ABOUT THE TENDENCY TO SYNCHRONIZATION OF THE EARTH AND THE MOON ROTARY MOVEMENTS

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The Hierarchical structure of the Earth–Moon system provides process of self-organizing. The principle of synchronization and symmetry of initial structure of the Earth–Moon system and the processes, breaking symmetry form the basis of self-organizing model. The theory of a nonlinear parametrical resonance allows finding the features as results of perturbation effects. In the present paper some ratio which determine stationary structure of the Earth–Moon system is given.

INTRODUCTION

Piphagor and J. Kepler ideas about world harmony have deep physical sense. The hypothesis about full resonance Solar System was put forward A. M. Molchanov [6].

E. A. Grebenikov has come to a conclusion [3]: "The Numerical analysis and theoretical generalizations have allowed stating a principle of the least interaction (or a principle of synchronism). According to this principle any planetary system, irrespective of an initial condition, sooner or later evolves to resonant condition where resonant ratio prevails between the basic frequencies systems.

Some calculations specify that the Solar System now is near to the first resonant condition and to this condition she has come for a time interval in some billions years".

Time of evolution of satellite systems to the first resonant condition is much less. It depends on initial conditions in parameters of system, *i.e.,* weights, forms of bodies, elements of orbits during formation of system as dynamic structure.

Classical example of resonant movement is the indignant movement of terrestrial axis. An attraction the Sun and the Moon brings the comparable contribution in this movement.

The movement of the Moon is described by empirical Cassini laws and has double 1:1 resonance: firstly, between axial and orbital rotations and, secondly, between precession orbits and the Moon's axes of rotation. Thus, one axis of the Moon inertia in the movement "traces" the radius-vector of an orbit, *i.e.,* takes place the law "a constant phase": at each passage of a pericenter of an orbit one of the main axes of inertia, normal to an axis of rotation, and a radius-vector of a pericenter have equal distances up to a line of knot [2].

Marked the ancients general harmony of the universe and existing tendencies to synchronization in the Earth–Moon system are shown in simple quantitative ratio between the sizes of forms and the orbits of both bodies. Result of synchronization is the coordination of parameters at formation of structure of the Earth–Moon system as single whole.

PROBLEM OF SYNCHRONIZATION. MODEL PARAMETRICAL OSCILLATOR

The Earth–Moon represents an open self-organizing subsystem of the Solar System. She models external influence and her parts.

For example, barycenter of the Earth–Moon system goes on an elliptic orbit around the Sun as a result of self-organizing system.

Displacement of the lunar orbit perigee under action of the Sun is responsible for the reduction of duration astral period of the Moon rotation and changes oscillatory energy in system. This process concerns to management at more high level.

The hierarchical structure allows to consider modelling of dynamic processes in the Earth–Moon system as the act of compression of the entrance information for reception of qualitative and quantitative estimations of considered process at various hierarchical levels. Thus, the question is not discussed, how might be carried out dynamically (at least at a theoretical level) procedure of self-organizing, or compression of the information. Such approach to modelling provides opportunity of the reduction of degrees of freedom quantity and application of the unified descriptions of the investigated process models.

The Earth–Moon system represents complex nonlinear oscillator. The mathematical decision of a problem of synchronization of movements of two connected nonlinear oscillators that are the Earth and the Moon includes, at least, two requirements:

1. opportunity of drawing up of two van der Pol equations with various managing parameters and mathematical description of their connections;
2. the description of the dissipation.

Performance of these conditions is the difficult problem and not solved till now.

Nevertheless, universal character of process of synchronization in a nature, in various mechanical and electric devices is reflected in a generality of existing mathematical descriptions.

Opportunities of application of these models for the description process of synchronization of oscillators with close frequencies in complex oscillatory systems are based on the assumption of sufficiency of weak interaction.

A simple model can described a parametrical resonance in the Earth–Moon system, concerning to parametrical excitation of the Chandler oscillation [5].

The equation of oscillation looks like

$$\ddot{\Theta} + \omega_0^2[1 + e\cos(2\omega t)] \cdot f(\Theta) = 0, \tag{1}$$

where Θ is the angular displacement of the Earth, ω_0 is the frequency of own oscillation of the Earth in the Earth–Moon system, 2ω is the frequency of parameter change ρ, ρ is the distances between the centers of masses, e is the eccentricity of the Moon's orbit, $f(\Theta) = \Theta + \frac{1}{2}\gamma\Theta^3$ is the function describing nonlinearity of system, $\gamma = \pm\sqrt{1 + (4\lambda)^2}$, $2\lambda = \dfrac{1+\mu}{1-\mu}$, $\mu = \dfrac{m_M}{m_E}$ (m_M is the mass of the Moon; m_E is the mass of the Earth).

The description of model (1) has universal character and represents a special case of the Hill's-equation – the Mathieu equation, which has the decision in the first area of instability ($\omega_0 = \omega$).

The parameter ρ in the Equation (1) changes under the periodic law:

$$\rho = \frac{\rho_0}{[1 + e\cos(2\omega t)]} \tag{2}$$

that to similarly periodic change of a gravitational field as

$$g_t = g_0 + g_1\cos(2\omega t). \tag{3}$$

We can discussed the qualitative analysis of process of the symmetry infringement in case of a parametrical resonance.

The equation (1) for small deviations from equilibrium movement ($x = \Theta - \Theta_0$) is given by

$$\ddot{x} + \omega_0^2(1 + \alpha)f(x) = 0, \tag{4}$$

where $1 > \alpha \geq 0$ and $f(x) = x + \frac{1}{2}\gamma x^3$.

The equation of fluctuation of system (4) is the elementary movement with one degree of freedom.

In this case, $F = -\omega^2(1 + \alpha)f(x)$ there is a restoring force which acts upon the point-mass. The force $F = -\dfrac{\partial V}{\partial x}$, where $V(x)$ is the potential. If $\alpha = 0$ and $f(x) = x$ the point-mass (system) is in a potential hole.

$$V_0(x) = \frac{1}{2}\omega_0^2 x^2. \tag{5}$$

This case is shown in Fig. 1: there is one stationary condition $\dfrac{dx}{dt} = 0$, $x = 0$, and it is steady.

In case of nonlinear function $f(x) = x + \dfrac{1}{2}\gamma x^3$ and $\alpha = 0$ the potential $V(x)$ looks like

$$V(x) = \frac{1}{2}(\omega_0^2 x^2 + \frac{1}{4}\omega_0^2\gamma x^4) \tag{6}$$

or

$$V(x) = V_0(x)(1 + \frac{1}{4}\gamma x^2). \tag{7}$$

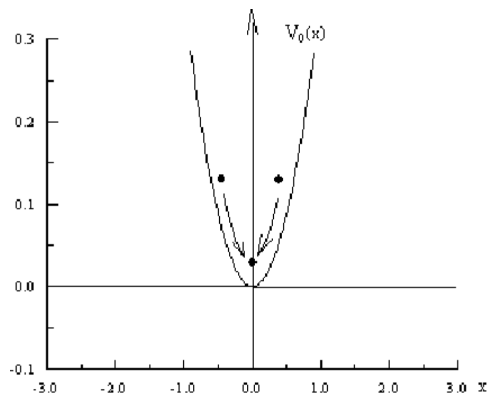

Figure 1. Potential $V_0(x) = \frac{1}{2}{\omega_0}^2 x^2$

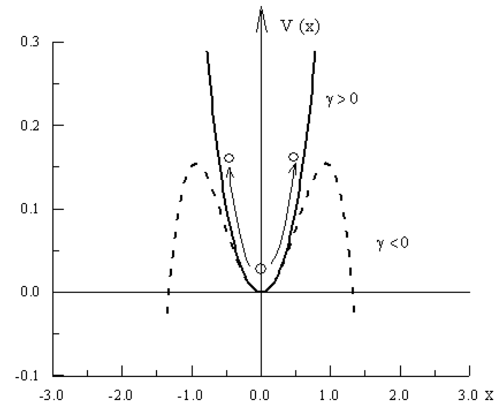

Figure 2. Potential $V(x) = V_0(x)(1 + \frac{1}{4}\gamma x^2)$

Now the number of special points is three. If $\gamma > 0$, the form of new potential is former (Fig. 1), *i.e.,* the unique stationary condition $x_0 = 0$ is steady. In a case $\gamma < 0$ the form of potential $V(x)$ changes, but symmetrically as it is shown in Fig. 2. Former steady condition $x_0 = 0$ becomes unstable and the varying particle (system) moves in one of equiprobable conditions x_1 or x_2.

The periodic potential ($\alpha \neq 0$) in case of nonlinear function $f(x)$ has difficult character.

From the analysis a conclusion was made that for occurrence of uncertainty concerning the following condition of system has not necessarily big number of degrees of freedom: only one nonlinearity does makes unpredictable behaviour of system.

The resonance arising under action of external periodic forces on nonlinear oscillator, results in increase of amplitude oscillations and, hence, to the exit of frequency of the oscillator from a resonance. In the Earth–Moon system (according to Gamilton) the asymptotic steady conditions or asymptotic steady cycles are absent [1]. Therefore, a bit later the system again comes back to a vicinity of a resonance.

Synchronization in system of nonlinear Earth–Moon oscillator is based on exchange of energy between amplitudes and the frequencies of fluctuations on all hierarchical structures.

The analytical description of changes of parameters of the Chandler oscillation is a description of process of self-organizing of the Earth–Moon system on various hierarchical levels under influence of external management. The model of self-organizing is based on symmetry of initial structure of the Earth–Moon system. Nonlinear processes break this symmetry and influence on evolution of system. Special methods find out the new features caused by perturbations.

One of these methods is the theory of a parametrical resonance.

CHARACTERISTICS OF STATIONARY STRUCTURE

As consequence of action of a principle of synchronism into the Solar System as a result of long evolution was defined the coordinated behaviour of planets and their satellites.

The steady structure of the Solar System and its parts was determined as a result of this. The Earth–Moon system is open, *i.e.,* the exchange of energy with an environment takes place. Due to this the homogeneous stationary condition transfers in non-uniform stationary condition which is steady concerning small perturbations. A formation of structures of the Earth–Moon system, most likely, had wave character. It has determined independence its characters from initial conditions.

Stationary connections between average parameters of figures and orbits of both bodies determine structure of the Earth–Moon system. Dependence between squares of own frequencies before and after an establishment of connection is:

$$\left| \frac{\omega_1^2 - \omega_2^2}{\nu_1^2 - \nu_2^2} \right| = \gamma, \tag{8}$$

where $\left|\nu_1^2 - \nu_2^2\right|$ is the difference of squares of own frequencies of the Earth and the Moon up to establishments of connection, $\left|\omega_1^2 - \omega_2^2\right|$ is the difference of squares of own frequencies of the Earth and the Moon in the Earth–Moon system, $\gamma = 2.280730667$ is the dimensionless constant determined through the relation of the Moon and the Earth masses.

Characteristics of constant structure were derived as ratios:

a) Between the Earth and the Moon radiuses

$$4 \cdot \frac{1-\rho'}{1+\rho'} = \gamma, \tag{9}$$

where $\rho' = \rho_0 k_R$, ρ_0 is the relation of the Moon average radius R_M to the Earth equatorial radius a_e. Dimensionless factor k_R looks like

$$k_R = \frac{1 - \frac{1}{2}\sqrt{5 - 2P_M}}{\frac{1}{2}\sqrt{5 - 2P_\odot}}, \tag{10}$$

where $P_M = 1 + \sin\pi'_M + \cos\pi'_M$, $P_\odot = 1 + \sin\pi_\odot + \cos\pi_\odot$, π'_M is the corner under which is seen an average radius of the Moon R_M on average distance between the centers of masses, $\pi_\odot$ is the constant of the Sun parallax. Sizes P_M and $P_\odot$ have geometrical analogue – the dimensionless perimeters of appropriate rectangular triangles.

b) Between constant inclinations equator the Earth ξ and the Moon I_M

$$2 \cdot \frac{1+h_0}{1-h_0} = \gamma, \tag{11}$$

where $h_0 = \dfrac{2I_M}{\varepsilon P_M}$.

c) Between the main moments of inertia of the Earth C_E and the Moon C_M

$$4 \cdot \frac{1 - 2Z\sqrt{C_0}}{1 + 2Z\sqrt{C_0}} = \gamma, \tag{12}$$

where $Z = \dfrac{1 + \sqrt{\sin\Delta}}{1 - \sqrt{\sin\Delta}}$, Δ is the difference of the coordinated inclinations equator of the Earth and the Moon to the ecliptic, $C_0 = C_M/C_E$.

d) Between constants general of the Earth's precession p and of the corner in top of internal cone of the Cassini 2κ [4]

$$\frac{2p}{\kappa} \cdot P_M = \gamma. \tag{13}$$

CONCLUSIONS

The hierarchical device of the Solar System, the features self-organizing provide stability of structure and of dynamic regimes in the Earth–Moon system. The self-organizing on various hierarchical levels is based on the principle of synchronization.

The theory of a nonlinear parametrical resonance allows finding the features as results of effects of perturbation.

Structure of the Earth–Moon system was generated as a result of long evolution of the Solar System. Characteristics of this structure are constant ratio between average parameters of the figures and orbits of the Earth and the Moon.

Acknowledgements. Researches were supported by grant RFFI (04–02–16633).

[1] *Arnol'd V. I.* Small denominators and problems of stability of movement in the classical and heavenly mechanics // Successes Mat. Sci.–1963.–**6**.–114 p.

[2] *Beletsky V. V.* Sketches about movement of space bodies.–Moscow: Nauka, 1977.–432 p. (in Russian).

[3] *Grebenikov E. A.* Introduction in the theory of resonant systems.–Moscow State University, 1976.–176 p. (in Russian).

[4] *Kulikov K. A., Gurevich V. B.* Bases lunar astrometry.–Moscow: Nauka, 1972.–392 p. (in Russian).

[5] *Kurbasova G. S., Rykhlova L. V.* // Astron. J.–2001.–**78**, N 11.–P. 1049–1056.

[6] *Molchanov A. M.* The resonant structure of the Solar System // Icarus.–1968.–**8**, N 2.