relation to the running time for two nearby stars observed simultaneously with a high-speed two-channel photometer. We calculate also the photometric error of pseudoobservations from the Poisson distribution with the mean equal to the mean of the individual observational data, which can be regarded as the lower limit to the precision of stellar photometry. We can improve the accuracy by searching for the integration time value for which the photometric error reaches a minimum or agrees within the Poissonian model. Improving the accuracy down to the fundamental limit needs a special technique presented in this paper. This technique should satisfy the following demands: (1) a program star and a comparison star should be measured simultaneously; (2) in order to overcome the effect of intensity variations we should search for a value of the characteristic integration time for which the r.m.s. noise error reaches a minimum. We can conclude that the effect of intensity variations can for the greater part be suppressed by integrating during a particular time $\tau$ equal to the some characteristic "period" of the variations. We must note, however, that finding of this characteristic "period" requires estimating the power spectrum of errors for the data and visual inspection of the diagram like that shown in Fig. 1.

## OBSERVATIONS

Photometric observations were carried out with the 2-m Zeiss RCC telescope and the Zeiss-600 telescope (equipped by a two-channel photometer) at the high-altitude Terskol Observatory ( 3100 m ), with the $1.25-\mathrm{m}$ Cassegrain telescope AZT-11 (equipped by a $U B V R I$ photometer-polarimeter) and the 50 -inch Cassegrain telescope (equipped by a two-channel photometer) at the Crimean Astrophysical Observatory. Bulk of observations was carried in $U$ band with integration times of 0.1 s . The program stars were the variables R CrB , OP And in quiescence and their reference ones distanced at about 20 arcmin . These long-term variables may be considered as constant photometric targets on time scales of interest. As we already mentioned, stellar image motions (SIM) can be detected only by the use of the observations gathered with several telescopes synchronously. Observations of stellar image motions were made with the above mentioned telescopes in a synchronous operating mode. The simultaneous operation of the telescopes was synchronized to an accuracy of 0.1 s . The measurements of SIM were carried out by taking registration of a star image near to the diaphragm edge which plays a role of an optical knife. The threshold sensitivity of this method may reach a few hundredths of arc second. This subject explored in detail by Zhilyaev et al. in [6].


Figure 1. The instrumental errors of the differential stellar photometry for pairs of constant stars observed simultaneously with the $60-\mathrm{cm}$ telescope (left side) and the $2-\mathrm{m}$ telescope (right side) at the Terskol Observatory

## MEASURING THE PHOTOMETRIC ERRORS

Experimental measurements of the photometric precision vs. run time are shown in Fig. 1. The relative error of the differential stellar photometry in magnitude is expressed as function of integration time. Figure 1 (left side) shows typical values of internal errors obtained with the $60-\mathrm{cm}$ Cassegrain telescope at the Terskol Observatory. The top panel demonstrates instrumental errors from two bright stars which have similar $U$ magnitude ( R CrB in quiescence and its reference star). The bottom panel shows a small part of the above curves. It illustrates that the photometric errors of two nearby stars do not decrease with time and oscillate around $\sim 0.003 \mathrm{mag}$ with a period of about 2.2 s . Moreover, the temporal behaviour of two nearby stars shows a significant correlation. Figure 1 (right side) presents typical values of errors obtained with the $2-\mathrm{m}$ RCC telescope at the Terskol Observatory. The top contour (squares) shows internal instrumental errors from two bright stars which have similar $U$ magnitude (OP And in quiescence and its reference star). The lower curve shows the errors of the simulated data. These are taken out from the Gaussian distribution with the same mean and covariance as the real data. It is straightforward to see that it decreases with the $1 / 2$ power of integration time. The smooth bottom curve corresponds to the exact solution for the strictly stationary random Poisson noises. This experimental measurement illustrates that the photometric errors do not decrease with time and oscillate after 60 s around $\sim 0.002 \mathrm{mag}$. The instrumental error curve is practically above the fundamental one, i.e., the errors are limited by counting statistics. This situation may be improved, perhaps, only for certain particular values of the run time. We can adopt them as the optimum estimates of the integration time to reach a maximum of the signal-to-noise ratio. A comparison between observations of bright stars and their simulations obtained from the Poisson distribution (see Fig. 1, right side) had revealed that the precision of ground-based photometry is limited by the quantum nature of light at a low integration time. At the mid and high integration time, as would-be hypothesis, we may imply that the actual stellar intensity cannot be longer considered as a stationary
random variable, but as an unsteady-state process. This leads to a number of interesting effects. In particular, the stellar magnitude of any star may have some fundamental uncertainty, which cannot be improved in principle. An experimental measurement of the photometric precision leaves unanswered the basic question: what limits the precision of ground-based stellar photometry? Correlated brightness variations of two nearby stars distanced at tens arc minutes reflect the novel phenomenon, i.e., stellar image motions in the focal plane of telescopes. We shall prove presently that the instrumental coordinates of a star are actually the bounded random variables.


Figure 2. Two-site monitoring of Jupiter on August 9, 1999 (see text)

## MEASURING THE STELLAR IMAGE MOTIONS

By searching the possibilities to attain the highest precision of stellar photometry, giant planets and their satellites were selected as photometric targets with an a priori constant irradiance for some short length of time. Surprisingly, it was found that they show clear signs of short-period brightness oscillations. The synchronous two-site monitoring has given us evidence for the correlated brightness variations of Jupiter and its satellite Europa, as well as of Saturn and its satellite Rhea. It has become evident that the variability of the brightness of solar planets provides an example of a certain unknown universality. The well-defined oscillations with a period from seconds to tens of seconds and the amplitudes of some thousandths of magnitude were found. The oscillations observed with distant telescopes show a close accordance not only in frequency but also in phase. They show a good fit of modulation properties as well. Further discussion of the problem can be found in our papers $[6,7]$. Figure 2 shows an example of the two-site monitoring of Jupiter on August 9, 1999. The instruments used were: the 2-m Ritchey-Chretien telescope at Terskol Peak in the Northern Caucasus and the $1.25-\mathrm{m}$ reflector AZT-11 at the Crimean Astrophysical Observatory. Observations were obtained in the $U$ passband with a sampling time of 0.1 s . The simultaneous operation of telescopes was synchronized to an accuracy of 0.1 s . As we already mentioned, there are some indications that the variability observed is due to the stellar image motions in the focal planes of telescopes, though the exact mechanism remains obscure. Figure 3 shows how SIM can be detected using observations obtained with two telescopes synchronously. The measurements of SIM were carried out by taking registration of a star image near to the diaphragm edge, which plays a role of an optical knife. Figure 3 reflects modulation of light from the field star PPM 119406 as seen simultaneously on September 30 and October 1, 2003 by the $1.25-\mathrm{m}$ reflector AZT-11 and the 50 -inch telescope of the Crimean Astrophysical Observatory which are separated by a distant of about 100 meters, respectively. An important point is that the image displacements measured synchronously in different telescope reference frames show significant correlations. The well-defined oscillations with a mean period of $20.3 \pm 1.0 \mathrm{~s}$ are obvious on both nights. In these cases the seeing was about 2 arcsec. Making use of the Gaussian fit to the star's image spread, the oscillation amplitudes of SIM may be found to be equal to about 0.2 arcsec. Such displacements remain below a detectable level with a single telescope because they are far much less than the stellar image $F W H M$. At the same time the significant correlation of displacements obtained from observations with several telescopes is a very strong argument in favour of the reality of SIM.


Figure 3. Modulation of light from the field star PPM 119406 as seen simultaneously on September 30 (left side) and October 1, 2003 (right side) by the $1.25-\mathrm{m}$ reflector AZT-11 and the 50 -inch telescope of the Crimean Astrophysical Observatory separated by about 100 meters

## DISCUSSION

The direct method of measuring the photometric errors versus run time allows one to optimally determine the brightness of any star in each observing run. This method can improve stellar photometry by a factor (say) from two to five over standard methods. In principle, the instrumental error can be reduced down to the photon count limit. In any case the method enables one to obtain the photometry with lower photometric errors. The difficulties begin with SIM. Can SIM be directly observed? The use of an appropriate method may prove this. It should be noted that all the results presented here are experimentally derived. We need the observations with several remote telescopes to detect SIM because their amplitudes have at least an order of magnitude less than the stellar image extent. Observations with a single telescope do not enable to derive an error source. As can be seen from Figs. 1-3 temporal variations of both the photometric and coordinate estimates can be considered as unsteady-state random processes. This means that both the brightness and instrumental coordinates of any star cannot be predicted with a perfect confidence and have some fundamental uncertainties. The internal uncertainty of differential stellar photometry determined above from pairs of constant stars is no better than $\pm 0.002 \mathrm{mag}$. The uncertainty of the instrumental coordinate measuring, as it is clearly shown in Fig. 3, is of the order of $\pm 0.2$ arcsec. One may conclude that these uncertainties are due to stellar image motions.

Our above mentioned tests have shown that far distant stars vary their brightness correlatively. Whether this effect reflects some particular feature of the telescope reference frame? It is generally known that global VLBI observations provide an angle resolution in the milliarcsecond (mas) range. The Hipparcos positions of stars are accurate to 30 mas or so. Most of the interferometer arrays for the visible and infrared imaging are aiming for $1-10$ mas resolution. Our conclusion concerning uncertainties in instrumental coordinates is not in conflict with the VLBI and Hipparcos results because these are differential in their character. For instance, in interferometer devices fluctuations in the tilting angle of the incident light are generally eliminated by tracking the light source with the tilt mirrors controlled by computer. The Hipparcos positions of stars are measured by recording the angle between the light sources too.

To summarize, we may infer that the position of a star, measured with respect to a telescope reference frame, is subject of low-frequency motions. Curiously enough, that image motions measured simultaneously with several telescopes spaced by hundreds of kilometers, show the significant correlation. The displacements themselves might typically amount to tenths of arc second on the scale from a few seconds to minutes. So, image motions may degrade the precision of both stellar photometry and position determination. The precision of the differential stellar photometry is no better than 0.002 mag . The position with respect to a telescope reference frame may have an error of about 200 mas. This novel phenomenon, recently discovered by the authors [6], may be the very barrier both to millimagnitude photometry and milliarcsecond astrometry with ground-based instruments.
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