SELF-ORGANIZATION AND SELF-CONSISTENT EQUILIBRIUM KINETICS IN ELECTRODYNAMIC SYSTEMS WITH INTENSE CHARGED BEAMS

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Equilibrium states rising as self-organization result in unstable plasma with fast charged beam system are considered. Self-consistent states described by non-linear Bolzmann-Vlasov equation are obtained for harmonic, solitary and shock waves. Kinetics of captured and slipping beam particles is discussed.

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1. INTRODUCTION

In present paper series of aspects of wide-spread subjects bound with phenomenon of self-organizing in open non-equilibrium systems are considered. Among last in modern physics different electrodynamic schemes, using of intensive beams of charged particles for fundamental and technological applications, are actively explored. Further we will be restricted to one of major applications of charged beams, in particular beams of relativistic electrons, for generation of intense microwave radiation.

It is known open systems under influence of external perturbations (for example, flows of energy and a matter) can lose stability, that is usually accompanied by the various phenomena of self-organizing, i.e. modification of an initial condition by formation of new structures [1,2]. Here process of new formation is already stopped owing to various mechanisms of nonlinear saturation and stabilization.

In the elementary view a scheme for generation of microwave radiation consists of the open system (base or "cold" system) in which a beam of fast charged particles is injected. As result the initial system loses a stability and in it the irreversible processes - generation of electromagnetic waves and nonlinear saturation of a wave amplitude - are developed.

The nonlinear saturation of a wave appears because of particle capture (trapping) by the same wave. In the total the new structure of self-consistent state a wave - modulated beam will be formed. New wave - beam state aquires the rather important property: capture of particles and their phase mixing translates a state of system in essentially nonlinear one. Therefore a process of generation is sated, and the new state results irreversible (even in absence of a true dissipation). Thus, our case is one of vivid examples of non-equilibrium systems above mentioned with nonlinear saturation and irreversibility.

In general new structure arises as the compromise of processes of generation, i.e. redistribution of initial beam energy, and nonlinear stabilization and, as the consequence, it is active dynamic system. Therefore for research its dynamics it is extremely important to establish equilibrium state, in which the average parameters of system are stationary.

Three cases are considered below, the distinctions between which are determined by the initial conditions: periodic (harmonic and quasi- harmonic) wave, solitary and shock waves. The description is limited to the most evident one-dimensional motion of a beam in plasma. It is supposed that the beam of particles is immersed in a strong longitudinal M electromagnetic slow waves is taken into account only. Density of plasma exceeds essentially a beam one, and plasma electrons are non-magnetized. Besides the plasma particles are not captured and can be described in hydrodynamic approximation.

Further, it is necessary to explain physical conditions at which there is a formation of equilibrium state. Physical reason for instability and self-organization is self-focusing of particles in systems with slow wave (plasma, travelling wave tubes and so on). So, in plasma Coulomb's force between the same charged moving particle is attractive if the effective frequency of signal, excited by both particles, is less than Langmuir's one and magnitude of the plasma dielectric constant is negative:

$$\varepsilon_p = 1 - \frac{\omega_L^2}{\omega^2} \to < 0 . \qquad (1)$$

2. SIMPLE WAVES

In ordinary conditions an injected beam has the extended length up to ten meters and more. Such beam excites "almost" harmonic electromagnetic wave (further harmonic wave is termed as a simple one). Usually this process has resonant character if wave frequency is close to frequency of a slow beam wave and of cold system intrinsic one (see details [2]). Here equilibrium state corresponds (meets) to a stage with complete phase mixing of particles and their synchronization with an excited wave. Thus the beam becomes energetically inhomogeneous.

In general an equilibrium state of magnetized beam - wave is described in phase space by Bolzmann-Vlasov equations:

$$\frac{\partial F}{\partial t} + eE_z\frac{\partial F}{\partial p} + v\frac{\partial F}{\partial z} = 0, \qquad (2)$$

$$div(\varepsilon_p E_я) = 4\pi\rho_b, \qquad (3)$$

$$\rho_b = e\int Fdp; \ j_b = e\int vFdp,$$

where F is the beam phase density, p and v are linear momentum and velocity of particles.

In equilibrium state $z \to z - v_b t$. So, phase density F is determined in beam frame by equation

$$eE_z\frac{\partial F}{\partial p} + v\frac{\partial F}{\partial z} = 0, \qquad (4)$$

and condition $\langle j_b E_z \rangle = 0$ is irreversibility one of non-linear state.

A solution of (4) is

$$F = F(H_0 - H), \qquad (5)$$

where $W = \int v dp$ is particle energy,

$U = e\int E_z dp$ is field potential.

Due to finiteness of field magnitude and energy of particle relative motion,

$$F = \begin{cases} \sum_{S \geq 0} F_S (H_0 - H)^{S+S_0} \to & H \leq H_0 \\ 0 & H > H_0 \end{cases} \qquad (6)$$

and

$$\rho_b = \sum_{S \geq 0} C_S U^{S+1} \to S_0 = \frac{1}{2} \qquad (7)$$

where $C_S \to$ are constants.

For simple (harmonic) wave in beam frame (because $\omega/k = v_{ph} = v_b$)

$E_Z \to E_0 \sin kz$.

Linearity of Maxwell's equation provokes linear connection between field and beam density

$$\hat{L}E_Z = \frac{\partial}{\partial z} e\int Fdp \to ConstE_Z \qquad (8)$$

Due to non-relativistic motion of particles in beam frame

$$H = \frac{p^2}{2m_0} - U, \qquad (9)$$

$$F = F_0\sqrt{H_0 - H} \to F_0\sqrt{\frac{p^2}{2m_0} - U}$$

at $H \leq H_0$. A typical particle distribution in beam coordinate system is presented in Fig. 1.

Note that the self-consistent state of beam contains not only captured particles, but also some fraction of slipping ones. And all the particles have a developed kinetics. So, half of particles overtakes a wave, and another lags behind. But in the whole beam and wave are synchronized. And it can be represented as a consequence of bunches harmonically modulated. Physically this type of kinetics becomes possible if centers of every beam bunches are located in points $kz_n = (2n + 1)\pi$ where n is integer. Thus the foremost parts bunches appear in a braking wave phase, and other particles are in accelerating one. It is seen from the field equation that in possible if $\varepsilon_p < 0$ that is confirmed by physical consideration (see above).

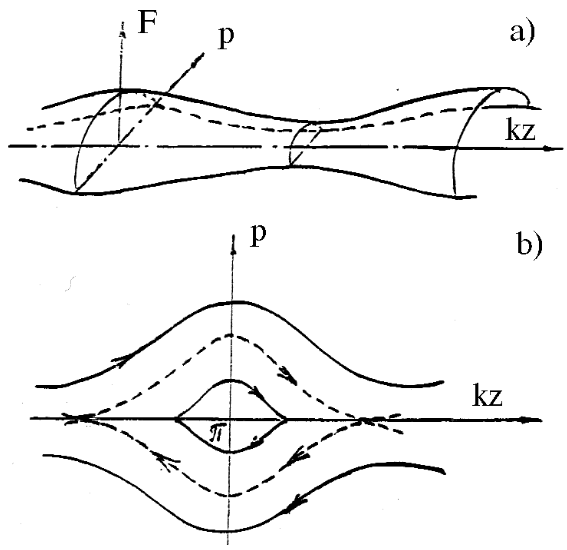

Fig. 1. *Phase density distribution of F (a)) and phase trajectories (b)) of beam particles. Dotted curve is separatrix*

The consideration of equilibrium states allows receiving a broad circle of estimates, useful for operational uses. So, utilizing the laws of conservation of energy and momentum in system, it is possible to estimate parameters of a limiting mode of generation of an electromagnetic radiation (generation efficiency, state and temperature of a beam, frequency shift at generation and so on, look more in detail for example [4]).

3. SOLITARY WAVE – SOLITION

Physical reason for forming of an equilibrium state "solitary wave - short particle bunch" is self-focusing of particles in systems with slow wave (plasma, travelling wave tubes and so on). But self-consistent state obtains new character including more complex non-linear field - beam density connection that provides, since [5], nonlinear stability of a soliton.

For wave potential U in beam frame

$$e\,div\vec{E} = \frac{d^2U}{dz^2} = 4\pi e(\rho_b + \rho_p) = 4\pi e\rho(U) \qquad (10)$$

where ρ_b (U) includes captured and slipping particles, but ρ_p (U) contains only slipping ones. Since (6) - (7)

$$\rho = \sum_{n \geq 1} \rho_n U^n, \qquad (11)$$

and we have the following result:

$$\frac{d^2U}{dz^2} = -\frac{d\Phi}{dU}, \qquad (12)$$

where "potential"

$$\Phi = -4\pi e \int^{U} \rho \, du \, . \qquad (13)$$

According to equation (12) periodic solutions arise if potential contains negative terms [5], Fig. 2.

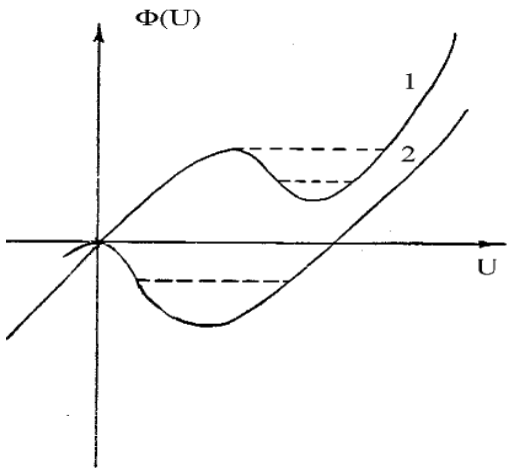

Fig. 2. Potential wells Φ(U) (solid lines) and energy levels of oscillator (dotted lines). Upper level in curve 1 corresponds to solitary wave with pedestal. Level in curve 2 and lower one in curve 1 correspond to periodical waves

In particular if non-linear part of sum density contains only a term of second order, the equation (12) transforms to Korteweg-De Vries kind one. In result

$$U = \frac{U_0}{ch^2(z/L)} + Const \qquad (14)$$

where solition length L is determined by beam parameters [4], Fig. 3.

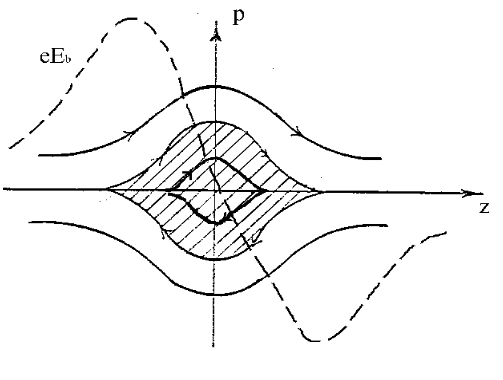

Fig. 3. Phase trajectories of particles and field force in solitary wave

4. SHOCK EQULIBRIUM WAVE IN FIELD ABSORBING MEDIUM

Consider a system consisting from "collision-less" fast beam propagating in absorbing plasma. Beam has a sharp front. The magnitude of plasma current is determined by a equation containing the collision frequency ν:

$$\frac{\partial j_p}{\partial t} + \nu j_p = \frac{\omega_L^2}{4\pi} E_z \, . \qquad (15)$$

Now we have for the equilibrium front propagation

$$\frac{d^3U}{dz^3} + \frac{\omega_L^2}{\nu^2}\frac{dU}{dz} - \frac{\omega_L^2 \nu}{\nu^3} U = 4\pi e \frac{d\rho_b}{dz} \, . \qquad (16)$$

Dissipation changes cardinally a solution character of (12): upper trajectory in Fig. 2 begins to oscillate but "falls" finally on the potential bottom [5]. Here slipping particles overtaking beam are "working" on wave for dissipation compensation. Then reaching the wave front are lagging, Fig. 4.

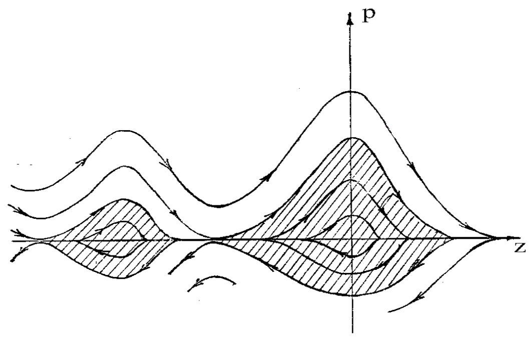

Fig. 4. Phase trajectories of beam particles in laboratory frame. Hatched areas are capture zones

In conclusion it is necessary to say that the results obtained do not hard restrictions on beam parameters. But all the equilibrium states are characterized by very developed kinetics. Therefore resulting energetic spread in beam states is sufficient for stability (with respects to initial kinds of perturbations). The latter permits to use proposed approach for estimations in particular of diverse beam schemes efficiency.

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