NON-THERMAL FLUCTUATIONS IN PLASMA DENSITY NEAR THE TEMPERATURE MINIMUM OF THE SOLAR ATMOSPHERE

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We consider the formation of plasma-density fluctuations by non-thermal motions of gas near the temperature minimum of the solar atmosphere. For the inertial wavenumber range of turbulent velocity field of gas, an analytic expression for the one-dimensional (1D) spectrum of the fluctuations is obtained. This expression is used to predict the shape of the 1D horizontal spectrum of the fluctuations under three values of magnetic field strength: 5, 50, and 250 G. Also, estimates of the rms level of the fluctuations are made. It is shown that the increase in magnetic field has to alter the shape of the horizontal spectrum and to increase the level of the plasma-density fluctuations.

INTRODUCTION

Intensive studies of the solar atmosphere structure and motions are motivated by their importance for obtaining a better understanding of basic solar phenomena such as atmospheric energy transport, turbulent diffusion of magnetic fields or chaotic excitation of solar oscillations (see, *e.g.*, [1–3, 8]). Observational data show that gas motions near the temperature minimum of the solar atmosphere have non-thermal (turbulent) nature [2]. It was established that the photospheric flows include both organized and stochastic motions. Spectra associated with the stochastic velocity fields obey power laws, which are consistent with the spectrum of Kolmogorov turbulence [1, 8]. It is generally agreed that the atmosphere near the temperature minimum ($T = 4170\,^\circ\mathrm{K}$) is weakly ionized (hydrogen number density $N_n = 2.1{\cdot}10^{21}\,\mathrm{m}^{-3}$, when electron number density $N_e = 2.5 \cdot 10^{17}\,\mathrm{m}^{-3}$) [4, 9]. In many cases it can be considered as a mixture of ion-electron plasma and the neutral gas. One can say that the plasma is a passive contaminant embedded in flows of the gas. The electrically charged components have no influence on the atmospheric gas motions near the temperature minimum excluding sunspots and active regions with very strong magnetic fields. The non-thermal motions of the photospheric gas have to result in non-thermal fluctuations in plasma density. The formation of these fluctuations (with scales smaller than the granular size, $l < L_g$) in the lower solar atmosphere was theoretically considered in [5, 6]. Expressions for their three-dimensional (3D) and one-dimensional (1D) omnidirectional spectra were obtained and analysed in [5, 7]. However, in experiments a 1D spectrum along the given direction is usually measured.

The aim of this report is to obtain the 1D spectrum of the plasma fluctuations induced by turbulent gas motions in the solar atmosphere near the temperature minimum, and to consider the shape of the spectrum and the rms fluctuation level under changes in the magnetic field strength.

BASIC ASSUMPTIONS AND EQUATIONS

Turbulent mixing of the weakly ionized plasma near the temperature minimum of the solar atmosphere is a slow process. The time-scales of the process are larger than an interval between ion-neutral collisions $t \gg \tau_i$ and the length-scales are larger than the ion mean free path $l \gg \Lambda_i$. It can be described by fluid equations for ions, electrons and neutrals. The neutral gas motion can be regarded as given (the gas is unaffected by collisions with the electrically charged components). The behaviour of charged components is of interest in this work. Then, a simplified system of equations for electrons and ions may be used [5]:

$$\partial N_s/\partial t + \vec{\nabla}(N_s \mathbf{v}_s) = 0, \tag{1}$$

$$\tau_s^{-1}(\mathbf{v}_s - \mathbf{u}) = q_s m_s^{-1}\mathbf{E} + \omega_{Bs}(\mathbf{v}_s \times \mathbf{b}) - v_{Ts}^2 \vec{\nabla} N_s/N_s. \tag{2}$$

In this set of equations, the variables are chosen as density N_s and velocity v_s for each species ($s \equiv i, e$), $\mathbf{u}$ is the velocity of neutral gas, τ_s is a characteristic time of charged particle collisions with neutrals, q_s is the particle charge ($q_e = -q_i = -e$), $\omega_{Bs} = q_s B/m_s c$ is the gyrofrequency, v_{Ts} is the thermal velocity, m_s is the particle mass, $\mathbf{b} = \mathbf{B}/B$ is the unit vector along the magnetic field $\mathbf{B}$, $\mathbf{E}$ is the electric field.

Assumptions $T_e \approx T_i \approx T_n = T$ and $N_e \approx N_i = N$ are valid for slow processes near the temperature minimum. In addition, $\tau_i \omega_{Bi} \ll 1$ in the quite regions of the solar atmosphere. If the only electric field considered is that required to prevent charge separation (due to the electric field $\mathbf{E}$ electrons tend to follow ions), and eliminating $\mathbf{E}$ from (2), we then obtain the local drift velocity of ions (or plasma in general)

$$\mathbf{v}_i(\mathbf{x},t) \approx \mathbf{u}(\mathbf{x},t) + \beta_i(\mathbf{u}\times\mathbf{b}) - D_a\nabla^2 N, \tag{3}$$

here $\beta_i = \tau_i\omega_{Bi}$, D_a is the ambipolar diffusion coefficient.

Using (3), the equation of continuity (1) then becomes

$$\partial N/\partial t + \vec{\nabla}\cdot(N\mathbf{u}) + \beta_i \mathbf{b}\cdot\vec{\nabla}\times(N\mathbf{u}) - D_a\nabla^2 N = 0. \tag{4}$$

The equation (4) describes the plasma-density behaviour when plasma is embedded in the flow of neutral gas and relates the plasma number density, N, with the neutral gas velocity, $\mathbf{u}$.

If the flow of neutral gas is turbulent, $\mathbf{u}$ and N can be divided into their ensemble mean parts $\mathbf{u}_0 = <\mathbf{u}>$, $N_0 = <N>$ and fluctuations around them $\mathbf{u}_1$, N_1 ($<\mathbf{u}_1> = 0$, $<N_1> = 0$): $\mathbf{u} = \mathbf{u}_0 + \mathbf{u}_1$ ($\mathbf{u}_0 > \mathbf{u}_1$), and $N = N_0 + N_1$ ($N_0 > N_1$). Here we consider the case when the length-scales of random ingredients $\mathbf{u}_1$ and N_1 are close to each other and smaller than the length-scales of mean quantities $\mathbf{u}_0$ and N_0. The same can be said about the time-scales. If l is the length-scale of random fluctuations, L_g stands for a granular size (the length-scale of mean gas velocity), and $L_N = N_0|\vec{\nabla}N_0|^{-1}$ is the length-scale of mean plasma-density gradient, then evidently the following inequalities are valid: $l \ll L_N \leq L_g$.

The velocity of gas motions near the atmosphere temperature minimum is usually small compared to the sound velocity, hence, the neutral gas can be considered as incompressible: $\vec{\nabla}\cdot\mathbf{u} = \vec{\nabla}\cdot\mathbf{u}_1 = 0$. Then, we can write the equation for the relative plasma-density fluctuations, $\delta N = N_1/N_0$, in the form [5, 6]:

$$\partial\delta N/\partial t + \vec{\nabla}\cdot(\delta N\,\mathbf{u}_1) - D_a\nabla^2\delta N = -L_N^{-1}(\mathbf{u}_1\cdot\mathbf{n}) - \beta_i\mathbf{b}\cdot(\vec{\nabla}\times\mathbf{u}_1), \tag{5}$$

where $\mathbf{n} = L_N(\vec{\nabla}N_0/N_0)$ is the unit vector along the mean plasma-density gradient.

The equation (5) describes the process of formation of plasma fluctuations with scales $l < L_N$. It may be seen that the first term on the RHS of (5) is more important at larger scales $l > \beta_i L_N$, and the second is more important for smaller scales $l < \beta_i L_N$. The process in which the neutral gas turbulence in conjunction with the background plasma-density gradient produce plasma fluctuations by mixing regions of high and low density dominates at the larger scales, while the interaction of the plasma embedded in the turbulent motions of neutral gas with the magnetic field is more important for generation of the small-scale fluctuations.

SPECTRA OF PLASMA DENSITY FLUCTUATIONS

Under the assumption that the random fields $\mathbf{u}_1(\mathbf{x},t)$ and $\delta N(\mathbf{x},t)$ are stationary functions of $\mathbf{x}$ and t with Fourier transforms: $u_{1j}(\mathbf{x},t) = \int \mathrm{d}\mathbf{k}\mathrm{d}\omega\, u_{1j}(\mathbf{k},\omega)\,\mathrm{e}^{\mathrm{i}(\mathbf{kx}-\omega t)}$, and $\delta N(\mathbf{x},t) = \int \mathrm{d}\mathbf{k}\mathrm{d}\omega\,\delta N(\mathbf{k},\omega)\,\mathrm{e}^{\mathrm{i}(\mathbf{kx}-\omega t)}$, the Fourier transform of (5) is

$$\begin{aligned}(D_a k^2 - \mathrm{i}\omega)\cdot\delta N(\mathbf{k},\omega) + \mathrm{i}k_j \int \mathrm{d}\mathbf{k}'\mathrm{d}\omega'\,\delta N(\mathbf{k}',\omega')\cdot u_{1j}(\mathbf{k}-\mathbf{k}',\omega-\omega') \\ = -L_N^{-1}(\mathbf{n}\cdot\mathbf{u}_1(\mathbf{k},\omega)) - \mathrm{i}\beta_i\mathbf{k}(\mathbf{u}_1\times\mathbf{b}).\end{aligned} \tag{6}$$

The convolution term on the LHS of (6) represents the contribution of mode interactions in the process of plasma fluctuation generation. If we take it into account through the coefficient of turbulent diffusion D_t, then

$$\delta N(\mathbf{k},\omega) = (-L_N^{-1}\mathbf{n} - \mathrm{i}\beta_i(\mathbf{b}\times\mathbf{k}))\,((D_a + D_t)\,k^2 - \mathrm{i}\omega)^{-1}\cdot\mathbf{u}_1(\mathbf{k},\omega). \tag{7}$$

For statistically homogeneous and stationary random fields the following relations are valid (see, *e.g.*, [10]):

$$\langle u_{1i}(\mathbf{k},\omega)\cdot u_{1j}^*(\mathbf{k}',\omega')\rangle = \Phi_{ij}(\mathbf{k},\omega)\,\delta(\mathbf{k}-\mathbf{k}')\,\delta(\omega-\omega'), \tag{8}$$

$$\langle \delta N(\mathbf{k},\omega)\cdot \delta N^*(\mathbf{k}',\omega')\rangle = \Psi(\mathbf{k},\omega)\,\delta(\mathbf{k}-\mathbf{k}')\,\delta(\omega-\omega'), \tag{9}$$

where the $(*)$ denotes a complex conjugate, $\Phi_{ij}(\mathbf{k},\omega)$ is the spatiotemporal spectrum tensor of the field $\mathbf{u}_1$, $\Psi(\mathbf{k},\omega)$ is the spatiotemporal spectrum of the relative fluctuations in plasma density.

The tensor $\Phi_{ij}(\mathbf{k},\omega)$ can be presented in the form [1, 10]:

$$\Phi_{ij}(\mathbf{k},\omega) = (\delta_{ij} - k_i k_j\, k^{-2})\,[4\pi^2 k^2 (1+\omega^2\tau_k^2)]^{-1}\tau_k E(k), \tag{10}$$

here $E(k) = C_1\varepsilon^{2/3}k^{-5/3}$ is the energy spectrum function in the inertial range $k_0 < k < k_d$ (the wavenumber $k_0 = 2\pi/L_g$ represents the outer scale of turbulence or the basic energy input scale, in our case the granular size L_g, $k_d \approx \nu^{-3/4}\varepsilon^{1/4}$ is the viscous wavenumber that corresponds to the viscous length-scale $l_d = 2\pi/k_d$ at which viscous dissipation is adequate to dissipate the energy at the rate ε, ν is the kinematic viscosity of the gas, the Kolmogorov constant C_1 is around 1.5 [10]), $\tau_k^{-1} = \nu k^2 + \varepsilon^{1/3}k^{2/3}$ is the decay rate of eddy with the wavenumber k.

Taking (7)–(10) into account, we obtain an expression for the spatiotemporal spectrum of δN [5, 7]:

$$\Psi(\mathbf{k},\omega) = \frac{L_N^{-2}k^{-2}(\mathbf{n}\times\mathbf{k})^2 + \beta_i^2(\mathbf{b}\times\mathbf{k})^2}{4\pi^2k^2((D_a+D_t)^2k^4+\omega^2)\,(1+\omega^2\tau_k^2)}\cdot\tau_k\,E(k). \tag{11}$$

The spatial spectrum of the fluctuations $S(\mathbf{k})$ is related with $\Psi(\mathbf{k},\omega)$ as $S(\mathbf{k}) = \int_{-\infty}^{\infty}\mathrm{d}\omega\,\Psi(\mathbf{k},\omega)$. As in the case of most fluids and gas, near the atmosphere temperature minimum the Schmidt number can be regarded about unity, *i.e.*, $D_a \approx \nu$, then $(D_a + D_t)\,k^2 \approx \nu k^2 + \varepsilon^{1/3}k^{2/3}$, and after integration, we have

$$S(\mathbf{k}) = \frac{L_N^{-2}k^{-2}(\mathbf{n}\times\mathbf{k})^2 + \beta_i^2(\mathbf{b}\times\mathbf{k})^2}{8\pi\,k^2(\nu k^2+\varepsilon^{1/3}k^{2/3})^2}\cdot E(k), \tag{12}$$

the 3D power spectral density of δN. Using (12) we may obtain the rms level of the fluctuations:

$$\langle(\delta N)^2\rangle^{1/2} = \langle(N_1/N_0)^2\rangle^{1/2} = \left(\int \mathrm{d}\mathbf{k}\,S(\mathbf{k})\right)^{1/2} = \left(\int_{k_1}^{k_2} S_0(k)\,\mathrm{d}k\right)^{1/2}, \tag{13}$$

where

$$S_0(k) = \frac{L_N^{-2} + \beta_i^2k^2}{2k^3(1+(k/k_d)^{4/3})^2} \tag{14}$$

is the 1D omnidirectional spectrum of the fluctuations, $S_0(k)\mathrm{d}k$ represents the contribution to the power level of relative plasma-density fluctuations from the wavenumber range $(k, k+\mathrm{d}k)$.

From the expression (13) for the 3D spectrum we can obtain 1D spectrum that may be measured along $\mathbf{z}$-direction, $S_1(k_z)$. If we use the cylindrical polar coordinates $(k_\perp,\varphi,k_z)$, $k^2 = k_\perp^2 + k_z^2$, then:

$$S_1(k_z) = \int_0^{\sqrt{k_d^2-k_z^2}} k_\perp\mathrm{d}k_\perp \int_0^{2\pi} S(\mathbf{k})\mathrm{d}\varphi = \int_0^{\sqrt{k_d^2-k_z^2}} \frac{L_N^{-2}\,f(k_\perp,k_z,A_1) + \beta_i^2k^2\,f(k_\perp,k_z,A_2)}{8\,k^7(1+(k/k_d)^{4/3})^2)}\,k_\perp\mathrm{d}k_\perp, \tag{15}$$

where $k_0 < k_z < k_d$, $f(k_\perp,k_z,A) = k_\perp^2 + k_\perp^2\cos^2A + 2k_z^2\sin^2A$ (A_1 is the angle between $\mathbf{z}$ and $\mathbf{n}$, and A_2 between $\mathbf{z}$ and $\mathbf{b}$). If we consider the region of solar atmosphere with $\mathbf{n}\parallel\mathbf{b}$, then for the horizontal direction $\mathbf{z}\perp\mathbf{n},\mathbf{b}$ ($A_1 = A_2 = 90°$), and the 1D spectrum takes the form:

$$S_1(k_z) = \int_0^{\sqrt{k_d^2-k_z^2}} \frac{(L_N^{-2}+\beta_i^2k^2)\,(k_\perp^2+2k_z^2)}{8\,k^7(1+(k/k_d)^{4/3})^2)}\,k_\perp\mathrm{d}k_\perp. \tag{16}$$

To estimate the rms level of relative fluctuations (13), and the shape of expected spectra (14), (16), we adopt the following values of parameters near the temperature minimum of the atmosphere [3, 4, 9]: the mean ion mass $m_i = 25.3$ a.m.u.; the length-scale of the mean plasma-density gradient $L_N \approx L_g \approx 940$ km, the mean gas velocity on the outer scale of turbulence (associated with the granular size L_g) $u_0 \approx 1.1\,\mathrm{km\,s^{-1}}$; the energy dissipation rate $\varepsilon \approx u_0^3L_g^{-1} \approx 1.5\cdot10^3\ \mathrm{m^2\ s^{-3}}$; $D_a \approx \nu \approx 11\ \mathrm{m^2\ s^{-1}}$; and the viscous length-scale $l_d = 2\pi/k_d \approx 6\,\mathrm{m}$. The level and the spectral shapes are estimated under three values of the magnetic field strength B: 5, 50, and 250 G. An increase in B results in the rise in $\beta_i = \tau_i\omega_{Bi}$, which respectively takes the values: $5.5\cdot10^{-4}$, $5.5\cdot10^{-3}$, and $2.7\cdot10^{-2}$. The rms fluctuation level in the wavenumber range $k_m \le k \le k_d$ ($k_m = 2\pi L_m^{-1}$, $L_m = 300$ km) is 2.5, 2.8, and 6.6 %, respectively. Figure 1 shows the shapes of the 1D fluctuation spectra. Figure 1A illustrates the omnidirectional spectra $S_0(k)$ (14), Figure 1B shows the horizontal spectra $S_1(k_z)$ (16) (the values of B are indicated near the lines), the dashed line represents the slope of the Kolmogorov spectrum $k^{-5/3}$.

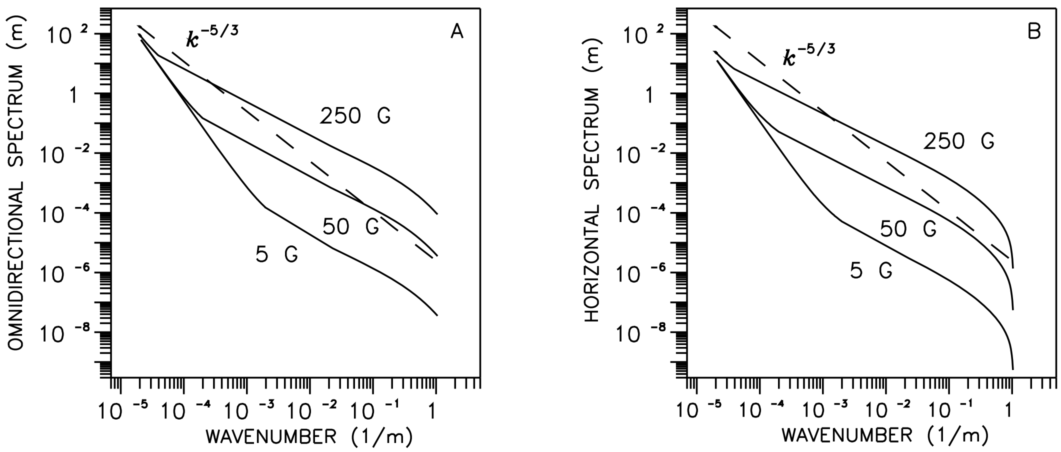

Figure 1. 1D spectra of plasma-density fluctuations expected near the temperature minimum of the solar atmosphere for three values of the magnetic field strength B indicated near the lines

CONCLUSIONS

In this work the analytic expression (15) for the 1D spectrum of small-scale plasma-density fluctuations, which may be generated by the microturbulence of gas near the temperature minimum of the solar atmosphere, was derived. This expression was used to predict the shape of the 1D horizontal spectrum (16) of the fluctuations for three values of magnetic field strength: 5, 50, and 250 G (see Fig. 1B), when parameters of the turbulence and the atmosphere were unchanged. The rms level of the fluctuations was estimated as well. It is shown that the increase in the magnetic field has to alter the shape of the 1D spectrum (see Fig. 1) and to increase the level of relative fluctuations in plasma density. Under the usual conditions for the temperature minimum of the quiet atmosphere, the fluctuation level for wavenumbers inside the inertial range of turbulence takes values 2.5% ($B = 5$ G), 2.8% ($B = 50$ G), and 6.6% ($B = 250$ G).

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