GEOMETRIC ATTRACTOR OF AN ELECTRON BEAM PASSING THROUGH A CRYSTAL

A.M. Gurin

B. Verkin Institute for Low Temperature Physics and Engineering
National Academy of Sciences of Ukraine
Kharkov, Ukraine
e-mail: gurin@ilt.kharkov.ua

The Lemmlein algorithm assigns a cyclic interaction of a mathematical point with other (n+1) points of the n - dimensional Euclidean space. In this paper generalization A of the Lemmlein algorithm for an arbitrary number of points m situated in the n - dimensional Riemann space is proposed. Algorithm A generates a Markovian chain consisting of the finite number of combinatorially different strongly convergent attractors. Algorithm A is generalized to describe the interaction of mass points, e.g., the motion of electrons in the real crystal medium. The strong convergence of the attractors provides stability of the electron trajectories in the vicinity of an attractor in the unit cell of the crystal after electron scattering at the crystal defects.

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1. INTRODUCTION

Some models of the motion of charged particles in the field witha non-uniform potential were considered in [1]. This permits theauthor to do only with a brief Introduction, where severalfurther comments are proposed.

The studies of equipotential surfaces can also include [2] the closed smooth convex surface inside the tetrahedron which is incontact with all the tetrahedron faces.

Models similar to those of elastic reflection [3,4] were used tosimulate conductivity in the normal metal - superconductor system [5,6].

The basic criterion of the model value [1] is the stability of the motion of charged particles, i.e. the conditions under which small changes in the particle paths cause small alterations in the initial parameters of the model. The computer region of model estimation [3,4] contains everywhere a dense set of directions with the fractal path measure over unity, which indicates possible instability of the paths.

The aim of this study is to supplement the models of Group [3] with an example of a model in which the region of parameter estimation does not contain points of instability.

2. THEORY

Let us accommodate the finite number of points k(x) (k is the point number, x is the vector of the point coordinates) in a compact region of the n-dimensional Riemann space. Similarly to the algorithm in [7], we can estimate the path of a point in respect to the rest of the points of the set k(x). For this purpose we employ Procedure A.

1. Let us mark one point of the set k(x) as ^{1} 1. The rest of the points are numerated with randomly ascribed natural numbers.The point numbers can then be normalized so that the highest number is max(k).

2. Let us connect point ^{1}1 of the set k(x) with point ^{1}2 with the shortest line and mark the point m(x) which divides the shortest line in the preassigned proportion P = [1(x) - 2(x)]/[2(x) - m(x)].

3. Let us connect the i-th point m(x) of the shortest line with the (i+2)th point of the set k(x) and mark the point in the shortest line which divides it in the proportion P. This is the (i+1)th point of the set m(x).

4. If in item 3 all points of the set k(x) are exhausted, Procedure A continues item 3 starting with point ^{1}1.

5. Going over to item 1 proceeds either through performing an arbitrary finite number of steps in item 3 or when the points m(x) reach the vicinity of their attractor [8].

Theorem 1. With any arbitrarily large number of cycles in item 3 of Procedure A and fixed P non-belonging to the section [-2,0], the sequence of points m(x) permits one and only one division into M(k) subsequences such that each M(x) subsequence converges to its limit point in the space metric.

With each j-th return of Procedure A to item 1 a new set of sequences M(k,j) is practically formed with its own limit points. Let us connect the limit points with the shortest lines MA(k,j). By definition [8,p.139], the polygonal line MA(k,j) is an attractor.

Theorem 2. The sequence M(k,j), where j = 1,..., is a Markovian chain. For each fixed P non-belonging to the section [-2,0] the number of metrically and combinatorically different attractors MA(k,j) is upper bounded with the number of transpositions of the elements in the set k(x).

3. RESULTS

Let us consider a geometric model of a crystal in the Euclidean space. There are some directions in the

crystal which form channels [1] free of the crystal atoms. Let us send the particle f(i) to one of these directions. Its velocity V is pre-assigned so that the particle could move over the distance L >> a, where a is the crystal lattice parameter. We pre-assign algorithm A3 of the interaction of the particle f(i) with the crystal atoms.The algorithm consists of two components: (i) the motion along the channel axis and (ii) Procedure A applied in item 1 to the real order of numeration of the atoms in the channel when the particle f(i) is moving along the channel. The reality of the order of atom numeration implies that the next number is assigned to the atom whose orthogonal projection onto the symmetry axis of the channel is next in the sequence. We put a screen at the end of the path L - the plane E which records the coordinates of the particle passing through it f(i). According to the Theorem 1, the path of the particle f(i) forms an attractor and is stable for all fixed P values in the region of P estimation. According to Theorem 2, all particles f(i) having the parameter P travel the distance L and cross the plane E within one small vicinity.

Let us consider a beam of particles f(i) directed into the crystal channel. Assume that the particles f(i) do not interact with one another. Then, according to Theorem 1, each particle trajectory is stable. As Theorem 2 suggests, if P(i) if the same for all particles, the beam of particles will become focused in the vicinity of one point of the plane E. If the distribution of P(j) among the set of particles f(i) is given, then, according to Theorem 2, j vicinities of the points corresponding to j different P(j)-values will be marked in the plane E.

Since Algorithm A is determined and valid for the non-zero curvature space, it is also valid to describe the motion of particles f(i) through a bent crystal. The model bent crystal can be specified in a limited region of the hypersphere of the four-dimensional space with a given curvature radius and the dimensions of the region ensuring uniqueness of the shortest line in it. The inference in this case is similar to that for a straight crystal.

Assume that the model crystal has a defect in the given channel. Because of the fast convergence of Algorithm A, the particles f(i) respond to the defect actively. Here there exists an alternative for the further path of the particle f(i) - either it restores fast and performs finite mothion or it chooses one of the possible neighboring channels.

Thus, the regular motion of each separate particle f(i) in the crystal may look like irregular in the f(i) beam for two reasons:

– the P(j) values one not equal among the set of f(i) particles;

– the crystal has defects.

4. CONCLUSION

Let us come back once again to Algorithm A and the conditions of Theorems 1 and 2, which say that P(j) does not belong to the interval [-2,0]. Let P increase unrestrictedly tending to infinity. We then can find a P-value at which Algorithm A3 transforms into the elastic scattering algorithm [2]. At P < -2 the path of particles f(i) will run beyond one channel.

Algorithm A3 can be applied to quasi-crystals and amorphous structures, but this falls outside the scope of this paper.

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