

QUARK-PARTON PHASE TRANSITIONS AND THE ENTROPY PROBLEM IN QUANTUM MECHANICS

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Since Feynman proposed his parton model in 1969, one of the most pressing problems in high-energy physics has been whether partons are quarks. It is shown that the quark model and the parton model are two different manifestations of one covariant entity. The nature of transition from the confined quarks to plasma-like partons is studied in terms of the entropy and temperature coming from the time-separation variable. According to Einstein, the time-separation variable exists wherever there is a spatial separation, but it is not observed in the present form of quantum mechanics. The failure to observe this variable causes an increase in entropy.

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1. INTRODUCTION

In 1969, Feynman proposed his parton model for hadrons moving with speed close to that of light [1]. He observed that the hadron appears as a collection of infinite number of partons. Since the partons appear to have properties quite different from those of the quarks, one of the most pressing puzzles in high-energy physics has been whether the partons are quarks, or whether the quark model and the parton model are two different manifestations of the same covariant formalism.

In 1970, at the April meeting of the American Physical Society held in Washington, DC (U.S.A.), Feynman gave an invited talk on a model of hadrons. His talk was published in a paper by Feynman, Kislinger and Ravndal in 1971 [2] There, the authors attempted to construct a covariant model for hadrons consisting of quarks joined together by an oscillator force. They indeed formulated a Lorentz-invariant oscillator equation. They also worked out the degeneracies of the oscillator states which are consistent with observed mesonic and baryonic mass spectra. However, their wave functions are not normalizable in the space-time coordinate system. They never considered the question of covariance.

In his 1972 book on statistical mechanics [3], Feynman says: *When we solve a quantum-mechanical problem, what we really do is divide the universe into two parts - the system in which we are interested and the rest of the universe. We then usually act as if the system in which we are interested comprised the entire universe. To motivate the use of density matrices, let us see what happens when we include the part of the universe outside the system.* Feynman's rest of the universe has been studied in detail in terms of two coupled oscillators [4].



Feynman's photograph hanging on the first floor of Jadwin Hall (physics building) of Princeton University

In this his report, we combine these three components of Feynman's research efforts to show that the quark and parton models are indeed two different manifestations of the same covariant entity. In order to achieve this purpose, we fix up first the mathematical deficiencies of the paper of Feynman et al. [2]. The idea is to construct a harmonic oscillator wave function which can be Lorentz-boosted.

We can first see whether the wave function is applicable to the quark model when the hadron is slow, and then see whether the same wave function describes the parton model when the hadron is boosted to an infinite-momentum frame. The 1971 paper by Feynman et al. [2] contains very serious mathematical flaws, but they have been all cleaned up within the framework of Wigner's little groups which dictate the internal space-time symmetries relativistic particles [5,6]. This covariant formulation solves the covariance problem. However, since we live in the three-dimensional world, it is possible that we miss something in the four-

dimensional world. The time-separation variable between the quarks is a case in point. In non-relativistic quantum mechanics, the Bohr radius is spacial separation between the quarks (or proton and electron). According to Einstein, there must be a time separation between the quarks, since otherwise the world will not be covariant.

Since we are not dealing with this time-separation variable in the present form of quantum mechanics, the failure to measure it leads to an increase in entropy [3]. In this report, we show that this entropy allows us to define the phase transition between the confined phase of the quark model and the plasma phase of the parton model.

In Sec. 2, we introduce the covariant harmonic oscillator formalism with normalizable wave functions which can be Lorentz boosted. In Sec. 3, we use the oscillator wave function to solve the quark-parton puzzle. In Sec. 4, we deal with the problems arising from measuring of four-dimensional physics in the three-dimensional world. The entropy plays a major role.

2. COVARIANT HARMONIC OSCILLATORS

Let us consider a hadron consisting of two quarks. Then there is a Bohr-like radius measuring the space-like separation between the quarks. There is also a time-like separation between the quarks, and this variable becomes mixed with the longitudinal spatial separation as the hadron moves with a relativistic speed. While there are no quantum excitations along the time-like direction, there is the time-energy uncertainty relation which allows quantum transitions. It is possible to accommodate these aspects within the framework of the present form of quantum mechanics. The uncertainty relation between the time and energy variables is the c-number relation [7], which does not allow excitations along the time-like coordinate, as illustrated in Fig. 1.

For a hadron consisting of two quarks, we can consider their space-time positions x_a and x_b , and use the variables

$$X = (x_a + x_b)/2, \quad x = (x_a - x_b)2\sqrt{2}.$$

The four-vector X specifies where the hadron is located in space and time, while the variable x measures the space-time separation between the quarks.

Since the three-dimensional oscillator differential equation is separable in both spherical and Cartesian coordinate systems, the wave function consists of Hermite polynomials of x , y , and z . If the Lorentz boost is made along the z direction, the x and y coordinates are not affected, and can be temporarily dropped from the wave function. Along the space-like longitudinal direction, there are excitations. On the other hand, along the time-like direction, there is an uncertainty relation even though there are no excitations. The covariant harmonic oscillator formalism accommodates this

space-time asymmetry [6]. However, since we are interested here only in Lorentz-boost properties of the wave function, we restrict ourselves to the ground-state wave function. The wave function then can be written as

$$\psi(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}(z^2 + t^2)\right\}, \quad (1)$$

which accommodates the uncertainty relations along the longitudinal and time-like directions.

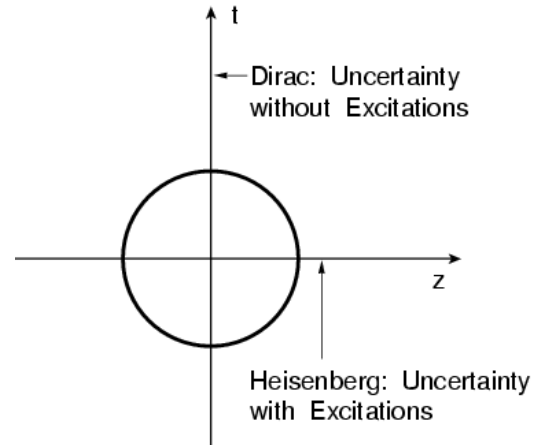


Fig. 1. Present form of quantum mechanics. There are excitations along the space-like dimensions, but there are no excitations along the time-like direction. However, there still is a time-energy uncertainty relation. We call this Dirac's c-number time-energy uncertainty relation. It is very important to note that this space-time asymmetry is quite consistent with the concept of covariance

The expression given in Eq. (1) is not Lorentz-invariant. It is covariant. This wave function describes the present form of quantum mechanics if the time-separation variable is factored out, integrated out, or ignored.

However, the time-separation variable is absolutely needed when we consider Lorentz covariance. The question is whether the above wave function can describe the parton model when it is boosted to an infinite-momentum limit. It is convenient to use the light-cone variables to describe Lorentz boosts. The light-cone coordinate variables are

$$u = (z + t)/\sqrt{2}, \quad v = (z - t)/\sqrt{2}.$$

In terms of these variables, the Lorentz boost along the z direction,

$$z' = (\cosh \eta)z + (\sinh \eta)t,$$

$$t' = (\sinh \eta)z + (\cosh \eta)t,$$

takes the simple form

$$u' = e^\eta u, \quad v = e^{-\eta} v,$$

where η is the boost parameter and is $\tanh^{-1}(v/c)$. Indeed, the u variable becomes expanded while the v variable becomes contracted. This is the squeeze mechanism illustrated discussed extensively in the literature [8,9]. This squeeze transformation is also illustrated in Fig. 2.

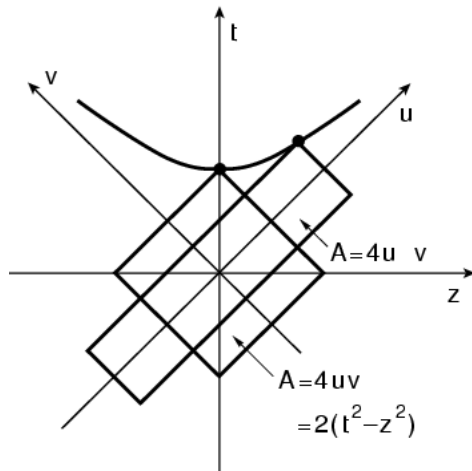


Fig. 2. Lorentz boosts in the light-cone coordinate system. One of the light-cone axes become expanded while the other becomes contracted

Thus, one way to combine quantum mechanics with relativity is to incorporate Fig. 1 into Fig. 2, and produce the elliptic deformation illustrated in Fig. 3.

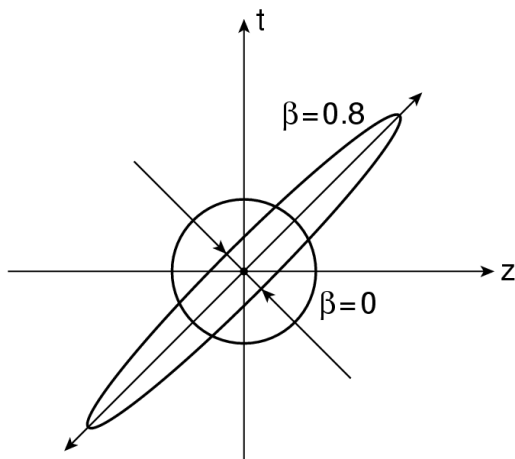


Fig. 3. Effect of the Lorentz boost on the space-time wave function. The circular space-time distribution at the rest frame becomes Lorentz-squeezed to become an elliptic distribution

If the system is boosted, the wave function becomes

$$\psi(z, t) = \sqrt{\frac{1}{\pi}} \exp\left\{-\frac{1}{2}(e^{-2\eta} u^2 + e^{2\eta} v^2)\right\}. \quad (2)$$

We note here that the transition from Eq. (1) to Eq. (2) is a squeeze transformation. The wave function of (1) is distributed within a circular region in the uv plane, and

thus in the zt plane. On the other hand, the wave function of Eq. (2) is distributed in an elliptic region. This ellipse is a «squeezed» circle with the same area as the circle, as is illustrated in Fig. 3.

QUARKS \longrightarrow PARTONS

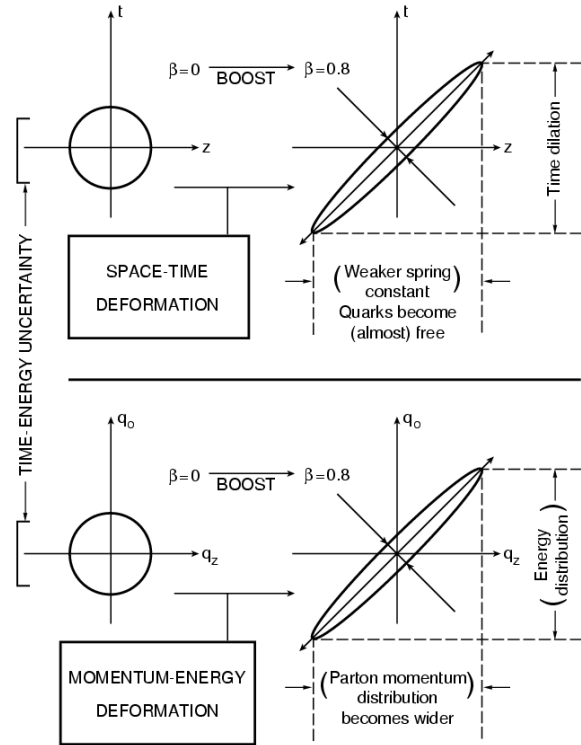


Fig. 4. Lorentz-squeezed space-time and momentum-energy wave functions. As the hadron's speed approaches that of light, both wave functions become concentrated along their respective positive light-cone axes. These light-cone concentrations lead to Feynman's parton picture

3. PARTON PICTURE

In 1969 Feynman [1] made the following systematic observations on hadrons moving with speed close to that of light:

- he picture is valid only for hadrons moving with velocity close to that of light.
- The interaction time between the quarks becomes dilated, and partons behave as free independent particles.
- The momentum distribution of partons becomes widespread as the hadron moves very fast.
- The number of partons seems to be infinite and much larger than that of quarks.

These observations constitute Feynman's parton picture. Because the hadron is believed to be a bound state of two or three quarks, each of the above phenomena appears as a paradox, particularly b) and c) together.

If the quarks have the four-momenta p_q and p_b , we can construct two independent four-momentum variables [2]

$$P = p_a + p_b, \quad q = \sqrt{2}(p_a - p_b).$$

The four-momentum P is the total four-momentum and is thus the hadronic four-momentum. q measures the four-momentum separation between the quarks.

Since we are using here the harmonic oscillator, the mathematical form of the above momentum-energy wave function is identical to that of the space-time wave function, and its transformation properties are the same. The Lorentz squeeze properties of these wave functions are also the same, as are indicated in Fig. 4.

When the hadron is at rest with η , both wave functions behave like those for the static bound state of quarks. As η increases, the wave functions become continuously squeezed until they become concentrated along their respective positive light-cone axes. Indeed, this figure provides the answer to the quark-parton puzzle [4].

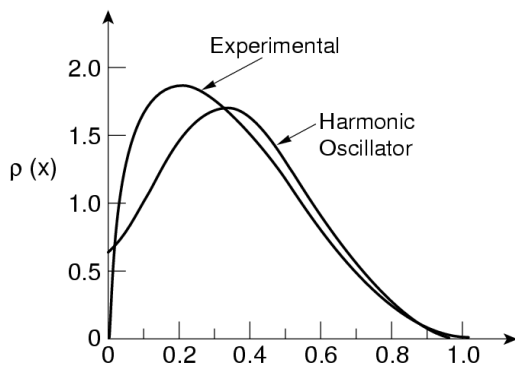


Fig. 5. Parton distribution. It is possible to calculate the parton distribution from the Lorentz-boosted oscillator wave function. This theoretical curve is compared with the experimental curve

The question then is whether the elliptic deformations given in Fig. 4 produce any quantitative results which can be compared with what we measure in laboratories. Indeed, according to Hussar's calculation [10], the Lorentz-boosted oscillator wave function produces a reasonably accurate parton distribution, as indicated in Fig. 5.

4. ENTROPY PROBLEMS

The covariant harmonic oscillator formalism presented in Sec. 2 produces the Lorentz squeeze property summarized in Fig. 4. This figure tells us that the quark model and the parton model are two different manifestations of one covariant formulation. In this figure, the time-separation variable plays the essential role. However, we are not able to deal with this variable in the present form of quantum mechanics.

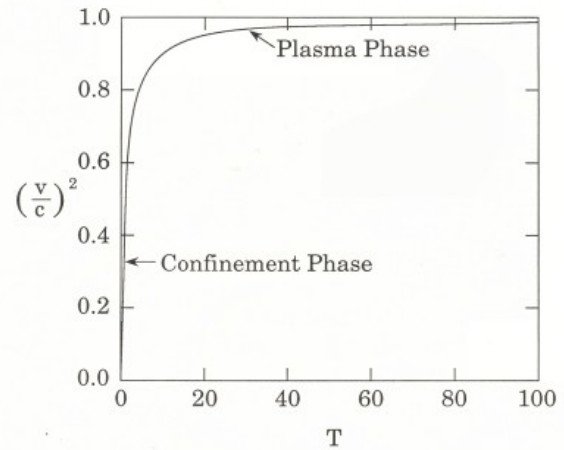


Fig. 6. The hadronic velocity versus the hadronic temperature given in Eq. (4). Here we used the unit system where we use the unit system where $\omega/k=1$

If there is a physical which we cannot measure, the variable certainly belongs to Feynman's rest of the universe [3,4] Then there is a well-defined procedure to deal with this problem: construct a density matrix from the wave function and integrate over the variable which we do not observe. In the present case, the variable we do not observe is the time-separation variable. This process leads to an increase in entropy [11]. It is straight-forward to calculate this entropy [4] and the result is

$$S = (\cosh \eta)^2 \ln(\cosh \eta)^2 - (\sinh \eta)^2 \ln(\sinh \eta)^2. \quad (3)$$

This form is identical to the entropy caused by thermally excited harmonic oscillators, if we write

$$\tanh^2 \eta = \exp(-\square\omega / kT). \quad (4)$$

The entropy of Eq. (3) takes the form [11,12]

$$S = \frac{\square\omega / kT}{\exp(\square\omega / kT) - 1} - \ln \left\{ 1 - \exp \left(\frac{-\square\omega}{kT} \right) \right\}.$$

Let us go back to Eq. (3). The $(\text{velocity}/c)^2$ is plotted against the temperature in Fig. 6. Its behavior makes a sudden change as the temperature rises. If the hadronic velocity is low, the temperature is relatively insensitive to the velocity, but for high velocities, it is in the other way around. We can use this behavior to tell the difference between the confinement phase of the quarks and the plasma phase of the partons.

We can summarize the result using the following figure. If a hadron is at rest, it appears like a bound or confined state of a finite number of quarks. If the hadron moves fast, it becomes like a collection of partons as observed by Feynman in 1969. Partons are like free particles, and the hadron is in a plasma state. This phase transition is illustrated in Fig. 7.

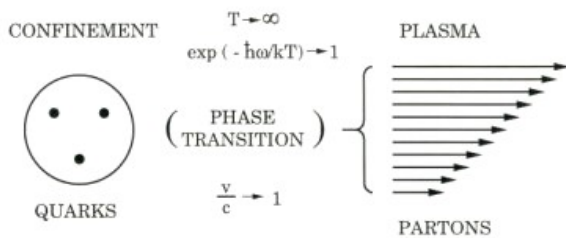


Fig. 7. Quark-Parton phase transition. This transition is accompanied by an increase of entropy. The ground-state harmonic oscillator goes through an excitation which is like the thermal excitation

It is possible to illustrate this effect using a harmonic oscillator being heated by a heater and the oscillator being carried by a truck, but observed by an observer in the rest frame, as illustrated in Fig. 8.

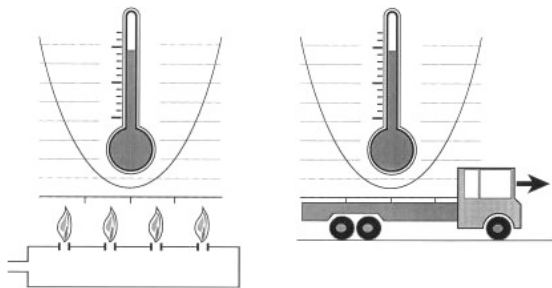


Fig. 8. Lorentz-boosted harmonic oscillator in its ground state. Let us boost the harmonic oscillator and assume that we are not measuring the time-separation variable as we do in quantum mechanics. Then the oscillator becomes like a thermally excited state

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I am grateful to Prof. Nikolai Shul'ga and the organizing committee for inviting me to this conference. I had and I still have a great respect for Prof. Alexander Akhiezer since I studied quantum electrodynamics from his book when I was a graduate student at Princeton (1958-61). This was my second visit to the city of Kharkov, and I was indeed happy to be with the citizens of Kharkov for their hospitality.

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